SIER Working Paper Series No. 101

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March, 2019

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March 5, 2019

Abstract

The paper studies incentives of low-quality sellers to disclose negative information about their product. We develop a model where one's quality can be communicated via cheap-talk messages only. This setting limits ability of high-quality sellers to separate as any communication strategy they pursue can be costlessly imitated by low-quality sellers. Two factors that can incentivize low-quality sellers to communicate their quality are buyers' risk-attitude and competition. Quality disclosure reduces buyers' risk thereby increasing their willingness to pay. It also introduces product differentiation softening the competition. We show that equilibria where low-quality sellers separate exist under monopoly and duopoly. Even though low-quality sellers can costlessly imitate high-quality sellers, equilibria where high-quality sellers separate can also exist but under duopoly only.

Keywords: Negative information, product differentiation, cheap talk, lemon markets

JEL classification: D21, L15

1 Introduction

Is honesty the best policy for sellers? Arguably, when customers cannot easily discover negative aspects of the products, low-quality sellers would be better off concealing information about those weaknesses. Many empirical studies have documented multiple instances where negative information damages sales and purchases likelihood, through various routes such as publicity, customer reviews, or word-of-mouth (Berger, Sorensen & Rasmussen 2010). Therefore, it seems natural for sellers to hide negative information about their products, in the presence of asymmetric information about product quality.

*We are grateful to Dongkyu Chang, Preyas S. Desai, David Dicks, Ginger Zhe Jin, Kyungmin Kim, Oded Koenigsberg, Donald R. Lehmann, Pei-yu Lo, William P. Putsis, Scott Shriver, participants of the AEI-Five conference, Asian Econometric Society meeting, the Singapore Economic Review Conference, and attendees of Economics seminars at various universtities for their comments and feedback. Vu Phu Tri provided an excellent research assistantship. The paper was supported by funds from the Department of Economics at Seoul National University.

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However, we do observe many sellers voluntarily sharing negative aspects about their products even when customers may not find them out (i.e., when products are high in experience or credence attributes). For example, Hans Brinker Hotel in Amsterdam, the Netherlands is famous for its strategy of honestly revealing its low quality and explaining negative aspects of their services such as rooms without a view and no hot water, through pictures and detailed descriptions. Chipotle Mexican Grill's website used to highlight drawbacks of their ingredients in its "Room for Improvement" section.[1] and Woot.com is famous for its preemptive revelation of the disadvantages of listed products, stating that they would prefer customers not buying from them to "regretting their purchases."[2] Voluntary disclosure of negative information on experience or credence attributes can also be found in many consumer-to-consumer online marketplaces. For example, sellers at eBay often voluntarily describe weaknesses of their listed products, both via cheap-talk messages such as "*the product is in fair condition*", and via verifiable information such as pictures of specific damages and scratches that are almost indiscernible otherwise. Jin and Kato (2006) have found that many eBay sellers of collectible baseball cards revealing low grades were actually honest about their claims, even though most buyers could not correctly evaluate the quality both prior to and after the purchase.

There are some interesting aspects about these phenomena as follows. First, voluntary honesty of sellers may not simply originate from their reputation concerns. For example, sellers at Craigslist often reveal negative information about their listings even though there is no reputation building mechanisms (like the one at eBay.com). Most sales at the Craigslist are one-time interactions with no repeated-game incentives of being honest, and customers may not find out that there is anything wrong with the product even after purchase. Second, voluntary honesty may provide some incentives that contribute to sellers' bottom line, unlike many people's common beliefs described above. For example, although Hans Brinker Hotel actively communicates negative aspects of their services, many travelers visiting Amsterdam choose to stay at this hotel and leave positive reviews such as "For the reputation of the world's worst hotel, it wasn't as bad as I thought."[3] Moreover, according to Craig Berman, Amazon's vice president of global communications, honest revelation of negative information through product reviews actually turned out to have positive effects on their performances despite initial fears.[4]

Although the phenomenon of sellers' honesty is not as uncommon as one would expect and also presents some interesting questions, it has not received much attention in the academic literature especially from theoretical perspective, as the extensive literature on information disclosure has focused primarily on the tension between consumers who want more information on quality and low-quality sellers who would like to hide it (Dranove & Jin, 2010). This paper thus attempts to fill this gap by investigating the incentives of low-quality sellers to disclose, rather than conceal,

[1] https://www.bloomberg.com/news/articles/2013-06-18/the-genetically-modified-burrito-chipotle-tells-all

[2] `http://www.woot.com/faq?ref=ft_wiw_faq`. Archived version: `https://archive.is/b9cxI`

[3] http://abcnews.go.com/Travel/proud-worlds-worst-hotels/story?id=17696356 (accessed on November 14, 2015). Archived version: https://archive.is/N9ZhL

[4] Webley, K. (2010, July 16). A Brief History of Online Shopping. Time. Retrieved from http://content.time.com/time/business/article/0,8599,2004089,00.html

negative information about their products.

For this purpose, we consider a model with the following characteristics. First, buyers cannot evaluate the quality of the products. In other words, there exists information asymmetry where the products are high in experience or credence attributes, and sellers thus do not have to worry that buyers may figure out negative aspects by themselves. Second, those standard tools that high-quality sellers can use to signal their quality are not available. For example, there are no repeated purchases, no reputation concerns and no warranties. The only tool available to sellers to affect buyers' beliefs about product's quality is cheap-talk messages. The cheap-talk setting limits the ability of high-quality sellers to separate, as any communication strategy they employ can be costlessly imitated by low-quality sellers, thereby shifting the focus to incentives of low-quality sellers' in any information transmission.

Under this setting, we identify two factors that can incentivize low-quality sellers to reveal their quality in our model. First, revealing one's quality, whether it is low or high, reduces the risk associated with the purchase, and increases buyers' willingness to pay. The literature has generally agreed that risk has a major negative influence on customers' purchase decisions (Bauer 1960; Dowling 1986; Markin, Jr. 1974; Ross 1975; Stone & Winter 1985; Taylor 1974). In an online setting, Dewally and Ederington (2006) showed empirically that risk reduction increases the valuation of listed products on online auctions. Second, revealing one's quality allows a seller to differentiate one's product from the competitor's, thereby softening the competition and increasing one's profits. Jin and Sorensen (2006) have shown that hospitals' decisions to disclose quality scores, either being positive or negative, are driven by incentives to differentiate themselves from competitors.

To separate the roles of these two factors — risk-reduction and softening competition via product differentiation — we first consider the setting with a monopolistic seller and then extend it by adding the second seller. The model's setup and timing are as follows. In the case of the monopolistic setting, the seller observes his type and sends a cheap-talk message. Buyers update their beliefs about the product's quality based on the received message, and then buyers' demand is determined by the updated beliefs. The seller sets the profit-maximizing price given the demand, and buyers decide whether to purchase the product. The duopoly setting is similar, except that there are two sellers. Sellers simultaneously send public cheap-talk messages, and then simultaneously set prices. Sellers are risk-neutral profit-maximizers while buyers are not risk-neutral and dislike uncertainty about product quality. We model buyers' risk-attitudes using two alternative frameworks: risk-aversion and loss-aversion. Buyers differ in their degree of loss- or risk-aversion. Other things being equal, buyers with higher (lower) loss/risk-aversion are more (less) likely to prefer the product with certain but lower quality over the product with uncertain but higher expected quality.

In our setting, sellers, when deciding whether to reveal their types or not, face a potential trade-off between a *quality effect* of pretending to be someone of a higher quality, and an *information effect* of disclosing information about their (lower) quality and removing uncertainty. A relative strength of these two effects determine whether revealing one's quality can occur in equilibrium. In the monopoly setting, we show that low-quality and only low-quality sellers can separate. High-quality sellers do not face trade-off between the quality and information effects as the separation would both reveal their high quality *and* remove quality uncertainty. Therefore, low-quality sellers always find it optimal to imitate them. For low-quality seller, on the other hand, this trade-off is non-trivial. Removing quality uncertainty reveals seller's low quality. As long as the benefits of information disclosure outweigh the benefits of a high-quality claim, it is possible to have equilibria where low-quality sellers separate.

In the second part of the paper, we study how incentives and opportunities to disclose quality information change in the duopoly environment with two sellers. It turns out that equilibria under the duopoly setting include outcomes that were not possible under monopoly setting. First, there are equilibria where the highest-quality type separates, and the low-quality type chooses not to imitate it. Second, in the case of risk-averse buyers, there are equilibria where the seller with the lowest quality separates. Both types of equilibria become possible because of the impact that competition has on the quality effect. Differently from the monopoly setting, sellers' profit depends not only on their expected quality but also on the competition intensity. If a message associated with higher quality is likely to result in a more intensive competition, it weakens the quality effect, which in turn weakens the incentives of low-quality sellers to pool with high-quality types. That allows for a possibility of high-quality sellers to separate in equilibrium and, in the case of risk-averse buyers, allows for an equilibrium where the lowest-quality seller chooses separation over pooling with higher-quality types.

