Arbitrage Comes Hand in Hand with the Risk of Market Crash

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March 2019

Our paper

- We propose a model that shows
  - hedge funds can initiate a sequence of arbitrage and a potential market crash even without any exogenous shock, and
  - an arbitrage opportunity comes with a chance of market crash
- The mechanism is coordination failure at the market level (endogenous price)
  - Coordination failure: multiple (two) equilibria
    - in one, everyone invests and asset is fair-valued
    - in the other, hedge funds do not make enough investments and asset is mispriced
  - Main cause of coordination failure: redemption risk
    - hedge fund industry features are also relevant: high leverage, information asymmetry between investors and hedge fund managers and a fee structure of hedge funds

- Implications
  - on hedge fund regulations (Volcker rule): Hedge funds reduce mispricing but may generate a crisis
  - on hedge fund behavior: hedge fund leverage decreases prior to the start of the financial crisis in mid-2007 (Ang, Gorovyy and Van Inwegen (2011)) and this is consistent with our model

Investors and managers

- Investors (of hedge funds): those who purchase hedge funds and do not have knowledge on the assets
- (Hedge fund) managers: those who trade assets on behalf of investors and specialize in the assets

Redemption risk

- Investors request redemption if the fund performance is poor
  - Investors do not know the hedge fund's strategy or the asset market
  - Observing fund performance, investors may update their belief on the skill level of managers
- The managers sell the asset to fulfill the redemption request
  - This may push down the asset price
- Empirical observations:
  - Buraschi, Kosowski and Sritrakul (2014): hedge funds experience sudden large outflows after experiencing 20% loss on average
  - Ben-David, Franzoni and Moussawi (2012): during the 2008-2009 financial crisis, the redemption of hedge funds was three times more intense than that of mutual funds

- Manager's action
  - Anticipating investors' redemption at the time of a financial crisis, managers make leverage decisions before a potential crisis
  - If every manager go aggressive, there is no crisis
  - A manager go defensive because others go defensive and there will be a crisis: coordination failure

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- Successful coordination
  - Hedge fund investment is enough to support the fair price
  - No mispricing at all
- Coordination failure
  - For fear of redemption risk, some hedge funds go defensive
  - The asset price goes below the fair value: arbitrage opportunity
  - Later, asset price may recover its fair value or drops even more: crash risk
  - These results are endogenously derived in equilibrium: Arbitrage comes with the crash risk!

Implications of this paper

- Implication 1: the hedge fund capital is more effective in alleviating the mispricing than other types of capital (e.g., pension fund)
  - In our model, if $1 is transferred into hedge funds from other types of capital before a crisis, the mispricing during the crisis is alleviated
  - This supports Stulz (2007): hedge funds can reduce mispricing more effectively than other funds
  - Kokkonen and Suominen (2015) empirically demonstrate that the aggregate size of hedge funds is more important than that of mutual funds in reducing the misvaluation of U.S. individual stocks
  - Volcker rule limits bank investments in hedge funds: a crisis may be more severe
- However,
  - our main argument is that hedge funds may generate a crisis without any exogenous shock

- Implication 2: A model prediction is consistent with empirical observation
  - Prediction: If coordination fails among hedge funds, hedge fund leverage is lower
  - Observation: Hedge fund leverages decrease prior to the start of the financial crisis in mid-2007 (e.g., Ang, Gorovyy and van Inwegen, 2011)

Relevant features of hedge fund industry:

Leverages

- Prime brokers make loans to hedge funds
  - Mostly short-term, 1 to 90 days
  - Prime brokers will not lend money any more if the hedge fund seems to be in trouble
- LTCM lost half of its AUM in August 1998
  - Extremely high leverage (at a ratio of 28 to 1 in 1995)
    - Risk management of customers/financiers $\Rightarrow$ forced liquidation of assets
  - The effect is larger when leverage is higher

