Does Income Assistance Increase Disposable Income? - Filling a Bottomless Pit?

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(Very Preliminary. Comments are Welcome.)

Abstract

This paper theoretically analyzes how income assistance/welfare benefits affect the aggregate disposable income of the benefit recipients when the government cannot observe the recipients' earning capability. If the benefit is linear to the wage income of the recipient and if the recipients' earning capabilities are uniformly distributed, the model shows that the means-tested linear benefits do not increase the aggregate disposable income of the benefit recipients regardless of the benefit size, as if filling a bottomless pit. Moreover, under a more realistic distribution of earning capabilities, the aggregate disposable income can decrease.

Keywords: Income Assistance; Disposable Income; Moral Hazard; Adverse Selection.
JEL Codes: H24, D82, H20

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1 Introduction

The main goal of the income assistance programs is to increase the disposable income of the recipients. Surprisingly, however, few studies have formally analyzed the effects of income assistance benefits on the *aggregate* disposable income of the benefit recipients.

It is well known that if the government cannot observe workers' earning capabilities, income assistance (or welfare) benefits can induce workers to work/earn less than their capability, called the *moral hazard problem*. Also, even the capable workers can reduce their work hours/wages in order to qualify for the (means-tested) welfare benefits, called the *adverse selection problem*. Then, it is ambiguous whether the income assistance benefits will increase the disposable income (i.e. the sum of wages and income assistance benefits) or not.

This paper provides a theoretical model to show that under the simple but standard assumptions, moral hazard and adverse selection problems can reduce the wage income of the benefit recipients significantly enough that the aggregate disposable income of the benefit recipients does not increase regardless of the benefit size.

Intuitively, if the benefit decreases fast as the wage increases, it would lead to more moral hazard problems. However, because the size of the benefit decreases fast, the benefit becomes less attractive to those high ability workers who can earn high wages on their own, and there will be less adverse selection problems. That is, there exists a trade-off between the moral hazard and the adverse selection problems.

Thus, if the income assistance benefit is linear to the wage income and if the benefit recipients' earning capabilities are uniformly distributed, I can show that the reduced aggregate wage income due to the moral hazard and the adverse selection problems *combined* is always equal to the aggregate income assistance benefits, regardless of the slope or the level of the linear benefit function. That is, the means-tested linear income assistance programs do not increase the aggregate disposable income regardless of the benefit size or the welfare expenditure.

Moreover, because income assistance benefits reduce wage income most for relatively higher ability recipients, if there exist relatively more higher ability workers among the benefit recipients, the aggregate disposable income can decrease. For example, if the earning capability distribution is single-peaked and if the benefit recipients are distributed on the left (or lower) side of the distribution, the means-teated linear income assistance benefits reduce the aggregate disposable income of the benefit recipients, regardless of the size of benefit expenditure.

It is worth emphasizing that these results do not imply that the income assistance programs are not effective, because they do change the income distribution of the benefit recipients. Especially, the income assistance programs can raise the disposable income of those with relatively lower earning capabilities. Therefore, even when the aggregate disposable income does not change or even decrease, if the society cares enough about the disposable income of the poorest, the income assistance or welfare benefits can still be justified.

The moral hazard and the adverse selection problems of welfare policies are already well-known (Walker 2005). However, most previous studies have focused on the utility of the benefit recipients (e.g. Saez 2010). Thus, few studies have analyzed the effects of welfare benefits on the disposable income of the benefit recipients. The studies on the taxable income elasticity are most closely related (Saez et al. 2012). However, they focus on the before tax individual income, while this paper focuses on the after benefit (or tax) aggregate disposable income. Moreover, the taxes are generally imposed on all workers, where the adverse selection problem is relatively small. But the income assistance programs target low-income workers only, where the adverse selection problem can be more serious.

2 Basic Model

Suppose that workers are risk-neutral and that their utility function is given as follows:

$$U(w) = w + B - \frac{1}{k}c(w) \tag{1}$$

where w is the (before-tax) wage; B is the income assistance benefit; and $\frac{1}{k}c(w)$ is the cost of earning wage w where $c(0) = 0$, $c' > 0$, $c'' > 0, c'(0) = 0$, and $c'(\infty) = \infty$.