Overall, this study contributes to the literature on information disclosure by focusing on the incentives of low-quality sellers to reveal negative information. First, we show that even when the ability to communicate one's quality is limited to cheap-talk messages, and when there are no market frictions, such as search or matching, information transmission is possible. Second, due to the limited ability of high-type sellers to communicate their quality, the information transmission is driven by incentives of low-quality sellers. Third, we identify two factors—buyers' risk-attitude and increased product differentiation—that can incentivize low-quality sellers to separate. We analyze the role of each factor on information disclosure and show how they interact with each other.

The paper is organized as follows. In Section 2 we review related literature. In Section 3 we introduce our benchmark model assuming monopoly, and in Section 4 we consider the effect on competition by examining the duopoly situation. We summarize and discuss the findings of this paper in Section 5. All proofs are provided in the appendix.

2 Literature Review

The literature on information disclosure and information asymmetry has been primarily centered on the analysis of conflicting interests of high-quality sellers who want to credibly communicate their quality information to customers, and low-quality sellers who want to hide it. Studies on unraveling, for example, have argued that full disclosure naturally begins from the seller with the highest quality and advances to the sellers with lower quality (Grossman 1981; Milgrom 1981; Viscusi 1978), without considering how low-quality sellers might initiate information disclosure.

Empirical literature has mainly examined how negative information hurts sellers, suggesting little basis for low-quality sellers' information disclosure. For example, many studies have shown that negative information decreases sales and purchase likelihood through publicity (Tybout, Calder & Sternthal 1981; Wyatt & Badger 1984), word-of-mouth (Arndt 1967; Engel, Kegerreis & Blackwell 1969; Haywood 1989; Laczniak, DeCarlo & Ramaswami 2001; Mizerski 1982; Wright 1974), and customer reviews (Basuroy, Chatterjee & Ravid 2003; Chevalier & Mayzlin 2006; Clemons, Gao & Hitt 2006; Dellarocas, Zhang & Awad 2007; Reinstein & Snyder 2005).

Our paper differs from that strand of literature and belongs to a smaller group of papers that study incentives of low-quality sellers to disclose quality information. Board (2009) has shown that in the framework with risk-neutral buyers where information disclosure is costless, credible, and verifiable, low-quality sellers may disclose their types if the loss from lower perceived value is smaller than the gain from decreased competition with high-quality sellers. Guo and Zhao (2009) have considered a duopoly setting where private information can be credibly and truthfully, though not costlessly, disclosed. In their setting, duopoly sellers consider voluntarily disclosing quality information even when the quality is not the highest, in order to achieve differentiation and avoid direct competition. Our paper has a similar trade-off between information disclosure and product differentiation, but in a different setting, where private information cannot be credibly communicated and buyers are not risk-neutral. In the setting where quality information is not verifiable, Gardete (2013) has applied a cheap-talk model to a market with a search good, where customers know the true quality before purchase. He shows that if customers differ in their marginal valuations for quality, then low-quality firms may want to reveal their types to attract those customers with low marginal valuations for quality. Kim (2012) has shown that, in the presence of search and matching frictions and when it is buyers who make offers, low-quality sellers can use cheap-talk messages to reveal their types in order to attract more buyers and intensify competition among them. Similar to Gardete (2013) and Kim (2012), we assume that quality information is communicated via cheap-talk messages. However, in our paper, buyers cannot learn quality prior to purchase, and there are no search and matching frictions. Finally, Tadelis and Zettelmeyer (2015) have performed a large-scale field experiment in wholesale automobile auctions and proved that disclosure of negative information can increase the revenue of sellers through matching buyers with different quality preferences to appropriate markets.

Our paper is also related to the literature on cheap-talk communication between customers and sellers when their incentives do not align, information disclosure is payoff-irrelevant, and there is no credibility cost (Aumann & Hart 2003; Gardete 2013; Li 2005; Yi Zhu & Dukes 2015). Various instruments, such as properly termed revenue-sharing contracts (Li 2005) and advertising (Gardete 2013), have been found to match the incentives of sellers and buyers, thereby making the cheap-talk communication credible. In our setting, the information transmission also occurs via cheap-talk messages, and the incentives of sellers and buyers are originally misaligned as sellers want to hide weaknesses of their products while buyers would like to have that information. However, the consideration of risk and risk-attitudes of customers play the role of matching buyers' and sellers' incentives and making credible information disclosure mutually beneficial. Therefore, our paper contributes to this literature by suggesting that consideration of risk can facilitate cheap-talk communications by aligning the incentives of both parties.

3 Monopoly

3.1 Basic setup

We first consider a model with a seller that sells a product with exogenously given quality that is unobserved by buyers. The quality is distributed with a cdf $F(v)$ on an interval $[v_L, v_H]$ and the distribution can be either discrete, or continuous with a positive density on $[v_L, v_H]$. The marginal cost of the product with quality v is c_v. Unless explicitly stated otherwise, we assume that c_v and $v - c_v$ are strictly increasing functions of v. If the seller of type v serves share s_v of buyers at price p_v, his expected profit is

$$U_v = (p_v - c_v)s_v.$$

There is a continuum of buyers with unit demand that we normalize to 1. Buyers' utility is determined by the price, p, at which they purchase the product, as well as their beliefs about the product's quality, μ. Buyers are not risk-neutral and dislike uncertainty about product quality. We model buyers' risk-attitude using two alternative frameworks. The first framework assumes that buyers are loss-averse with the references point endogenously determined by the expected quality. Among a variety of reference-dependent models with endogenous reference points (see e.g. Gul, 1991; Shalev, 2000; or Köszegi & Rabin, 2006) the framework that we employ in the paper *"... has proven quite popular in applications, as the reference point is neither stochastic nor recursively defined, but is simply the expected consumption utility of the lottery."* (Masatlioglu and Raymond, 2016, p. 2765).[5] Specifically, if, given beliefs μ and price p, the buyer purchases the product of quality v, then his utility is

$$u_b(v, p, \mu) = \begin{cases} v - p & \text{if } v \geq E_\mu v \\ v - p + b(v - E_\mu v) & \text{if } v < E_\mu v \end{cases}, \tag{1}$$

where $E_\mu v$ is a reference point that determines whether the outcome is viewed as a gain or a loss. Parameter $b \geq 0$ measures the degree of loss-aversion. If the purchased product has the quality below what the buyer had expected, then the buyer experiences the loss. When $b = 0$, the buyer is risk-neutral. Higher b means higher degree of loss-aversion. In what follows, we assume that buyers

[5]To see how our framework is related to Köszegi & Rabin (2006), consider a simple case of two qualities (v_L, v_H) with probabilities (q_L, q_H). Define the reference point as getting the product with quality Ev with probability 1. Ignoring price p, for the sake of example, the agents' utility from getting the product is $Ev + q_L(\lambda - 1)(v_L - Ev)$. Here λ corresponds to notations from Köszegi & Rabin (2006), and is the slope in the domain of losses. According to notations in our paper $\lambda - 1 = b$. Expression $Ev + q_L(\lambda - 1)(v_L - Ev)$ is exactly the equation (2) in our paper. The difference between preferences used in our paper and Köszegi & Rabin (2006) is in the utility of not buying the product. In our paper it is 0. For the KR-agents, utility from not buying is not zero but $-\lambda Ev$. Thus for the KR preferences, it is a personal equilibrium to have reference point "quality Ev with probability 1" and decision "to buy" whenever $Ev + q_L(\lambda - 1)(v_L - Ev) > -\lambda Ev$. In our paper, buyers buy whenever $Ev + q_L(\lambda - 1)(v_L - Ev) > 0$.

differ in their degree of loss-aversion. We assume that b is distributed with a positive differentiable log-concave density $\phi(b)$ and support $[0, B]$. The last remark is that, as equation (1) indicates, all buyers value quality equally and have the same price-quality trade-off. In this sense our model is that of a horizontal differentiation where buyers differ in their expected quality vs risk trade-off, and not of a vertical differentiation where buyers differ in their price-quality trade-off.

Taking expectations of (1) over v we get

$$U_b^{LA}(p, \mu) = E_\mu v - p + b \int_{v_L}^{E_\mu v} (v - E_\mu v) f(v) dv = E_\mu v - p - b \cdot ELoss_\mu, \tag{2}$$

where $ELoss_\mu = -E[(v - E_\mu v) \cdot \mathbf{1}_{v<E_\mu v}] > 0$ is the buyer's expected loss, which is defined to be positive.

The second framework is the expected utility framework with risk-averse buyers. Buyers have concave Bernoulli utility function, $u(\cdot)$, with constant absolute risk-aversion. Buyers' utility from purchasing a product with uncertain quality v at price p is

$$U_b^{EU}(p, \mu) = E_\mu u(v - p). \tag{3}$$

Given the CARA assumption, we do not need to specify the initial wealth. Buyers differ in the degree of absolute risk-aversion, γ. With a slight abuse of notations, we will denote the distribution of buyers' risk-aversion as $\Phi(\gamma)$. Density $\phi(\gamma)$ is assumed to be positive, differentiable, and log-concave.[6]

Sellers can communicate their product's quality to buyers using cheap-talk message, $m \in \mathcal{M}$. We assume that $\mathcal{M}$ is rich enough that it includes the support of $F(v)$. The timing is as follows. First, the seller learns his type, v. Second, the seller sends a publicly observable cheap-talk message $m \in \mathcal{M}$. Third, buyers observe the message and form posterior beliefs $\mu(m)$ about the quality distribution, which determines their demand for the seller's product. Fourth, the seller chooses the price p, and, finally, buyers decide whether to purchase the product or not.