Relevant features of hedge fund industry: Fee structure

- Hedge funds charge fees
  - Management fee: a fraction of AUM, typically, 2%
  - Incentive fee: a fraction of profits, typically, 20%
  - When losses occur, only management fee is charged
- Fund managers are not responsible for the loss of the funds
  - After a severe loss, hedge fund managers often shut down the fund and open a new fund
  - Reputation cost or cost of starting a new fund

Relevant features of hedge fund industry: heterogeneous strategies

- There are more than 10,000 hedge funds as of 2016
- 1,040 new fund launches and 864 liquidations in 2014 (Hedge Fund Research)
    - This is related to the incentive structure of hedge fund's managers
    - Losses are not shared by hedge funds
    - When opening a new fund, reputation may matter
- Some funds go aggressive and others wait for (or even bet on) a crisis
    - Steve Eisman, who was portrayed the movie "Big Short", is an example
        - He bet on the collapse of the subprime mortgage market (short on CDO and CDS) while others go the other way around
- Our equilibrium is consistent with the above

Difference from the literature

- Our paper: No exogenous shock is assumed
- Bernanke and Gertler (1989), Kiyotaki and Moore (1997): financial leverage amplifies a shock to generate a business cycle
- Global games: shocks on fundamental is essential

- Bank run literature: Diamond and Dybvig (1983), Allen and Gale (1998)
    - contractual linkages such as deposit contracts
- Our paper: through endogenous price

Contribution to the literature

- Liu and Mello (2011): model of redemption among investors at a particular hedge fund as a global game
  - Our paper: market-level interaction among hedge funds
- Brunnermeier and Pedersen (2008): margin requirements
  - (shock⇒lower price⇒sell to keep leverage⇒lower price)
  - Every fund's margin requirement is binding
  - Our paper: going defensive is voluntary
- Shleifer and Vishny (1997): performance-based fund flow
  - exogenous shock, insufficient funding
  - Our paper: no exogenous shock, sufficient funding

Outline

Setup

- There are one (possibly risky) asset and cash in the market
  - The asset supply is 1
- There are 3 periods, $t = 0, 1, 2$
  - Each unit of the asset pays off 1 (cash value) at its maturity at $t = 2$
  - No interim cash flow
  - p_t: price at time $t = 0, 1, 2$. To be determined endogenously
    - $p_2 = 1$
- If $p_0 < 1$ or $p_1 < 1$, there is an arbitrage opportunity

Participants

- There are 3 types of market participants: (hedge fund) managers, investors, and long-term holders
- Managers
  - know the market and $p_2 = 1$ at $t = 0, 1$
    - (other participants know $p_2 = 1$ at $t = 2$ only)
  - may invest in the asset and cash at $t = 0, 1$
  - funding from investors (capital) and financiers (debt)
  - if the loss of the portfolio at $t = 1$ is too large, it may collapse at $t = 1$ (detail later)
- Long-term holders
  - abstraction of pension funds, insurance companies, banks, and maybe mutual funds
  - for simplicity, their demand (in quantity) is assumed to be exogenously given

Long-term holders and states

- At $t = 0$, long-term holders' total demand is exogenously given $X < 1$
- At $t = 1$, there are two states, $\omega = g$ (good) and $\omega = b$ (bad)
  - In state g (with prob $1 - q$), long-term holders' demand stays the same, X
  - In state b (with prob q), long-term holders' demand is $X(1 - \varepsilon)$ with $0 \le \varepsilon \le 1$
- ε is a constant and allowed to be 0
  - If $\varepsilon = 0$, the two states are essentially identical (no fundamental shock)

Fund managers

- Continuum of risk-neutral managers of mass 1, indexed by $i \in [0,1]$
  - No heterogeneity across managers but equilibrium behaviors may differ (to be seen later)
- $W_{i,t}$: asset under management (AUM) of fund $i \in [0,1]$ at time $t = 0, 1, 2$
  - Initial AUM, $W_{i,0} = W_0$, in cash supplied by investors
  - $\int_0^1 W_0 di = W_0$ is the total asset value in the market at $t = 0$