Note that the (marginal) cost of earning wage w decreases in k. Therefore, I interpret k as *earning capability*. For now, assume that k is uniformly distributed over an interval $[0, 1]$. Later, I will consider other distributions. Also, assuming that wage is the only source of earned income, $w + B$ can be defined as the disposable income. As I will show below, both the wage and the benefit are the functions of k. Thus, the aggregate disposable income can be defined as follows:

$$D = \int_0^1 (w(k) + B(k))dk$$

If there exist no welfare benefits (i.e. $B(k) = 0$),then, from utility maximization, it is straightforward to show that the optimal wage is:

$$w^* = c'^{-1}(k) \equiv g(k), \tag{2}$$

where $g \equiv c'^{-1}$. That is, without welfare benefits, a worker with earning capability k will work enough to earn wage equal to $g(k)$ where $g(0) = 0$ and $g' > 0$. Thus, without the welfare benefits, the aggregate disposable income is

$$D^N = \int_0^1 g(k)dk \tag{3}$$

For simplicity, I assume that the goal of the income assistance is to guarantee the minimum income level (denoted by $\underline{w}$) for everyone. I also assume that the government provides the income assistance to those below the minimum income level only, that is the benefits are means-tested. Let us define $\underline{k}$ such that

$$\underline{w} = g(\underline{k}). \tag{4}$$

Then, from (2), individuals with earning capability less than $\underline{k}$ must be supported by the income assistance programs.

3 Symmetric Information

For a benchmark, consider the first-best case where the government can observe each worker's earning capability k. That is, there is no information asymmetry problem. From (2), workers with earning capability k can earn the wage $w = g(k)$ by themselves. Then, the first-best income assistance benefits would be

$$B^*(k) = \begin{cases} \underline{w} - g(k) & \text{if } k \leq \underline{k} \\ 0 & \text{if } k > \underline{k} \end{cases}. \tag{5}$$

Note that because the benefit depends on the earning capability k only, $B^*(k)$ does not change workers' incentives to earn wages. Thus, the aggregate disposable income in the first-best case is

$$D^* = \int_0^1 w^*(k) + B^*(k)dk = \int_0^{\underline{k}} (g(k) + \underline{w} - g(k))dk + \int_{\underline{k}}^1 g(k)dk. \quad (6)$$

Then, by the definition of $\underline{k}$, $D^* - D^N = \int_0^{\underline{k}} (\underline{w} - g(k))dk = c(\underline{w}) > 0$.[1] That is, the income assistance program $B^*(k)$ increases the aggregate disposable income of the benefit recipients. Also, the aggregate benefit size or the required welfare expenditure is $\int_0^1 B^*(k)dk = \int_0^{\underline{k}} (\underline{w} - g(k))dk = D^* - D^N$. To summarize,

Proposition 1 *When workers earning capabilities* (k) *is observable,* $B^*(k)$ *increases the aggregate disposable income of the benefit recipients by* $c(\underline{w})$*. Also, the required aggregate benefit expenditure is* $c(\underline{w})$*.*

Proof. From the discussion above. ■

Note that if the government raises the minimum income level $\underline{w}$, it raises the benefit level $B^*(k)$ for a given k, and also increases $\underline{k}$ so that more workers can receive the benefits. In the first-best case, since the workers' incentives to earn wage income are not affected by the income assistance benefits, the more the government spend on the benefits by raising $\underline{w}$, the more the aggregate disposable income of the benefit recipients increase.

Example 1 *(The First Best) Suppose that* $c(w) = \frac{1}{2}w^2$*. Then,* $w^* = k$ *and* $\underline{k} = \underline{w}$*. From Figure 1, both the increase in the aggregate disposable income and the aggregate benefit expenditure can be represented by the area* $E = \frac{1}{2}\underline{w}^2$*.*

[Figure 1 here]

[1] The last equality is from the formular $\int f^{-1}(y)dy = yf^{-1}(y) - F \circ f^{-1}(y) + C$ and $g = c'^{-1}$.

4 Asymmetric Information and Wage Income

Now suppose that the government cannot observe each worker's earning capability k, but that workers themselves know their earning capability. Thus, there exists an information asymmetry problem. Note that the first-best income assistance program in (5) is no longer feasible because the government cannot observe k any more.

4.1 Linear Benefits

Even though the government cannot observe earning capability k, it can still observe a worker's wage income w. Thus, I consider the linear income assistance benefit, $B^L(w)$, as follows:

$$B^L(w) = \begin{cases} \underline{w} - bw & \text{if } w \leq \underline{w} \\ 0 & \text{if } w > \underline{w} \end{cases} . \tag{7}$$

where $0 \leq b \leq 1$.

Note that as wage income increases, the benefit decreases by b. If $b = 0$, the benefit is fixed and does not decrease in wage income, as in the basic pension for the elderly in Korea. If $b = 1$, the benefit decreases as much as the wage increases, as in the national basic livelihood security payment in Korea.