Definition 1 *An equilibrium is a quadruple $(m(v), p(m, v), \mu(m), s(\mu, p))$ where $m(v)$ is the seller's messaging strategy, $p(m, v)$ is the seller's pricing strategy, $\mu(m)$ is the buyers' beliefs about the quality distribution, and $s(\mu, p)$ is the share of buyers who purchase the product such that the following conditions hold:*

a) given m and buyer's demand $s(\mu(m), p)$, the seller of type v chooses price, $p(m, v)$, that solves his profit-maximization problem:

$$\max_p (p - c_v) s(\mu(m), p);$$

[6] Both loss-aversion and expected-utility frameworks capture the idea that buyers receive dis-utility when the product's quality is uncertain. The loss-aversion framework has a simpler functional form, making it more tractable. However, just like many non-EU frameworks such as variance-aversion frameworks, the loss-aversion preferences can violate state-dominance. For example, when b is high, the buyer might prefer a product with known low quality over the product whose quality can be either low or high. It violates state-dominance as the latter option will result in either the same (low) quality as the former option, or in the better (high) quality. Yet, buyers with high b will prefer the former. Our results, however, are not based on the violation of state-dominance, as state-dominance holds in the expected-utility framework.

b) the seller of type v sends message m that maximizes his profit given buyers' beliefs and purchasing decisions

$$\max_{m \in \mathcal{M}} (p(m, v) - c_v) s(\mu(m), p);$$

c) buyers' purchasing decision is optimal, that is

$$s(\mu, p) = \Pr(\{U_b(p, \mu) \geq 0\});$$

d) if message m is sent with positive probability, buyers beliefs $\mu(m)$ are derived from $m(v)$ by the Bayes' rule.

Some remarks are due here. First, we assume that prices are determined after the message, and not jointly. This assumption is not essential in the case of a monopolistic seller. We do impose it for the sake of the similarity with the duopoly setting. Second, buyers' beliefs, μ, depend on cheap-talk message m only. The model is stripped away from standards mechanisms that are identified in the literature as a way for high-quality sellers to credibly signal their quality. There are no repeated purchases, warranties, reputation, and buyers do not use prices to infer a product's quality (though the latter requirement, as we argue below, is not essential for our results). Limiting the ability of high-quality sellers to signal their quality puts focus on the incentives of low-quality sellers who, if they choose to, can costlessly imitate any strategy pursued by high-quality sellers. If any information about low-quality product is revealed, therefore, it is driven not by high-quality sellers' ability to separate, as is the case in unraveling or education-as-signaling models, but by low-quality sellers' intent not to pool with high-quality sellers.

The assumption that only cheap-talk messages affect buyers' beliefs implies that buyers do not use prices to infer information about product's quality. Empirical literature has shown that the relationship between prices and *perceived* quality — that is, buyers' beliefs about the product's quality rather than the actual quality — is weak in products that are high in experience and credence attributes, which are the types of product that we focus on in the paper. Rao and Monroe (1989) and Lichtenstein and Burton (1989) have found that customers are not capable of predicting quality from price signal for durable, higher-priced, or non-frequently purchased products. Similarly, Vöckner and Hofmann (2007), in a meta-study that covered 23 papers on price-perceived quality relationship, have shown that durable goods as well as services — which are generally low in search attributes and high in experience and credence attributes — have much weaker price-perceived quality relationships than fast-moving consumer goods.

From a theoretical point of view, a side effect of this assumption is that given the same message types with lower cost (and lower qulaity) find it optimal to charge lower prices, yet buyers infer quality information from messages only, and their quality's perception is not affected by prices. To show that our results are not driven by this assumption on price-perceived quality relationship, even though it has empirical support, in Appendix B we develop an alternative framework where it is prices that are informative signals about the product's quality. The results are not qualitatively affected.[7]

[7] To be specific, in Appendix B we consider a similar framework to the one in the main part of the paper except that

3.2 Equilibrium Analysis

We now begin analysis of our framework. As a benchmark, we first look at the case of risk-neutral buyers. Proposition 1 shows that there is no equilibrium with any relevant information being transmitted, where by relevant information we mean information that changes buyers' valuation of the product. This proposition is trivial, so no formal proof is provided. Intuitively, if two on-equilibrium messages result in different beliefs about expected quality, then no seller would find it optimal to send the message with lower expected quality.

Proposition 1 *If buyers are risk-neutral, then for any on-equilibrium message m, posterior beliefs, μ, are such that $E_{\mu(m)}v = Ev$. For any equilibrium message, m, the seller, regardless of the product's quality, charges price $p = Ev$ and serves the whole market, $s(\mu, p) = 1$.*

Consider now a case when buyers are not risk-neutral. We will classify messages in $\mathcal{M}$ as follows. Any on-equilibrium message $m \in \mathcal{M}$ that results in deterministic beliefs $Pr(v = v_s|m) = 1$ for some type v_s will be called a *separating* message. Any on-equilibrium message that does not result in deterministic beliefs, i.e., any message that is sent by more than one type, will be called a *pooling* message. Sellers' messaging strategy, $m(v)$, can be mixed, i.e., sellers can randomize over messages in $\mathcal{M}$. We say that a seller with quality v_s *separates* if he sends a separating message with positive probability. Otherwise, we say that a seller *pools.*

It is straightforward to derive the buyers' demand and the seller's optimal price as a function of buyers' beliefs. We will do it for the case of loss-averse buyers, and the case of risk-averse buyers is similar. If a seller with quality v sends a separating message m_s, then buyers no longer face uncertainty about the seller's quality. Buyers believe (correctly in equilibrium) that product's quality is v with probability 1. The seller will set price $p = v$, all buyers will purchase the product, and the seller will earn the profit of $v - c_v$. If a seller with quality v sends a pooling message m_p, then the customer indifferent between purchasing and not has loss-aversion b^0 such that:

$$E_{\mu(m_p)}v - p - b^0 ELoss_{\mu(m_p)} = 0,$$

where $\mu(m_p)$ are buyers' beliefs about the quality distribution conditional on m_p. Only buyers with $b < b^0$ will purchase the product, and therefore the demand function is given by $\Phi\left(\dfrac{E_{\mu(m_p)}v - p}{ELoss_{\mu(m_p)}}\right)$.

instead of cheap-talk messages sellers use prices to signal their quality. As a solution concept we use Sender-preferred subgame perfect equilibrium (see e.g. Kamenica and Gentzkow, 2011) which, in addition to standard requirements, assumes that if a Receiver is indifferent between some actions at a given belief, he takes an action that maximizes Sender's expected utility. In our model it means that when buyers are indifferent between purchasing and not they purchase. It changed results as follows. Proposition 1 did not change. In Proposition 2 it is always the case that $\bar{v} = v_L$, i.e. there exist $v^1, \ldots, v^{N+1}$ such that $v_L = v^1 \leq v^2 < \cdots < v^{N+1} = v_H$ where all sellers with $v \in (v^i, v^{i+1})$ choose the same price with probability 1. Proposition 3 changes in that now *only* type v_L can separate. Proposition 4 is not affected. In the duopoly section, which is a notably more complex framework, all the Propositions are proved for the case of constant costs. Therefore, the issue of different types charging different prices does not arise in the duopoly part of the paper.

The optimal price for type v is then determined from $\max_p \Phi\left(\frac{E_{\mu(m_p)}v - p}{ELoss_{\mu(m_p)}}\right)(p - c_v)$.

The next Proposition characterizes properties of equilibrium's messaging strategy in the case of loss-averse buyers. There exists threshold $\bar{v}$ such that types with higher quality, $v \geq \bar{v}$, send messages according to the partition $v_L \leq \bar{v} = v^1 < \cdots < v^{N+1} = v_H$. All sellers in the interval $[v^i, v^{i+1}]$ send the same message m_i, and for sellers with $v \in (v^i, v^{i+1})$ message m_i is strictly optimal. All types with $v < \bar{v}$ are indifferent between on-equilibrium messages they send. More precisely, if type $v' < \bar{v}$ sends message m' and type $v'' < \bar{v}$ sends message m'' then, in fact, both v' and v'' are indifferent between m' and m''. In equilibrium, all types with $v < \bar{v}$ serve the whole market and charge the same price. Otherwise a seller charging a lower price could profitably deviate. When it is not possible to serve the whole market, i.e. when B is sufficiently high, then $\bar{v} = v_L$, i.e., the equilibrium messaging strategy has the interval structure over the entire support, $[v_L, v_H]$. The last part of Proposition 2 is to show that all prices charged by sellers in $[v^{i-1}, v^i]$ are lower than prices charged by sellers in $[v^i, v^{i+1}]$. That is, even though buyers do not infer quality from prices, in equilibrium one cannot have a seller claiming "*my quality is in interval* $[1, 2]$" charging a price higher than a seller claiming "*my quality is in interval* $[2, 3]$".