- Compensation structure for fund managers
    - management fees: $\beta W_{i,2}$ with $\beta > 0$
    - incentive fees: $\alpha \max\left(W_{i,2} - W_0, 0\right)$ with $\alpha > 0$
    - fund managers do not share the loss with investors but:
        - If customers withdraw their money, the fund closes and it costs the manager $C \geq 0$
        - C: liquidation cost (including cost of setting up a new fund and bad reputation)
    - fund i maximizes

$$U = \Pr\left(\text{survive}\right)\left(\alpha \max\left(W_{i,2b} - W_0, 0\right) + \beta W_{i,2b}\right) - \Pr\left(\text{not survive}\right) C$$

Fund managers at $t = 0$

- Fund managers can invest in the asset or hold cash
- Can also borrow
  - Short-term borrowing: Has to pay back at $t = 1$
  - (May borrow again at $t = 1$, to be seen later)
  - Borrowing interest rate is normalized to 0
- Let $l_{i,0}$ be the leverage ratio of manager i at $t = 0$
  - $l_{i,0} = \frac{\text{(amount of borrowing)}}{\text{(wealth at } t=0,\ W_0)}$
  - Invest $W_0 (1 + l_{i,0})$ in the asset
  - For example, $l_{i,0} = -1$: investing in cash only
- Borrowing limit at $t = 0$: $l_{i,0} \leq \bar{l}_0$
- No short-sales of the asset: $l_{i,0} \geq -1$

Fund managers at $t = 1$

- Wealth of manager i at the beginning of period $t = 1$ in state $\omega = g, b$

$$W_{i,1} = W_0 \left((1 + l_{i,0}) \frac{p_{1\omega}}{p_0} - l_{i,0} \right)$$

  - if $p_1 > p_0$, leverage gives profit
  - if $p_1 < p_0$, leverage gives loss
- Risk management rule or liquidation condition
  - if $W_{i,1} < W_0 (1 - s)$, arbitrage fund i is liquidated
  - s is exogenously given
  - $s = 0$ means no loss is allowed

Motivation of liquidation condition

- s is exogenous
  - so that we can focus on managers' behaviors and the endogenous prices
- Non-transparency: Arbitrage markets are highly specialized, customers/financiers do not understand the market or the manager's strategy
  - It is common that a hedge fund manager does not reveal his strategy (to prevent competitors from copying the strategies)
- Motivation
  - Investors request redemption based on the past performance, possibly by updating beliefs on the manager's ability

Be liquidated or survive

- Arbitrage fund i is liquidated if $W_{i,1} < W_0(1-s)$
  - If a manager used high leverage at $t=0$, it may cause redemption in the bad state at $t=1$
  - Then, the manager's payoff is assumed to be $-C$
- If manager i survives at $t=1$, she trades the asset
  - Borrowing interest rate: 0
  - Borrowing limit at $t=1$: $l_{i,1} \leq \bar{l}_1$
- At $t=2$, one unit of the asset pays 1 and managers receive the fees

Market clearing

- The asset supply is 1
- At time 0,

$$\text{(demand of long-term investors and arbitragers)} = 1$$

- In each state at time 1,

$$\text{(demand of long-term investors and fund managers who have survived)} = 1$$

- To be determined endogenously:
    - p_0, p_{1g}, p_{1b} ($p_2 = 1$ is assumed)
    - $l_{i,0}$, $l_{i,1b}$, $l_{i,1g}$: leverages

Outline

1. Setup
2. Equilibrium
3. Conclusion

Undervalued only

Lemma 1. In any equilibrium, $p_{1g} \leq 1$**,** $p_{1b} \leq 1$ **and** $p_0 \leq 1$.