With the linear benefits, *if* a worker is qualified for the benefits (i.e. $w \leq \underline{w}$), the optimal wage would be determined as follows:

$$U'(w) = (1 - b) - \frac{1}{k}c'(w) = 0$$

or

$$w_1^L(k) = g((1 - b)k). \tag{8}$$

Note that compared with the first-best case in (2), the wage income is smaller as long as $b > 0$. That is, if income assistance benefits decrease in wage income, the benefit recipients work/earn less, known as the moral hazard problem.

4.2 Benefit Choice

To analyze the wage income and the level of income assistance of the benefit recipients, however, one must check whether the optimal wage income $w_1^L(k)$

is qualified for the benefit (i.e. $g((1-b)k) \leq \underline{w}$), and whether the worker will choose the benefits (i.e. choose to earn wages less than or equal to $\underline{w}$) in the first place.

Let us define k_1 such that

$$g((1-b)k_1) = \underline{w}. \tag{9}$$

Since $g' > 0$, workers with $k \leq k_1$ can earn the optimal wage $w_1^L(k)$ and still qualified for the benefits.[2]

Let us define the level of utility when workers with $k \leq k_1$ receive the benefits $B^L(w)$ as

$$U_1(k) = \underline{w} + (1-b)g((1-b)k) - \frac{1}{k}c(g((1-b)k)) \tag{10}$$

Instead, if a worker earns a wage greater than $\underline{w}$ and does not receive the benefits, he would earn $w^* = g(k)$ which must be greater than $\underline{w}$. Let us define the level of utility when a worker does not receive the benefits as

$$U^*(k) = g(k) - \frac{1}{k}c(g(k)). \tag{11}$$

Lemma 1 *There exists $k_0 \in [\underline{k}, 1)$ such that $U^*(k) \geq U_1(k)$ iff $k \geq k_0$.*

Proof. See appendix. ■

Therefore, even with the information asymmetry problem, workers with high enough earning capability would not receive the income assistance benefits, because it would induce them to reduce their wages too much

If $k_0 \leq k_1$, then workers with $k \leq k_0$ would prefer receiving the benefits and make wage income $w_1^L(k)$. Also their optimal wage income $w_1^L(k)$ would be small enough to be qualified for the benefits. The following lemma derives the condition for $k_0 \leq k_1$.

Lemma 2 *There exist $b_0 \in (0,1)$ such that $k_0 \leq k_1$ iff $b \geq b_0$.*

[2] I assume that $\underline{w}$ is small enough that $k_1 \in (0,1)$.

Proof. See appendix. ■

That is, if b is large enough, the size of the benefit is smaller for higher k. Thus, only those workers with very low earning capability will choose to receive the benefits. Also, because the benefit decreases fast as wage increases, workers have less incentive to earn wage income. Thus, the optimal wages of the benefit recipients are low enough to qualify for the benefits.

4.3 Wage Income

From lemma 1 and 2, there are two cases to consider. First, suppose that $b \geq b_0$ or $k_0 \leq k_1$. Then, as discussed above, if $k \leq k_0$, $U_1 \geq U^*$ and $w_1^L(k) = g((1-b)k) \leq \underline{w}$. That is, workers with $k \leq k_0$ prefer receiving the benefits and earn $w_1^L(k)$. Also, $w_1^L(k)$ is feasible as it is low enough to be qualified for the benefit.

If $k > k_0$, a worker would prefer not receiving the benefits and earn $w^*(k)$ only. Also, since $\dot{k}_0 \geq \underline{k}$ from lemma 1, $w^*(k) > \underline{w}$. That is, $w^*(k)$ is not qualified for the benefits if $k > k_0$.

Second, now suppose that $b < b_0$ or $k_0 > k_1$. If $k \leq k_1$, from lemma 1, the optimal wage of the benefit recipients $w_1^L(k)$ qualifies for the benefits and the worker prefers receiving the benefits.

If $k > k_1$, however, a worker may prefer receiving the benefits, but the optimal wage of the benefit recipients $w_1^L(k)$ is greater than $\underline{w}$ and does not qualify for the benefits. Therefore, in order to qualify for the benefits, the worker would have to reduce the wage to $\underline{w}$. Let us define the level of utility in this case as follows:

$$U_2 = \underline{w} + (1-b)\underline{w} - \frac{1}{k}c(\underline{w}).$$

A worker would reduce his wage to $\underline{w}$ in order to receive the benefits if it is still better than not receiving the benefits or $U_2 \geq U^*$.