The proof of Proposition 2 relies on the fact that consumers' demands, given different buyers' beliefs, satisfy the single-crossing condition. The proof does not depend on whether the quality distribution is discrete or continuous. That is, nowhere in the proof it is assumed that type $\bar{v}$ or thresholds $\{v_i\}$ belong to the support of the quality's distribution $F(v)$.

Proposition 2 *Assume that buyers are loss-averse. Assume that no two messages m and m' result in beliefs such that $E_{\mu(m)}v = E_{\mu(m')}v$ and $ELoss_{\mu(m)} = ELoss_{\mu(m')}$. Let $\bar{v}$ be the highest quality type that with positive probability serves the whole market, if it exists, and let $\bar{\mathcal{M}}$ be the set of equilibrium messages sent by types with $v \leq \bar{v}$. Then*

i) all types $v \leq \bar{v}$ charge the same price, serve the whole market, and are indifferent between any message from $\bar{\mathcal{M}}$;

ii) there exists $v^1, \ldots, v^{N+1}$ such that $v_L \leq \bar{v} = v^1 < v^2 < \cdots < v^{N+1} = v_H$, where the number of intervals, N, is either finite or countable, and all sellers with $v \in (v^i, v^{i+1})$ send the same message with probability 1.

iii) for any v' and v'' such that $\bar{v} \leq v' < v''$, type v' will charge a strictly lower price.

In this framework, the set of possible equilibria is very rich. Proposition 2 can be used to characterize what kind of information can be transmitted in equilibrium, and to derive conditions that guarantee existence of a particular type of equilibrium. To see the application of Proposition 2, consider the case of three types $v_L < v_M < v_H$ with costs $c_L < c_M < c_H$. When $\bar{v} = v_L$ one can show that, depending on parameters, the following two outcomes are possible in equilibrium. Under the first outcome there are three on-equilibrium messages: message L which is sent by type v_L only and is, therefore, a separating message; message M which is sent with positive probabilities by types v_L and v_M; message H which is sent with positive probabilities by types v_M and v_H. In notations of Proposition 2: $v^1 = v_L$, $v^2 = v_M$ and $v^3 = v_H$. Under the second outcome there are two on-equilibria messages: message L which is sent by type v_L only and is, therefore, a separating message; and message H which is sent with positive probabilities by type v_L and with probability 1 by types v_M and v_H. In notations of Proposition 2: $v^1 = v_L$, $v^2 = v_H$.[8] In addition, there exist an equilibrium where $\bar{v} = v_M$ and there are two on-equilibria messages: message M that is sent by type v_M only; message H that is sent by types v_L and v_H with probability 1, and by type M with non-negative probability. In notations of Proposition 2: $v_L < \bar{v} = v^1 < v^2$, where $v^1 = v_M$ and $v^2 = v_H$.

It is straightforward to find conditions on parameters such that each of the three equilibrium outcomes exists. As an example, we will show how to derive conditions on parameters for the last outcome, though similar reasoning can be applied for the other two outcomes as well. Let $0 = v_L < v_M < v_H$ and $c_L = 0 < c_M < c_H$. Assume that each quality level has equal probability of 1/3. Furthermore, assume that $v_M < Ev$, that is medium quality is below average. As Proposition 3 will show this condition is necessary for the equilibrium to exist. In our example, condition $v_M < Ev$ is equivalent to $2v_M < v_H$. Buyers are loss-averse with $b \sim U[0, B]$. We are interested an equilibrium where v_M separates and there are two on-equilibria messages: message M that is sent by type v_M only; message H that is by types v_L and v_H with probability 1 and by type M with non-negative probability.

In this equilibrium, the posterior belief given H is $\mu(H) = (\sigma, \tau, \sigma)$ where $\sigma = \Pr(v = v_L|H) = \Pr(v = v_H|H)$. Given the prior, $1/3 \leq \sigma \leq 1/2$. For example, if v_M sends M with probability 1 then $\sigma = 1/2$; if v_M sends M with probability 0 then $\sigma = 1/3$. The posterior belief given M is $\mu(M) = (0, 1, 0)$.

Conditional on M, type v_M sets optimal price $p_M = v_M$ and serves the whole market earning profit $v_M - c_M$. To calculate optimal price given H, we first calculate buyers' demand, which is determined by $ELoss_\mu$ and Ev_μ. It is immediate to calculate $E_\mu v = \sigma v_L + \tau v_M + \sigma v_H = v_M + \sigma(v_H - 2v_M)$ and

$$\begin{aligned} ELoss_\mu &= -[\sigma(v_L - E_\mu v) + \tau(v_M - E_\mu v)] \\ &= \sigma[(v_H - v_M) - \sigma(v_H - 2v_M)]. \end{aligned}$$

When calculating $ELoss_\mu$ we used that $v_L = 0$ and that $v_M < Ev$, which implies that $v_M < E_\mu v$ and, therefore, buyers experience loss if the product's quality is v_M. Given H, the demand function faced by a seller is

$$s(\mu, p) = \frac{1}{B} \frac{E_\mu v - p}{ELoss_\mu} = \frac{1}{B} \frac{v_M + \sigma(v_H - 2v_M) - p}{\sigma[(v_H - v_M) - \sigma(v_H - 2v_M)]}, \tag{4}$$

if $E_\mu v - p < B \cdot ELoss_\mu$, and $s(\mu, p) = 1$ if $E_\mu v - p \geq B \cdot ELoss_\mu$. The optimal price given μ is

$$p^* = \frac{E_\mu v + c_v}{2}, \tag{5}$$

if $s(\mu, p^*) < 1$, and $p^* = E_\mu v - B \cdot ELoss_\mu$ otherwise.

[8] In the case of three types the thresholds have to coincide with the qualities in the support of $F(v)$, though, in general it is not the case.

In notations of Proposition 2 set $\bar{\mathcal{M}} = \{M, H\}$, and, given that $\bar{v} = v_M$ both v_L and v_M should be indifferent between both messages. More precisely, given buyers' beliefs, $\mu(M)$ and $\mu(H)$, both types should find it optimal to serve the whole market and they should charge the same price after either message.

Consider message M. Since $\mu(M) = (0, 1, 0)$, there is no quality uncertainty. Both sellers will find it optimal to charge the same price v_M and will serve the whole market. By Proposition 2 i) it must be also the case that both types will find it optimal to charge the price v_M given message H, and that the whole market is served when $p = v_M$ and beliefs are $\mu(H)$. Type v_M has a higher cost, so it is sufficient to verify the two conditions for v_M only. Given μ, the highest price to serve the whole market is $p_w = E_\mu v - B \cdot ELoss_\mu$. Thus, price $p = v_M$ is optimal for the medium-quality seller given H and will result in the seller serving the whole market when

$$\sigma(v_H - 2v_M) \leq v_M - c_M. \tag{6}$$

From $E_\mu v - B \cdot ELoss_\mu = v_M$, we can solve for σ, which is equal to

$$\sigma = \frac{v_H - v_M}{v_H - 2v_M} - \frac{1}{B}, \tag{7}$$

and by plugging it into (6) we get

$$c_M \leq (v_H - 2v_M)\frac{1 - B}{B}. \tag{8}$$

It is easy to verify that if parameters are such that $\sigma \in [1/3, 1/2]$ and (8) is satisfied then an equilibrium where type v_M separates exists. We already established that v_L and v_M are indifferent between pooling and separating. The only thing left is to show that it is optimal for v_H to pool. If v_H deviates and sends message M the optimal price is $p = v_M$ and the profit is $v_M - c_H$. If v_H sends message H, it can guarantee itself the profit of $v_M - c_H$ by setting $p = v_M$. Thus, it is at least weakly optimal for v_H to send H. Note that for this particular outcome to be an equilibrium B can be neither too high nor too low. It cannot be too high as otherwise too many loss-averse buyers would make separation attractive for other types. It cannot be too low since otherwise separation is not profitable.

3.3 Equilibria with Separation

As mentioned earlier, the set of equilibria in this framework is very rich. In what follows, we will focus on equilibria which satisfy an additional property that there exists a type that separates with a positive probability, just like in the example considered in the previous section.

Proposition 3 shows that in any equilibrium at most one type can separate, and, if a seller separates, his quality must be below average. The first part is trivial. If two types, $v_1 < v_2$, separate then type v_1 will optimally deviate and imitate v_2. To see the intuition behind the second part notice that high- and low-quality sellers differ in their incentives and ability to separate. When low-quality sellers, those with quality below average, separate, they face the trade-off between the positive *information effect* of removing quality uncertainty by disclosing quality information, and the negative *quality effect* of revealing one's low quality. High-quality sellers, on the other hand, do not face such a trade-off. For them, if they can separate, both the information and the quality effects are positive. But then their separation is impossible in an equilibrium as there will always exist a low-quality type that would find it profitable to imitate the separating high-quality seller.

The next proposition formalizes this argument. It holds for both loss- and risk-averse buyers.