- Intuition:
  - If $p_{1g} > 1$, no manager will buy because $p_2 = 1$ for sure.

$$\text{demand} = X(1-\varepsilon) \leq X < 1 \text{ and supply} = 1$$

  - Same for p_{1b}
  - If $p_0 > 1$, demand of managers is 0 because the price will drop for sure: $p_{1b} \leq p_{1g} = p_2 = 1$

Assumption 1

Assumption (1). There are enough funds in the market:

$$X + W_0\left(1+\bar{l}_0\right) \geq 1, \textbf{ and}$$

$$X(1-\varepsilon) + W_0\left(1+\bar{l}_1\right) \geq 1.$$

- Will make this assumption throughout
- $X + W_0\left(1+\bar{l}_0\right) \geq 1$: Sufficient funds in the market at $t=0$
  - $p_0 = 1$ (the fair price) can be supported because (the maximum possible demand)$=X + \frac{W_0\left(1+\bar{l}_0\right)}{p_0} \geq 1=$(asset supply)
- $X(1-\varepsilon) + W_0\left(1+\bar{l}_1\right) \geq 1$: Enough funds in the market in state b (as well as state g) at $t=1$
  - If $p_0 = p_{1b} = p_{1g} = 1$, then $W_{i,1} = W_0$ for any leverage level, $l_{i,0}$
  - (the maximum possible demand)$=X(1-\varepsilon) + \frac{W_0\left(1+\bar{l}_1\right)}{p_0} \geq 1=$(asset supply)

Equilibria

Theorem 1. Under Assumption 1, if q and s are sufficiently small, the followings hold:

(i) there is no liquidation if and only if $p_0 = p_{1g} = p_{1b} = 1$, and

(ii) there are some liquidations if and only if $p_{1b} < p_0 < p_{1g} = 1$.

- We call the equilibrium with $(p_0, p_{1g}, p_{1b}) = (1, 1, 1)$ *a calm equilibrium*
  - No mispricing, no crisis
- The equilibrium with $p_{1b} < p_0 < p_{1g} = 1$ is called *a crisis equilibrium*
  - Some funds are liquidated
  - Investors request redemption
  - Viewed as a crisis
- Arbitrage opportunity $(p_0 < 1)$ comes hand in hand with the risk of market crash $(p_{1b} < p_0)$

Lemma 2. Under Assumption 1, if q and s are sufficiently small, it holds that $p_{1g} \geq p_{1b}$.

- When $\varepsilon = 0$, states g and b are essentially the same
- According to the lemma, we consider $p_{1b} < p_0 < p_{1g} = 1$ only, not $p_{1g} < p_0 < p_{1b} = 1$

Theorem 2. A calm equilibrium exists if Assumption 1 holds.

- Existence of a calm equilibrium is straightforward
- A more importance question is if a crisis equilibrium exists as well
  - Will show existence of a crisis equilibrium, in particular when $\varepsilon = 0$
  - That is, is it possible that $p_{1b} < p_0 < p_{1g}$ in equilibrium?

Managers who survive at $t = 1$

- If manager i survives, the final wealth will be

$$W_{i,2} = W_{i,1\omega} \left(\frac{1}{p_{1\omega}} (1 + l_{i,1}) - l_{i,1} \right)$$

  at each state $\omega = g, b$

  - Recall that fees are increasing in $W_{i,2}$
  - A manager tries to maximize $W_{i,2}$, once he survives

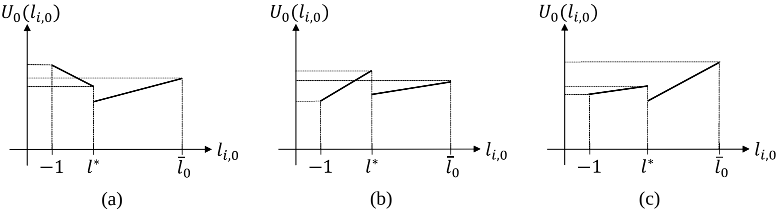
- In good state (g), $p_{1g} = 1$ in a crisis equilibrium
  - $p_0 < p_{1g}$. Thus, positive profit and no liquidation
- Bad state (b)
  - $p_0 > p_{1b}$. Some funds may have been liquidated
  - $p_{1b} < 1$. If a fund survives, the maximum leverage $(l_{i,1} = \bar{l}_1)$ will be optimal at $t = 1$
  - Demand$= X\left(1-\varepsilon\right) + \int_0^1 W_{i,1b}\left(1+\bar{l}_1\right)\cdot \mathbf{1}\left(W_{i,1b} \geq W_0\left(1-s\right)\right) di/p_{1b}$
  - Supply=1