Lemma 3 *If $b < b_0$ and $k > k_1$, there exist $k_2 \in (k_1, 1)$ such that $U^*(k) \geq U_2(k)$ iff $k \geq k_2$.*

Proof. See appendix. ■

That is, if $k_1 < k \leq k_2$, then $U^*(k) \leq U_2(k)$ and the worker will earn just $\underline{w}$ in order to qualify for the benefit.

Finally, if $k > k_2$, the worker would not choose the benefits and earn $w^* = g(k)$.

To summarize, I can fully characterize the workers' wage income when the means-tested linear benefit $B^L(w)$ is available as follows:

Proposition 2 *With linear benefits $B^L(w)$ in (7), there exist $b_0 \in (0,1)$ such that the wage income by a worker with earning capability k is determined as follows:*

(i) If $b_0 \leq b \leq 1$, then there exist $\underline{k} \leq k_0 < 1$ such that

$$w^L(k) = \begin{cases} g((1-b)k) & \text{if } k \leq k_0 \\ g(k) & \text{if } k > k_0 \end{cases} . \tag{12}$$

(ii) If $0 \leq b < b_0$, then there exist $\underline{k} \leq k_1 < k_2 < 1$ such that

$$w^L(k) = \begin{cases} g((1-b)k) & \text{if } k \leq k_1 \\ \underline{w} & \text{if } k_1 < k \leq k_2 \\ g(k) & \text{if } k > k_2 \end{cases} . \tag{13}$$

Proof. From the discussion above. ∎

Note that as long as $b > 0$, the income assistance benefits decrease in wage income. Therefore, those who receive the benefits earn less wages than the first-best level, known as the moral hazard problem. Moreover, both k_0 and k_1 in proposition 2 are larger than $\underline{k}$. Therefore, those workers who can earn wages greater than $\underline{w}$ are receiving the benefits and earn wages less than $\underline{w}$, known as the adverse selection problem. Thus, both the moral hazard and the adverse selection problems reduce wage income of the benefit recipients. Then, it is *a priori* ambiguous whether the sum of wage income and the income assistance benefits (i.e. disposable income) would increase for the benefit recipients.

5 Income Assistance and Disposable Income

From proposition 2, the disposable income of a benefit recipient is $w^L(k) + \underline{w} - bw^L(k) = \underline{w} + (1-b)w^L(k)$. Since the wage income without the benefits is

$w^* = g(k)$, the change in the *individual* disposable income due to the income assistance benefit can be defined as

$$\Delta(k) = (\underline{w} + (1-b)w^L(k)) - g(k). \tag{14}$$

Then, I can characterize the change in individual disposable income as in the following proposition.

Proposition 3 $\Delta(0) > 0$, $\Delta'(k) < 0$, $\Delta(k_0) < 0$ *if* $b_0 \leq b \leq 1$ *and* $\Delta(k_2) < 0$ *if* $0 \leq b < b_0$.

Proof. See appendix. ■

That is, among the benefit recipients, the disposable income of those workers with relatively lower earning capability increases. However, the disposable income of those with relatively higher earning capabilities decreases.

Intuitively, workers with relatively lower earning capability earn lower wage income without the benefits. Thus, the moral hazard problem due to the income assistance benefit (i.e. decrease in the wage income) is smaller in absolute magnitude. Also, the size of the benefit is larger for lower wage incomes. Therefore, the disposable income increases for workers with relatively lower earning capability. By the same intuition, the disposable income decreases for workers with relatively higher earning capability.

The following theorem shows that the increase in the disposable income among the relatively lower capability benefit recipients is exactly cancelled out by the decrease in the disposable income among the relatively higher capability benefit recipients.

Theorem 1 *If* k *is uniformly distributed, the aggregate disposable income of the benefit recipients do not increase for all* $\underline{w}$ *and* $b \in [0, 1]$.

Proof. See appendix. ■

Note that the theorem holds for all $\underline{w}$ and $b \in [0, 1]$.[3] If the government relaxes the condition for the income assistance benefits by raising $\underline{w}$, more workers will receive the benefits. Thus, one might think that the aggregate

[3] Recall, however, that throughout the paper, I assume $\underline{w}$ is low enough that some workers do not choose the benefits, that is, $k_0 < 1$ and $k_2 < 1$.

disposable income of all workers or the benefit recipients would increase. Surprisingly, however, theorem 1 shows that regardless of the level of the benefit ($\underline{w}$) or the slope of the benefit function (b), the income assistance benefits do not increase the aggregate disposable income. That is, the aggregate disposable income stays the same regardless of the size of the income assistance benefits, as if one is filling a bottomless pit.