Proposition 3 *In equilibrium, at most one type separates. There is no equilibrium where type $v > Ev$ separate.*

Consider an equilibrium where type $v_s < Ev$ separates. Since $v_s < Ev$ there must exist an on-equilibrium message m, and corresponding beliefs μ, such that $E_\mu v \geq Ev > v_s$. If the seller with quality v_s separates, he gets profit of $v_s - c_s$. It should be weakly higher than the profit from pooling with other sellers. If type v_s sends message m and charges price $p^0 = E_\mu v - B \cdot ELoss_\mu$ he will serve the whole market — since the most loss-averse buyer is indifferent between purchasing at p^0 and not — and will earn profit of $E_\mu v - B \cdot ELoss_\mu - c_s$. For separation to be optimal, it has to be the case that $E_\mu v - B \cdot ELoss_\mu - c_s \leq v_s - c_s$. Taking into account that $Ev \leq E_\mu v$ we have that $B \geq \dfrac{Ev - v_s}{ELoss_\mu}$. Let $MLoss$ be the highest expected loss possible. The maximum exists due to the interval structure of equilibria and boundedness of $[v_L, v_H]$. Then a necessary condition for equilibrium where v_s separates to exist is $B \geq \dfrac{Ev - v_s}{MLoss}$. This necessary condition puts a lower boundary on B, and, notably, lower v_s requires higher B. In order for type v_s to be willing to separate the positive information effect of separation should outweigh benefits of pooling with higher types. For lower v_s the benefits of pooling with higher types are greater. To outweigh it, the information effect that comes with separation must be greater as well which requires that there must be buyers with sufficiently high-degree of loss-aversion.[9]

Up until now all the results we derived held for both loss-averse and risk-averse cases. One instance, however, where the two frameworks differ is whether the lowest quality type can separate in equilibrium or not. In the case of the loss-averse buyers there exist equilibria where the lowest quality type separates. In the case of risk-averse buyers, however, such equilibria do not exist. To illustrate this point, we first present two examples of equilibria with loss-averse buyers where v_L separates.

Example 1 *Let $v_L = 1, v_M = 2$ and $v_H = 3$, and $c_L = 1/4, c_M = 1/2, c_H = 3/4$. Let $Pr(v = v_L) = 1/2$, $Pr(v = v_M) = Pr(v = v_H) = 1/4$, and $b \sim U[0, 147/16]$. In an equilibrium v_L mixes between separating message, $m = L$, and pooling message, $m = H$, with equal probabilities. Types v_M and v_H send message $m = H$ with probability 1.*

Example 2 *Let $v_L = 1, v_M = 2$ and $v_H = 3$ and $c_L = 0, c_M = 1/2, c_H = 3/4$. Let $Pr(v = v_L) =$*

[9] As the example from the previous section have shown sometimes one might need to impose an upper bound on B as well. This might be necessary to make sure that the information effect is not too strong so that other types would not imitate the separating type.

$0.194, Pr(v = v_M) = 0.473$ and $Pr(v = v_H) = 1/3$. Finally, let $b \sim U[0, 10]$. There are three equilibria messages: L, M and H with message L being a separating message. In this equilibrium type v_L sends message L with probability 0.82 and message M with probability 0.18. Type v_M sends message M with probability 2/3 and message H with probability 1/3. Finally, type v_H sends message H with probability 1. If we interpret message $i \in \{L, M, H\}$ as "my quality is v_i" then in equilibrium type v_L is indifferent between revealing his quality and pretending to have medium quality, type v_M is indifferent between being honest about his quality or pretending to be of a high quality, type v_H is honest about his quality with probability 1.

Next, we prove that with risk-averse buyers, type v_L cannot separate in equilibrium. Proof by contradiction. Assume the lowest type separates with message m_L. It then charges price $p = v_L$ and earns profit $v_L - c_L$. Consider another on-equilibrium message, call it m, that generates beliefs μ. Given μ, the product has a positive probability of having quality above v_L and, by definition of v_L, zero probability of having quality below v_L. Thus $E_{\mu(m)}u(w + v - v_L) > u(w)$, where w is initial wealth. It then turns out that type v_L has a profitable deviation from equilibrium message m_L. The seller can send message m and charge the price $v_L + \varepsilon$ where ε is sufficiently small. Just as in the case of sending separating message m_L, the seller will serve the whole market but at a higher price. Indeed, for any risk-averse buyer $E_{\mu(m)}u(w + v - v_L - \varepsilon) > u(w)$ as long as ε is small enough, so that all buyers prefer purchasing the product with uncertain quality at price $v_L + \varepsilon$ over not purchasing it. Then the seller's deviation profit is $v_L + \varepsilon - c_L$ and the deviation is profitable.

This establishes the following proposition.

Proposition 4 *Let v_L be the lowest possible quality. When buyers are loss-averse, there exist parameter values such that there exists an equilibrium where the seller with v_L separates. When buyers are risk-averse, there is no equilibrium where the seller with v_L separates.*

The difference between loss-aversion and risk-aversion cases comes from the fact that risk-aversion respects state-dominance and loss-aversion does not. Risk-averse buyers will always have a higher willingness to pay for a product whose quality can be either v_L or better than for a product with certain quality of v_L. This is why it is never optimal for v_L to separate. For loss-averse buyers, on the other hand, if degree of loss-aversion is high enough, a buyer can have higher willingness to pay for a product of certain quality v_L over a product that can be either v_L or better. That makes separation of v_L possible in equilibrium.

4 Duopoly

4.1 Basic Setup.

In this section, we extend the monopoly framework by assuming that there are two sellers on the market. The seller's quality is exogenously given and is pure private information, i.e., it is unobserved by buyers and by the other seller. The quality distribution is the same for both sellers and is given by a cdf $F(v)$. The distribution can be either discrete or continuous with a positive density on $[v_L, v_H]$.

The game has three stages. The first stage is the messaging stage, where both sellers simultaneously send costless messages (m_i, m_j) that are publicly observed. The second stage is the pricing stage. Given (m_i, m_j), sellers simultaneously determine prices for their products (p_i, p_j). The third stage is the purchasing stage. Buyers observe messages and prices of both sellers and choose which product to purchase. Utility of buyers and sellers is the same as in the monopoly case. Sellers are risk-neutral maximizers of their expected profit, and buyers can be either loss- or risk-averse. Furthermore, we assume that buyers' valuation of the product is high enough that they always purchase a product. The equilibrium in this model is defined as follow.

Definition 2 *An equilibrium is a quadruple of sellers' messaging and pricing strategies:* $(m_i(v_i)$, $m_j(v_j)$, $p_i(m_i, m_j, v_i)$, $p_j(m_i, m_j, v_j))$*; buyers' beliefs* $(\mu_i(m_i), \mu_j(m_j))$*; and buyers' purchasing strategies* $(s_i(\mu_i, \mu_j, p_i, p_j)$, $s_j(\mu_i, \mu_j, p_i, p_j))$ *such that*

a) messaging strategy $m_i(v_i)$ *maximizes seller i's profit given seller j's messaging and pricing strategies, and buyers' beliefs and purchasing strategies:*

$$\max_{m_i \in \mathcal{M}} E_{v_j}(p_i(m_i, m_j, v_i) - c_{v_i}) s_i(\mu_i, \mu_j, p_i, p_j);$$ [10]

b) pricing strategy $p_i(m_i, m_j)$ *maximizes i's profit given* m_i, m_j, *seller j's pricing strategy, and buyers' beliefs and purchasing strategies:*

$$\max_{p_i} E_{v_j}(p_i - c_{v_i}) s_i(\mu_i, \mu_j, p_i, p_j);$$

c) buyers purchasing decisions are optimal:

$$s_i(\mu_i, \mu_j, p_i, p_j) = \Pr(\{U_b(p_i, \mu_i) \geq U_b(p_j, \mu_j)\}),$$

and they always purchase a product: $s_i(\mu_i, \mu_j, p_i, p_j) + s_j(\mu_i, \mu_j, p_i, p_j) = 1$*;*

d) if message m_i *is sent with positive probability, buyers' beliefs on the quality of seller i,* $\mu_i(m_i)$*, are derived from* $m_i(v_i)$ *by Bayes' rule.*

As the definition of equilibrium reveals, we have made several simplifying assumptions. First, we assume that prices are determined after the message, not jointly. The assumption is reasonable as long as it is quicker to adjust pricing strategy so that sellers can react with their pricing decision to the type of information disclosed (Janssen & Teteryatnikova 2016). Second, we ignore participation constraints and assume that buyers' valuation is sufficiently high so that all buyers make purchases. Thus, sellers always directly compete with each other. Finally, as in the monopoly case, buyers do not use prices to update their beliefs about a seller's quality.

The general duopoly setup, as defined above, is quite complicated to analyze as it is a cheap-talk model with multiple senders and with a non-trivial subgame — the Hotelling model with incomplete

[10] For the sake of brevity, we omit arguments of pricing and messaging strategies.

information — that follows the messaging stage.[11] For our purposes, however, it will be sufficient to use a simpler setting. Throughout this section, we will assume that sellers have two quality types v_L and v_H, where $v_L < v_H$, and costs of both types are zero. The probability of the product being high-quality is q. There are two possible messages $\mathcal{M} = \{L, H\}$. We will use words "*low*" or "*high*" when referring to the actual quality; and labels L or H when referring to cheap-talk messages. For example, expression an H-seller will refer to a seller who sends message H, regardless of the actual product's quality.