Leverage decision at $t = 0$

- $l_{i,0}$ will determine $W_{i,1b}$, which determines
  - whether the fund is liquidated in state b,
  - p_{1b} (by the market clearing in b)
  - the final wealth and the fee
- $U_0 (l_{i,0}) =$

  $\Pr(\text{survive})(\alpha \max(W_{i,2b} - W_0, 0) + \beta W_{i,2b}) - \Pr(\text{not survive}) C$
  - The expected payoff as a function of $l_{i,0}$

- $p_0 < 1$ opens up the possibility of an arbitrage opportunity
- The textbook argument on arbitrage implies
  - $l_{i,0}$ should be as large as possible, and
  - the equilibrium price should be $p_0 = 1$
- But we have frictions
  - $l_{i,0}$ has an upper limit
  - at $t = 1$, the mispricing may get larger and the investors may request redemption
    - So, arbitrage opportunity at $t = 0$ is not perfectly riskless to the manager



- Let l^* be the maximum leverage level that makes a manager survive in b
  - -1 or l^*: defensive strategy. Betting on b. If g realizes, small profits obtain. If b realizes, large profits obtain.
  - $\bar{l}_0$: aggressive strategy. Betting on g. If g realizes, large profits obtain. If b realizes, liquidated

Candidate for equilibrium strategy

- A manager takes $l_{i,0} = -1$, l^* or $\bar{l}_0$
  - Different from the textbook argument because of the frictions
  - There is a chance that the arbitrage fund is liquidated when $l_{i,0} = \bar{l}_0$

Definition 1. The bang-bang strategy profile refers to a strategy profile in which $h \in (0,1)$ proportion of managers take $l_{i,0} = \bar{l}_0 > l^*$ and the other managers take $(l_{j,0}, l_{j,1b}) = \left(-1, \bar{l}_1\right)$ or $(l_{j,0}, l_{j,1b}) = \left(l^*, \bar{l}_1\right)$

- h proportion of managers go aggressive and bet on g. They will be liquidated in state b
- $1 - h$ proportion of managers go defensive and bet on b (wait for a crisis)
- Consistent with actual hedge fund behaviors

Leverage decision at $t = 0$

- (In this presentation, we will focus on $l_{j,0} = l^*$ and ignore $l_{j,0} = -1$ for simplicity of notations. Both cases are taken care of in the paper.)
- $U_0(l^*) > U_0(\bar{l}_0)$: Everyone goes defensive ($h = 0$)
  - Everyone survives in b, funds are enough in b and thus $p_{1b} = 1$
  - Then, no reason to be defensive
- $U_0(l^*) < U_0(\bar{l}_0)$: Everyone goes aggressive ($h = 1$)
  - All funds in the market are invested in the asset at $t = 0$
  - $p_0 = 1$ because of sufficient funds
  - No reason to be aggressive because there is no gain from $p_0 = 1$ to $p_2 \leq 1$
- So, in equilibrium, $U_0(l^*) = U_0(\bar{l}_0)$