This result is in contrast with the first-best outcome in proposition 1. When the workers' earning capabilities are observable, greater income assistance benefits increase the aggregate disposable income more. However, when the workers' earning capabilities are not observable, the aggregate disposable income does not increase regardless of the size of the benefit (expenditure).

Example 2 *(Decreasing Benefits) Suppose that* $c(w) = \frac{1}{2}w^2$ *and* $b = 1$*. That is, the benefits decrease as much as the wage income increase. If a worker receives the benefit, his utility function is* $U(w) = w + \underline{w} - w - \frac{1}{2k}w^2 = \underline{w} - \frac{1}{2k}w^2$*. Therefore, the optimal wage for the benefit recipient is* $w_1^L = 0$ *(which qualifies for the benefit) and the utility level is* $U_1 = \underline{w}$*. If the worker does not receive the benefits, from example 1, the optimal wage is* $w^* = k$ *and the utility level is* $U^* = \frac{1}{2}k$*. Therefore, a worker would receive the benefit if* $U_1 \geq U^*$ *or* $k \leq 2\underline{w}$*.*

[Figure 2 here]

In figure 2, the thick solid line represents the disposable income (= wage + benefit), and the dashed line represents the wage income when there exists no benefits. Note that for workers with $k \leq \underline{w}$, the disposable income increases by area E. However, for those workers with $\underline{w} < k \leq 2\underline{w}$, the disposable income decreases by area D. Since $E = D = \frac{1}{2}\underline{w}^2$, the aggregate disposable income does not change regardless of $\underline{w}$.

Example 3 *(Fixed Benefits) Suppose that* $c(w) = \frac{1}{2}w^2$ *and* $b = 0$*. That is, the benefits do not decrease in wage income as long as the wage is less than* $\underline{w}$*. If a worker receives the benefit, his utility function is* $U(w) = w + \underline{w} - \frac{1}{2k}w^2$*. Therefore, the optimal wage for the benefit recipient is* $w_1^L = k$ *And if* $k \leq \underline{w}$*,*

w_1^L qualifies for the benefits. If $k > \underline{w}$, to receive the benefits, workers would earn just $\underline{w}$, and their utility level is $U_2 = \underline{w} + \underline{w} - \frac{1}{2k}\underline{w}^2$. From example 2, without the benefit, the utility level is $U^* = \frac{1}{2}k$. Therefore, a worker would reduce their wage to $\underline{w}$ to receive the benefit iff $U_2 \geq U^*$ or $k \leq (2 + \sqrt{3})\underline{w} \approx 3.73\underline{w}$.

[Figure 3 here]

In figure 3, for workers with $k \leq 2\underline{w}$, the disposable income increases by area $E' + E$. However, for those workers with $2\underline{w} < k \leq 3.73\underline{w}$, the disposable income decreases by area D'. Since $E + E' = D'$, the aggregate disposable income does not change regardless of $\underline{w}$.

In example 2 (or figure 2), the benefits decreases as much as wage income. Thus, worker who receive the benefits would make zero wages. That is, with larger b, there exist relatively larger moral hazard problems. In example 3 (or figure 3), the benefits do not decrease in wages. Thus, the benefit recipients would like to earn up to their first-best wage levels. That is, there is less moral hazard problem. However, since the benefits do not decrease in income, those who have relatively higher ability would earn wage income just enough to qualify for the benefit at lower marginal cost and receive the benefits. That is, with smaller b, there exist relatively smaller moral hazard problems, but larger adverse selection problems. Therefore, the aggregate disposable income does not increase regardless of b.

Theorem 1 depends on a key assumption that k is uniformly distributed in the economy. More realistically, suppose that the distribution of k, denoted by $f(k)$, is single-peaked (e.g. normal or log-normal distribution) at $k = \overline{k}$. Also, the benefit recipients are distributed on the left (or lower) side of the distribution. That is, k_0 or k_2 in proposition 2 is smaller than $\overline{k}$.

Then, among the benefit recipients, there will be relatively more high ability workers, that is, $f'(k) > 0$. From proposition 3, since disposable income decreases for relatively high ability workers, if there exists relatively more high ability workers, the aggregate disposable income must decrease.