4.2 Equilibria with Separation.

Similar to the monopoly framework, we will focus on equilibria with separation, i.e., equilibria where there exists at least one type that separates with a positive probability. In the monopoly setting, we identified two effects that affect benefits of separation. The information effect, which is that separation provides complete information about the product's quality for buyers and, therefore, increases, their willingness to pay. Second, the quality effect, which is that separation changes buyers' beliefs about the product's quality. For high-quality sellers the second effect is positive, for low-quality sellers it is negative. It turns out that competition introduces another factor that affects seller's incentives to separate. In addition to removing buyers' uncertainty about the product, separation affects the intensity of competition. This changes the relative strength of both the quality and information effects, as sellers' profit depends not only on buyers' beliefs about the product's quality but also on the competition intensity.

Consider, for example, an equilibrium where all types send the same messages. In this equilibrium, buyers' information about both products' quality is identical and they will purchase the cheapest product, resulting in intense Bertrand competition. In contrast, if seller i sends a separating message while seller j sends a pooling message, buyers have different beliefs about those two products, introducing product differentiation. Some buyers, those with a low degree of loss/risk-aversion, have stronger preferences for higher expected quality; other buyers, those with a high degree of loss/risk-aversion, have stronger preferences for better information about the quality. Therefore, buyers from the latter group are more likely to purchase seller i's product, even if it has lower quality or is more expensive. This product differentiation softens the competition, benefiting both sellers.

The duopoly setting allows for two outcomes that were not possible under the monopoly setting: in the case of risk-averse buyers there are equilibria where the lowest-quality seller can separate;

[11] First, as mentioned earlier, even though product's quality differs our model is that of a horizontal differentiation, and not a vertical differentiation. The reason is that all buyers have exactly the same price-quality trade-off and the differentiation comes from differences in buyers' attitude towards risk. That does not meat that the pricing subgame is identical to the Hotelling model but one can show that it is very similar. Second, as far as the Hotelling model with incomplete information goes, even in the simplest case of risk-neutral buyers the pricing subgame is a rather complex model of Bertrand competition with unknown costs. It is similar to the standard auction setting, and the symmetric case has been analyzed by Spulber (1995). The asymmetric case, which in our setting naturally arises after different messages, is similar to auctions with asymmetric bidders, where with few exceptions the explicit solution does not exist.

there are equilibria where the highest quality seller can separate. In both cases, low-quality sellers choose not to imitate high-quality sellers and instead disclose negative information.

Both outcomes can happen in equilibria because competition can make imitating high-quality sellers less attractive if it is likely to result in intense competition and low profit. That is, competition weakens the quality effect of pooling with high-quality types. At the same time, for such equilibrium to exist the information effect of separation has to be positive. That is, it cannot be the case that the H-seller outprices the L-seller out of the market. As we will show, it will impose conditions on how buyers' risk/loss-aversion is distributed. Intuitively, the market segment that the L-seller will compete for cannot be too populous but has to have strong preference for the L-product. The combination of these two conditions will make the H-seller unwilling to compete for that segment allowing the L-seller to earn positive profit. For example, in the case of the separation of the low-quality type such segment is most risk-averse buyers. As Proposition 6 will show for an equilibrium to exist there must be sufficiently risk-averse buyers but their share must be sufficiently low. The former condition guarantees that there are buyers who detest risk and have preferences for certain quality offered by the L-seller, the second condition guarantees that there are not too many of them so that it is suboptimal for the H-seller to compete for them.

4.2.1 Risk-Averse Buyers. Separation of the Lowest-Quality Type

Consider the following messaging strategy $m^*(v)$: a high-quality seller sends message H with probability 1, and a low-quality seller mixes between messages L and H with probabilities λ and $1-\lambda$. Given $m^*(v)$, message L is the separating message which is sent by the lowest-quality type, v_L. We will determine conditions when $m^*(v)$ is a messaging strategy in a symmetric equilibrium.

We solve for equilibrium using backward induction. First, we look at the purchasing stage. If both sellers send the same messages (L, L) or (H, H) then, from buyers' point of view, the two sellers are identical. Therefore, buyers will purchase the product with the lowest price. Consider now the purchasing decision given messages (L, H), and prices (p_L, p_H). The L-product has a certain quality of v_L. The H-product may be of low quality with probability $q_{LH} = Pr(v = v_L|H)$, or of high quality with probability $1 - q_{LH}$. Given m^*:

$$q_{LH} = \frac{(1-\lambda)(1-q)}{(1-\lambda)(1-q)+q}.$$

A buyer is indifferent between L- and H-products if

$$u(v_L - p_L) = q_{LH}u(v_L - p_H) + (1 - q_{LH})u(v_H - p_H),$$

or equivalently

$$e^{-\gamma^0(p_H - p_L)} - q_{LH} - (1 - q_{LH})e^{-\gamma^0(v_H - v_L)} = 0, \tag{9}$$

where γ^0 is the risk-aversion degree of an indifferent buyer. Given the CARA utility function, a buyer's initial wealth does not affect the indifference condition so we do not specify it here. Proposition 5 characterizes buyers' behavior in the (L, H)-case.

Proposition 5 *The indifference condition*

$$e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)},$$

has at most one solution $\gamma^0 > 0$. When the solution exists, all buyers with $\gamma > \gamma^0$ prefer an L-product while all buyers with $\gamma < \gamma^0$ prefer H-product.

Next, we consider the pricing stage. Suppose at the messaging stage both sellers sent the same messages (L, L) or (H, H). Then, as established earlier, buyers will purchase the cheapest product. Thus, the pricing equilibrium is for both sellers to charge $p_L = p_H = 0$. Consider now the (L, H)-subgame. From Proposition 5 follows that demand for the L-product is $(1 - \Phi(\gamma^0(p_L, p_H))$, and demand for the H-product is $\Phi(\gamma^0(p_L, p_H))$. The L-seller chooses price p_L to maximize $\max_{p_L}(1 - \Phi(\gamma^0(p_L, p_H)) \cdot p_L$, and the H-seller chooses price p_H to maximize $\max_{p_H} \Phi(\gamma^0(p_L, p_H)) \cdot p_H$. The corresponding first-order conditions are

$$-\phi(\gamma^0)\frac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_L + (1 - \Phi(\gamma^0)) = 0, \tag{10}$$

and

$$\phi(\gamma^0)\frac{\partial\gamma^0(p_L, p_H)}{\partial p_H}p_H + \Phi(\gamma^0) = 0. \tag{11}$$

Finally, at the messaging stage, a low-quality seller should be indifferent between L and H when the competitor plays the equilibrium strategy. This condition is

$$\lambda(1-q)\pi_{LL} + (1-\lambda(1-q))\pi_{LH} = \lambda(1-q)\pi_{HL} + (1-\lambda(1-q))\pi_{HH}. \tag{12}$$

Here, the LHS is the expected profit of the low-quality seller from sending message L and the RHS is the expected profit from sending message H; π_{m_i,m_j}, where $m_i, m_j \in \{L, H\}$, is the profit of seller i after messages m_i and m_j; $\lambda(1-q)$ is the probability that the competitor sends message L, and $(1 - \lambda(1-q))$ is the probability that the competitor sends message H. As we established earlier, $\pi_{LL} = \pi_{HH} = 0$. Notice that since both types have the same cost, the high-quality seller is also indifferent between L and H; therefore, message H is optimal for high-quality type. Combining equations (9), (10), (11) and (12), an equilibrium is determined by the following system of equations:

$$\begin{cases} e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}. \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \\ (1-\lambda(1-q))(1-\Phi(\gamma^0))p_L = \lambda(1-q)\Phi(\gamma^0)p_H. \end{cases} \tag{13}$$

As the next proposition shows, whether the solution to system (13) exists depends on the distribution of γ.

Proposition 6 *Assume that there are two quality types, and buyers are risk-averse with CARA utility function. Then the equilibrium where the lowest-quality type separates does not exist if* *i) $\Phi(\gamma)$ is a uniform distribution; or*

ii) $\Phi(\gamma)$ is a convex function.

The equilibrium where the lowest-quality type separates exists if

iii) $\Phi(\gamma)$ has infinite support.

Furthermore,

iv) for any concave $\Phi(\gamma)$ there exists $\alpha^0 > 0$ such that for any $\alpha \in (0, \alpha^0)$, if risk-aversion is distributed with cdf $\Phi(\alpha\gamma)$, the equilibrium where the lowest-quality type separates exists.

The intuition is as follows. Consider the pricing stage after messages (L, H). In terms of quality, the H-product is superior to the L-product, The L-product is guaranteed to be of low quality, while the H-product can be of either low or high quality. Regardless of γ, all risk-averse buyers have higher willingness to pay for the H-product. However, for those who are more risk-averse, the difference in willingness to pay between the H- and the L-products is smaller. Thus, the only way the L-seller can get a positive share of the market is by competing with the H-seller for risk-averse buyers with high γ (see also Proposition 5). The H-seller's willingness to compete for those buyers depends on two factors: a) how high γ can get; and b) how large the share of buyers with high γ is. If Γ is low or if there are too many buyers with high γ, then it is optimal for the H-seller to simply outprice the L-seller away, and serve the whole the market. In the first case, when Γ is low, even the most risk-averse buyers are not too concerned about quality uncertainty. In the second case, when there are sufficiently many buyers with high risk-aversion, it is suboptimal for H to choose not to serve them. But then from v_L's point of view, if either of the two conditions is satisfied, it is not optimal to send message L, as every pricing subgame results in zero profits. Thus, only when there are sufficiently risk-averse buyers *but* their share is sufficiently low, it is possible to have an equilibrium where v_L separates.