Equilibrium characterization

- Collect all the equilibrium conditions so far:
  - (p_0, p_{1g}, p_{1b}, h) satisfies

$$X + \frac{W_0\left(1 + h\bar{l}_0 + (1-h)\, l^*\right)}{p_0} = 1$$

$$p_{1g} = 1$$

$$X(1-\varepsilon) + \frac{(1-h)\, W_0 (1-s)\left(1+\bar{l}_1\right)}{p_{1b}} = 1$$

$$U_0(l^*) = U_0\left(\bar{l}_0\right)$$

- Does (p_0, p_{1g}, p_{1b}, h) exist such that $p_{1b} < p_0 < p_{1g} = 1$ and $h \in (0,1)$?
- Some go aggressive and others wait for a crisis
  - Homogeneous managers generate heterogeneous behaviors

Main theorem

Theorem 3. Suppose Assumptions 1 and $\varepsilon \geq 0$. If q, s and W_0 are sufficiently small, a crisis equilibrium exists.

- $\varepsilon \geq 0$: demand shock, q: prob of bad state, s: tolerable loss rate by risk management, W_0: initial wealth of hedge funds
    - small q: a crisis is less likely to happen
    - small s: investors do not tolerate small losses
- A crisis can arise without any exogenous shock ($\varepsilon = 0$ is allowed)

- Coordination failure
    - There is enough funding liquidity in the market (a calm equilibrium is possible)
    - Managers are selling only because others are selling
        - No one has to sell if no one else sells
        - Failure to coordinate leads to a crisis equilibrium
    - Coordination failure like in Diamond and Dybvig (1983), but ours is at the market level
        - Endogenous price

Theorem 4. Under Assumption 1, only a calm equilibrium exists if q is sufficiently large.

- One potential explanation of why some arbitrage trading strategies appear to 'pick up nickels in front of steamrollers.'
  - The aggressive strategy in the crisis equilibrium is characterized by a high probability of small arbitrage gains coupled with a low probability of huge losses.

Implication 1

Theorem 5. Under Assumption 1, if q, s and W_0 are sufficiently small, it holds that $\frac{dp_0}{dW_0} > \frac{dp_0}{dX} > 0$ and $\frac{dp_{1b}}{dW_0} > \frac{dp_{1b}}{dX} > 0$ in a crisis equilibrium.

- With enough size of hedge funds, the coordination works better
  - Less profitable to go aggressive
- Hedge funds are more effective at reducing mispricing than long-term funds
  - Stultz (2007): hedge funds can reduce mispricing more effectively than other funds.
  - Kokkonen and Suominen (2015): empirically demonstrate that the aggregate size of hedge funds is more important than that of mutual funds in reducing the misvaluation of U.S. individual stocks
  - Volcker Rule may lead to more severe crisis by limiting bank investments in hedge funds

- But hedge funds can generate a crisis by an earlier theorem
  - The previous theorem assumes a crisis equilibrium
  - Brunnermeier and Nagel (2004) and Griffin et al (2011): hedge funds might accentuate mispricing of technology stocks in the tech bubble/burst from 1997 to 2002

Implication 2

Theorem 6. Under Assumption 1, if q, s and W_0 are sufficiently small, it holds that $\frac{dh}{dC} < 0$, $\frac{dp_0}{dC} < 0$ and $\frac{dp_{1b}}{dC} > 0$ in a crisis equilibrium.

- C: liquidation cost
  - If liquidation is costlier, a crisis is less severe but the pre-crisis mispricing is more severe
- After the 2008 crisis, bankers/traders did not suffer much but receive bonuses from bailout money
  - A crisis will be more severe

Implication 3

Corollary 1. The aggregate leverage of hedge funds at time 0, $\int l_{i,0} di$, is lower in a crisis equilibrium than in a calm equilibrium.

- Low leverage of hedge funds prior to a financial crisis
- Ang et al. (2011): Leverage in the hedge fund industry decreases prior to the start of the financial crisis in mid-2007
- In our model, lower leverage may indicate an immediate crisis

Outline

Conclusion

- We show that a financial crisis can occur even when
    - there is enough funding liquidity in the market, and
    - there is no exogenous shock
- because of
    - manager's coordination failure at the market level
- the latter is caused by
    - redemption risk