Theorem 2 *If the distribution of k is single-peaked, and the benefit recipients are distributed on the left side of the distribution, the aggregate disposable income of the benefit recipients decreases for all $\underline{w}$ and $b \in [0, 1]$.*

Proof. From the discussion above. ■

From the aggregate disposable income perspective, theorem 2 shows very pessimistic results. Even though the goal of income assistance programs is to increase the disposable income of the benefit recipients, theorem 2 states that the income assistance programs can do exactly the opposite.

6 Conclusion

It is well-known that income assistance or welfare benefits would lead to the reduced earned wages because of the moral hazard and the adverse selection problem. Even so, it has been unclear whether the sum of (reduced) wages and welfare benefits, i.e. the disposable income, will increase. This paper shows that the extent of the moral hazard and the adverse selection problems are far more severe than one might expect. Thus, if the earning capabilities are uniformly distributed, the *aggregate* disposable income of the benefit recipients does not increase at all regardless of the size of the benefits. Moreover, if the distribution of earning capabilities is single-peaked, the aggregate disposable income can even decrease.

As emphasized in the beginning, these results do not necessarily imply that income assistance programs are ineffective. From proposition 3 (and examples 2 and 3), for those benefit recipients with relatively lower capabilities, the income assistance benefits do increase their disposable income. Thus, when the society puts greater emphasis on the income of the poorest, these income assistance programs can still be justified. Also, if the welfare benefits can be given based on workers' earning capabilities, not on their earned wage income, income assistance benefits can increase the aggregate disposable income.

I should also note that these results are based on the linear benefit function. It would be interesting for future research how these results extend to more general benefit functions. In particular, a typical earned income tax credit (EITC) has an increasing, fixed, and then decreasing benefit structure. Also, a worker may be able to select one benefit among different benefit functions.

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Appendix

Proof of Lemma 1 Define $F(k)$ such that

$$F(k) \equiv U^*(k) - U_1(k).$$

Then, from the envelope theorem, $\frac{\partial F}{\partial k} = \frac{1}{k^2}(c(g(k)) - c(g((1-b)k)) > 0$. If $\underline{w}$ is small enough, $F(1) > 0$ since U^* is the maximum of $w - \frac{1}{k}c(w)$. Also, $\lim_{k\to 0} F(k) = -\underline{w} < 0$ since $g(0) = 0$ and $\lim_{k\to 0} \frac{1}{k}c(g(k)) = \lim_{k\to 0} \frac{c'g'}{1} = \lim_{k\to 0} \frac{kg'}{1} = 0$ from L'Hospital's rule. Therefore, there exist a unique $k_0 \in (0,1)$ such that $F(k_0) \geq 0$ iff $k \geq k_0$.

Also, $k_0 \geq \underline{k}$ since $F(\underline{k}) = -(1-b)g((1-b)\underline{k}) - \frac{1}{k}(c(g(\underline{k})) - c(g((1-b)\underline{k})) < 0$.

Proof of Lemma 2 Since $\frac{\partial F}{\partial k} > 0$ and $F(k_0) = 0$, $k_0 \leq k_1$ iff $F(k_1) \geq 0$. Since k_1 is a function of b from (9), define

$$F_b(b) = F(k_1(b)) = g(k_1(b)) - \frac{1}{k_1}c(g(k_1(b))) - \underline{w} - (1-b)\underline{w} + \frac{1}{k_1(b)}c(\underline{w}).$$

If $b = 0$, then $g(k_1) = \underline{w}$. Thus, $F_b(0) = -\underline{w} < 0$. Also, if $b \to 1$, then $k_1 \to \infty$. Thus, $\lim_{b\to 1} F_b(b) > 0$ if $\underline{w}$ is small enough, since $g(k) - \frac{1}{k}c(g(k)) > 0$ and increasing in k for all $k > 0$. Since $\frac{\partial F_b}{\partial b} = \frac{1}{k^2}\left(c(g(k_1)) - c(\underline{w})\right) + \underline{w} > 0$, there exist $b_0 \in (0,1)$ such that $F(k_1) \geq 0$ iff $b \geq b_0$.

Proof of Lemma 3 Define $H(k)$ such that

$$H(k) = U^* - U_2 = g(k) - \frac{1}{k}c(g(k)) - \left(\underline{w} + (1-b)\underline{w} - \frac{1}{k}c(\underline{w})\right).$$

Note that $\frac{\partial H}{\partial k} = \frac{1}{k^2}(c(g(k)) - c(\underline{w})) > 0$ if $k > k_1$. Also from lemma 2 , $H(k_1) = F(k_1) < 0$ if $b < b_0$. And $H(1) > 0$ if $\underline{w}$ is small enough. Therefore, if $b < b_0$, there exist $k_2 \in (0,1)$ such that $H(k) \geq 0$ iff $k \geq k_2$ and $H(k_2) = 0$.Also note that $k_2 > k_1$ iff $H(k_1) = F(k_1) < 0$. Thus, from lemma 2, $k_2 > k_1$ iff $b < b_0$.