Proposition 6 captures that notion of having "*sufficiently risk-averse buyers but their share is sufficiently low*" using convexity and support of $\Phi(\gamma)$. For any $\Phi(\gamma)$ with infinite support, the equilibrium with the lowest-type separation exists. First, there are sufficiently risk-averse buyers. Second, there exists $\tilde{\gamma}$ high enough that $\phi(\gamma)$ can be made arbitrarily small for any $\gamma > \tilde{\gamma}$. The marginal benefit of decreasing p_H to serve buyers with $\gamma > \tilde{\gamma}$ then will be too small due to low increase in the market share. In equilibrium L and H will split the market.

For uniform or convex cdfs, no matter how high Γ is, $\phi(\gamma)$ never approaches to zero for high values of γ. Marginal decrease in p_H to attract buyers with high-risk aversion is always optimal, and seller L will be priced out of the market. With concave distributions, more buyers have a low degree of risk-aversion. Concavity alone, however, is not enough to guarantee the existence. One also needs to have $\phi(\gamma)$ to be sufficiently small when γ is close to Γ. One way to do this is to stretch $\Phi(\gamma)$ to a larger support, and Proposition 6 offers one way how it can be achieved. Notably, as the next example shows, one does not need unrealistic levels of risk-aversion for the equilibrium to exist.

Example 3 *Let the degree of risk-aversion γ be distributed with $\Phi(\gamma) = \sqrt{\gamma}$ on $[0, 1]$. Let $v_H - v_L =$*

3 and $q \approx 0.484$.[12] *One can verify that the following is equilibrium: $\gamma^0 \approx 0.713$, $p_L \approx 0.184$, $p_H \approx 1$, $\lambda \approx 0.063$ and $q_{LH} = 1/2$.*

In this example, the probability of buying from a low-quality seller is slightly above 1/2, $1 - q \approx 0.52$. The low-quality seller reveals the negative information with probability 6.3%. If one seller announces L and the competitor announces H then both prices are above the marginal cost and both sellers make positive profits. The indifferent buyer is located at $\gamma^0 \approx 0.713$. The market share served by the L-seller is $1 - \sqrt{\gamma^0} \approx 15.6\%$, and the market shares served by the H-seller is 84.4%. In the (L, H) subgame, sellers profits are $\pi_H \approx 0.84$ and $\pi_L \approx 0.03$.

It is worth highlighting that it is the competitive environment that makes it possible for the lowest-quality seller to separate. As argued earlier, in the monopoly case with risk-averse buyers, it is never optimal for v_L to separate, as he is guaranteed to get higher profit by pooling with the high-quality sellers. In a symmetric equilibrium of duopoly case, if types v_L of sellers i and j pool with v_H with probability 1, they earn zero Bertrand profit. This makes pooling with high-quality sellers less attractive. By separating with positive probability, type v_L creates product differentiation, which allows it to earn a positive profit.

4.2.2 Separation of the High-Quality Type

In this section, we assume that buyers are loss-averse. We will derive conditions for existence of an equilibrium where the high-quality seller separates, and we will show that these conditions are conceptually similar to those derived in the previous subsection.

Consider the following messaging strategy $m^*(v)$: a high-quality seller randomizes between sending messages L and H with probabilities λ and $1 - \lambda$, and a low-quality seller sends message L with probability 1. Given $m^*(v)$, message H is the separating message which is sent by the highest-quality type v_H only. We will determine conditions when $m^*(v)$ is a messaging strategy in a symmetric equilibrium.

As before, buyers will purchase the product with the lowest price after (L, L) or (H, H) messages, so that both sellers charge prices equal to the marginal cost and earn zero profit. Consider now the purchasing decision given message profile (L, H) and prices (p_L, p_H). The H-product has a certain quality of v_H. The L-product may be of low quality with probability $q_{LL} = Pr(v = v_L|L)$, or of high quality. Given m^*, q_{LL} is equal to $\dfrac{1-q}{1-q+q\lambda}$. The loss-aversion of the indifferent buyer is given by

$$v_H - p_H = q_{LL}v_L + (1 - q_{LL})v_H - p_L + b^0 q_{LL}(v_L - q_{LL}v_L - (1 - q_{LL})v_H).$$

Buyers with $b > b^0$ will purchase from the H-seller and buyers with $b < b^0$ will purchase from the L-seller, so that the profit of the H-seller is $(1-\Phi(b^0))p_H$, and of the L-seller is $\Phi(b^0)p_L$. Combining the buyers' indifference condition and the FOCs for L and H-sellers, we get

[12] Numerically it is simpler to start with q_{LH} instead of q. The equilibrium in this example was calculated based on $q_{LH} = 1/2$. Values of q and λ were then calculated from (γ^0, p_L, p_H) and q_{LH} using the fourth equation of (13).

$$\begin{cases} b^0 = \dfrac{p_H - p_L}{q_{LL}(1-q_{LL})\Delta v} - \dfrac{1}{1-q_{LL}} \\ \phi(b^0)\dfrac{\partial b^0(p_L,p_H)}{\partial p_L} p_H + (1-\Phi(b^0)) = 0 \\ \phi(b^0)\dfrac{\partial b^0(p_L,p_H)}{\partial p_L} p_L + \Phi(b^0) = 0 \end{cases} \tag{14}$$

We do not specify the indifference condition, as one can use the equivalent of Lemma 2 in the appendix to show that, if a solution to (14) exists then the equilibrium exists.

By subtracting the second equation from the third equation, and using the expression for b^0 to calculate its derivative with respect to prices, we get

$$\frac{1-\Phi(b^0)}{\phi(b^0)} - \frac{\Phi(b^0)}{\phi(b^0)} - b^0 = \frac{1}{1-q_{LL}}.$$

For a given q_{LL}, the solution to (14) exists if and only if

$$\max_b \left\{ \frac{1-\Phi(b)}{\phi(b)} - \frac{\Phi(b)}{\phi(b)} - b \right\} > \frac{1}{1-q_{LL}}.$$

For distributions with log-concave densities, the expression inside the parenthesis is a decreasing function of b^0. By Theorem 1 of Bergstrom and Banoli (2006), if $\phi(b)$ is log-concave, then so is $\Phi(b)$. Term $(1-\Phi(b))/\phi(b)$ is a decreasing function of b by Corollary 2 in Bergstrom and Banoli (2006). Term $\Phi(b)/\phi(b)$ is an increasing function of b by definition of log-concavity, and is decreasing when multiplied by minus one. Therefore, the maximum of the expression in the parenthesis is reached at $b = 0$. Thus, for a given q_{LL}, the equilibrium exists if and only if

$$\frac{1}{\phi(0)} > \frac{1}{1-q_{LL}}. \tag{15}$$

Since $q_{LL} \geq 1-q$, a necessary condition for the equilibrium where high type separates to exist is

$$\frac{1}{\phi(0)} > \frac{1}{q}. \tag{16}$$

Whether (16) puts any restriction on B or not depends on the underlying distribution. For example, when $\Phi(b)$ is uniform, condition (16) becomes $B > 1/q$. When $\Phi(b) = (b/B)^2$, condition (16) is satisfied for any B.

The intuition is similar to that in the previous subsection. The H-product is superior to the L-product in that its quality is both higher *and* certain. In order for the L-seller to be able to have positive profit, two conditions have to be satisfied: the L-product should be sufficiently differentiated from the H-product, and there should be sufficiently few buyers with low loss-aversion to make it suboptimal for the H-seller to compete for them and price the L-seller out of the market. The first condition requires q to be sufficiently high. When q is low, the L-product is very likely to have low quality, in which case there is not enough product differentiation between H- and L-sellers to generate positive profit for the L-seller. Indeed, the product differentiation comes from the difference in riskiness of the H- and L- products, and when the riskiness is similar the product differentiation is not sufficient for the L-seller to compete against the H-seller.[13] The second condition requires $\phi(0)$ to be sufficiently low. The intuition is similar to that behind Proposition 6 from the previous section. The only difference is that in the previous case the seller of the inferior product, L-seller, was competing for buyers with high-degree of risk-aversion, whereas now it competes for buyers with a low degree of loss-aversion. In order for the H-seller to find it suboptimal to serve the whole market, value of $\phi(0)$ should be sufficiently small.

5 Conclusion

This paper studies the incentives of low-quality sellers to voluntarily disclose negative information to buyers. We focus on markets where customers cannot assess the quality of the products and there is no reputation concern. Unlike previous literature, our model limits the ability of high-quality sellers to separate themselves, based on the assumption that buyers only update their beliefs through cheap-talk messages. As low-quality sellers can costlessly imitate any messages sent by high-quality sellers, the quality is only credibly revealed when low-quality sellers voluntarily choose not to imitate high-quality sellers. This is different from classical information asymmetry models such as unraveling or education-as-signaling, where the separation is driven by high-quality types.