Proof of Theorem 1 First, suppose that $b \geq b_0$. Note that

$$\int_{k_0}^{1} g(k)dk = G(1) - k_0 g(k_0) + c(g(k_0))$$

where $G() = \int g(k)dk$. Likewise,

$$\begin{aligned}\int_0^{k_0} [\underline{w} + (1-b)g((1-b)k)]dk &= \underline{w}k_0 + (1-b)\int_0^{k_0} g((1-b)k)dk \\ &= \underline{w}k_0 + \int_0^{(1-b)k_0} g(z)dz \\ &= \underline{w}k_0 + (1-b)k_0 g((1-b)k_0) - c(g((1-b)k_0)\end{aligned}$$

Therefore, from the definition k_0, the aggregate disposable income is

$$D_1 \equiv \int_0^{k_0} [\underline{w} + (1-b)g((1-b)k)]dk + \int_{k_0}^1 g(k)dk = G(1) = D^N,$$

where D^N is the disposable income when there exists no benefits, as defined in (3).

Second, now suppose that $b < b_0$. Note that

$$\int_{k_2}^1 g(k)dk = G(1) - k_2 g(k_2) + c(g(k_2))$$

$$\int_{k_1}^{k_2} [\underline{w} + (1-b)\underline{w}]dk = [\underline{w} + (1-b)\underline{w}](k_2 - k_1)$$

$$\begin{aligned}\int_0^{k_1} [\underline{w} + (1-b)g((1-b)k)]dk &= \underline{w}k_1 + (1-b)\int_0^{k_1} g((1-b)k)dk \\ &= \underline{w}k_1 + \int_0^{(1-b)k_1} g(z)dz \\ &= \underline{w}k_1 + (1-b)k_1 g((1-b)k_1) - c(g((1-b)k_1))\end{aligned}$$

From the definitions of k_1 and k_2, the aggregate disposable income is

$$D_2 \equiv \int_0^{k_1} [\underline{w}+(1-b)g((1-b)k)]dk + \int_{k_1}^{k_2} [\underline{w}+(1-b)\underline{w}]dk + \int_{k_2}^1 g(k)dk = G(1) = D^N.$$

Therefore, for all b and $\underline{w}$, the linear income assistance benefits $B^L(w)$ does not increase the aggregate disposable income

Proof of Proposition 3 Suppose that $b \geq b_0$. For the benefit recipients (i.e. $k \leq k_0$), from proposition 2(i), $\Delta(k) = (\underline{w}+(1-b)g((1-b)k) - g(k)$. Note that $\Delta'(k) = (1-b)^2 g'(k) - g'(k) < 0$ since $b \geq b_0 > 0$. Also $\Delta(0) = \underline{w} > 0$ and $\Delta(k_0) = -\frac{1}{k}\left(c(g(k)) - c(g((1-b)k)\right) < 0$.

Suppose that $b < b_0$. For $k \leq k_1$, $\Delta(k) = (\underline{w} + (1-b)g((1-b)k) - g(k)$. From above, $\Delta(0) > 0$ and $\Delta'(k) < 0$. For $k_1 < k \leq k_2$, $\Delta(k) = (\underline{w} + (1-b)\underline{w}) - g(k)$. Thus, $\Delta'(k) = -g'(k) < 0$ and, from definition of k_2, $\Delta(k_2) = -\frac{1}{k}\left(c(g(k_2)) - c(g(\underline{k}))\right) < 0$ since $k_2 > \underline{k}$.

Figure 1 First-Best Case

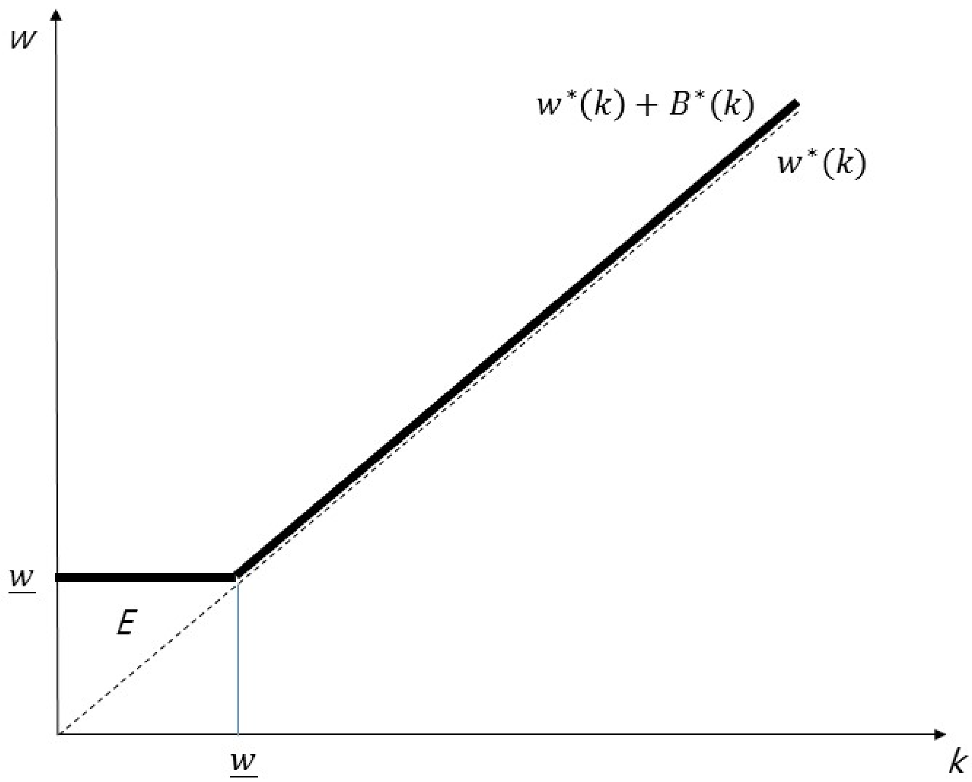

Figure 2 Decreasing Benefits (b=1)

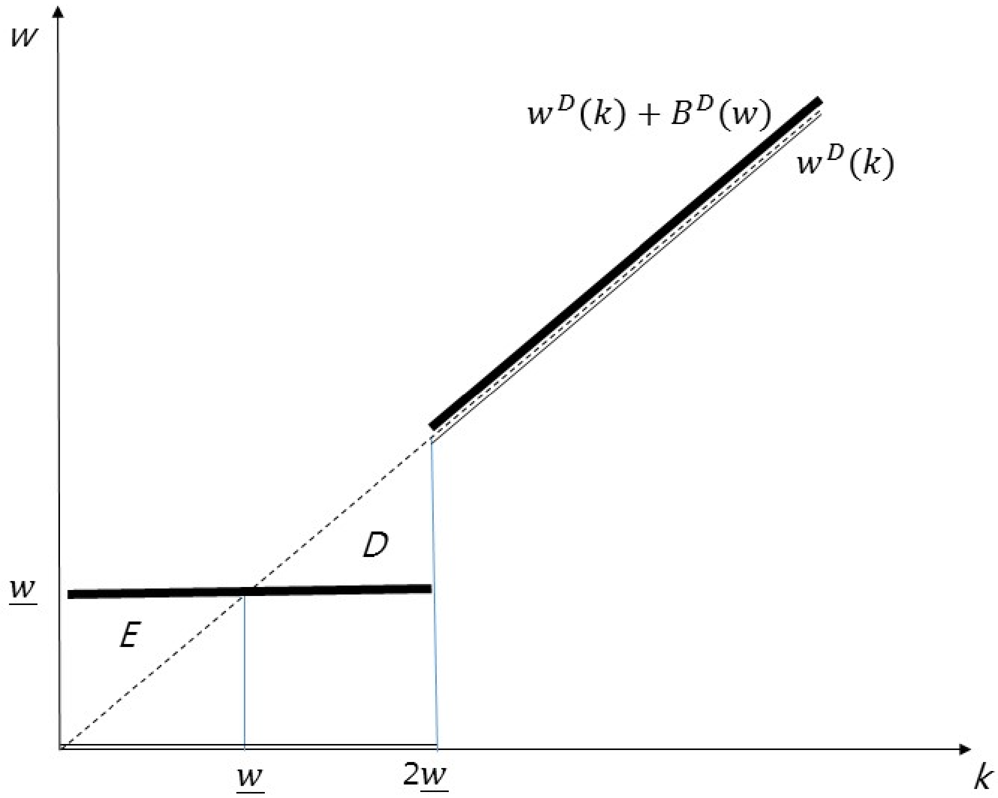

Figure 3 Fixed Benefits (b=0)