We identify two factors that can allow low-quality sellers to prefer separation over pooling with high-quality sellers: buyers' aversion to quality uncertainty and product differentiation. When choosing to reveal one's type, low-quality sellers face the trade-off between a positive *quality effect* of pretending to be of a higher-quality type by pooling with high-quality sellers, and a positive *information effect* of removing quality uncertainty which increases risk/loss-averse buyers' willingness to pay. In the monopoly setting, as long as there are sufficient loss- or risk-averse buyers, the information effect can be strong enough to outweigh the quality effect, making separation optimal for the low-quality seller. In the duopoly setting, the trade-off between quality and information effects is further affected by the fact that different information about competitors' products results in product differentiation and weaker competition. This leads to two new outcomes that were impossible in the case of the monopolistic sellers: high-quality types can separate and, in the case of risk-averse buyers, the lowest-quality seller can also choose to separate. Both can happen in equilibrium if the competition makes quality effect too small so that it is not optimal for the low-quality sellers to pretend to be of high quality.

[13] With risk-averse buyers, as in the previous subsection, Proposition 6 did not depend on q. Since the risk-averse model is harder, conditions required for existence were only established for sufficiently large Γ. For a given q, one can always find Γ large enough so that the equilibrium exists, which is, effectively, what Proposition 6 did. With loss-averse buyers, a closed-form solution is easy to derive. The existence of equilibrium can be established without requiring B to be arbitrarily large, which makes it possible to highlight the role of q and the role of the product differentiation motive. The fact that this subsection looks at the case when the highest-quality type separates is not crucial. An earlier version of the paper showed that with loss-averse buyers there is an equilibrium where the lowest-quality type separates iff $B + \dfrac{1}{\phi(B)} > \dfrac{1}{1-q}$. Therefore, again, the existence of the equilibrium depends not only on B and $\phi(B)$, but also on q.

Overall, this paper shows how honesty may work as an effective communication strategy. We believe our finding can be widely applied to various market settings where information asymmetry is high, providing valuable insights to both the field and the academia. For managers, this paper can provide helpful strategic implications on how to wisely consider buyers' uncertainty aversion while communicating the information about the weaknesses of their products. For academic researchers, this paper provides a theoretical explanation about an important market phenomenon that has been somewhat neglected in the literature. More specifically, this paper suggests that the consideration of risk might establish the incentive of low-quality sellers to voluntarily reveal their types.

On a final note, we hope that this study provides motivations for further empirical and theoretical research regarding the effect of revealing negative information under various other settings, which can lead to a better understanding about information asymmetry in markets.

6 Appendix A: Proofs

Proof of Proposition 2: Proof of i): First, we show that if type v_0 serves the whole market then any types $\nu < v_0$ also serves the whole market. Let p_0 denote the price charged by type v_0, and c_0 its marginal cost. Let s_ν denote the share of the market served by type ν, p_ν its price and c_ν its marginal cost. Proof by contradiction. Assume that $s_\nu < 1$. Then,

$$\begin{aligned}
(p_\nu - c_\nu)s_\nu &\geq (p_0 - c_\nu) \cdot 1 \\
(p_\nu - c_0)s_\nu + (c_0 - c_\nu)s_\nu &\geq (p_0 - c_0) \cdot 1 + (c_0 - c_\nu) \cdot 1 \\
(p_\nu - c_0)s_\nu &\geq (p_0 - c_0) \cdot 1 + (c_0 - c_\nu)(1 - s_\nu) > (p_0 - c_0) \cdot 1.
\end{aligned}$$

Here the first inequality comes from the fact that type ν prefers its equilibrium strategy over imitating type v_0 and charging price p_0. The last inequality is strict because $s_\nu < 1$. The inequalities above imply that type v_0 has a profitable deviation of sending the same message as type ν and charging price p_ν. Contradiction as then type v_0 could profitably deviate by imitating type ν. Thus $s_\nu = 1$ for any $\nu < v_0$. Since s_ν has to be equal to 1 after every on-equilibrium message, the price charge by ν should be the same as well for every on-equilibrium message. Otherwise, messages that result in lower prices would be suboptimal to send.

Proof of ii):[14] To guarantee the interval structure of the equilibrium messaging strategy it is sufficient to show that if there are two types v_1 and v_2 such that $\bar{v} \leq v_1 < v_2$ and that weakly prefer message m over any other messages, then any $v \in (v_1, v_2)$ will strictly prefer m over any other messages.

[14] As mentioned in the main body of the paper, the proof does not depend on whether the quality distribution is discrete or continuous. In equilibrium, the only thing that quality distribution affects is buyers' posterior beliefs $\mu(m)$. Given $\mu(m)$, one can then calculate optimal sellers' behavior for any quality v regardless of whether the quality belongs to the support of $F(v)$ or not. In particular, nowhere in the proof it is required that variables v_1, v_2, v_0 or $\tilde{v}$ that we introduce during the proof belong to the support of the quality distribution.

Consider the two types ν_1 and ν_2 such that $\bar{v} \leq \nu_1 < \nu_2$, and that weakly prefer message m to all other messages. Let μ denote buyers' beliefs given message m. Consider now a different message m' and let μ' be buyers' beliefs given m'. Let $\pi(\mu, v) = (p_v^* - c_v)s(\mu, p_v^*)$, where p_v^* is optimal price for type v given μ, and $s(\mu, p_v^*) = \Phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right)$.

Lemma 1 *Take beliefs μ and μ' and assume that either $E_\mu v \neq E_{\mu'} v$ or $ELoss_\mu \neq ELoss_{\mu'}$, or both. Then there exist at most one type with quality above $\bar{v}$ such that $\partial\pi(\mu, v)/\partial v = \partial\pi(\mu', v)/\partial v$.*

Proof. By the envelope theorem, $\partial\pi(\mu, v)/\partial v = -(c_v)' s(\mu(m), p_v^*) = -(c_v)'\Phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right)$. When for a given quality type, profit derivatives are equal it implies that $\Phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right) = \Phi\left(\frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}\right)$. From monotonicity of $\Phi(\cdot)$ then

$$\frac{E_\mu v - p_v^*}{ELoss_\mu} = \frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}. \tag{17}$$

Since p_v^* and $p_v'^*$ are optimal prices and the quality is above $\bar{v}$, they must satisfy the FOCs

$$\Phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right) - \frac{1}{ELoss_\mu}(p_v^* - c_v)\phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right) = 0 \tag{18}$$

$$\Phi\left(\frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}\right) - \frac{1}{ELoss_{\mu'}}(p_v'^* - c_v)\phi\left(\frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}\right) = 0. \tag{19}$$

Combined with (17) it follows from the FOCs that

$$\frac{p_v^* - c_v}{ELoss_\mu} = \frac{p_v'^* - c_v}{ELoss_{\mu'}}. \tag{20}$$

Adding equations (17) and (20) we have

$$\frac{E_\mu v - c_v}{ELoss_\mu} = \frac{E_{\mu'} v - c_v}{ELoss_{\mu'}}. \tag{21}$$

Notice that $\frac{E_\mu v - c_v}{ELoss_\mu}$ and $\frac{E_{\mu'} v - c_v}{ELoss_{\mu'}}$, as functions of c_v, intersect at most ones. It means that there can exist at most one type where $\Phi\left(\frac{E_\mu v - p^*}{ELoss_\mu}\right) = \Phi\left(\frac{E_{\mu'} v - p'^*}{ELoss_{\mu'}}\right)$, and that's the type that has cost that satisfies (21). ■

Take types ν_1 and ν_2 that weakly prefer m to any message, and assume that there is type $v_0 \in (\nu_1, \nu_2)$ that weakly prefers message m' over m. There are two possibilities: either there is more than one type that weakly prefers m', or such type is unique. Consider the first possibility. By continuity then, there exist at least two types in $[\nu_1, \nu_2]$ that are indifferent between m and m'. Denote them as v_1 and v_2 so that $\nu_1 \leq v_1 < v_2 \leq \nu_2$. By the mean value theorem there exists type $v \in (v_1, v_2)$ such that $\pi_v'(\mu, v) = \pi_v'(\mu', v)$ and by Lemma 1 such type is unique.

First, type v cannot be indifferent between m' over m. By the mean value theorem and Lemma 1, since derivatives of $\pi(\mu,\cdot)$ and $\pi(\mu',\cdot)$ are equal to each other at most once there cannot be three types that are indifferent between m and m'.

Second, it cannot be the case that type v strictly prefers m' over m. From, $\pi(\mu,v)<\pi(\mu',v)$ and $s(\mu,p_v^*)=s(\mu',p_v'^*)$ follows that $p_v^*<p_v'^*$. Consider now type v_2. By its definition, $\pi(\mu,v_2)=\pi(\mu',v_2)$. From the fact that $v_2>v$ and Lemma 1 follows that $\pi'(\mu,v_2)>\pi'(\mu',v_2)$. By the envelope theorem then $s(\mu,p_{v_2}^*)<s(\mu',p_{v_2}'^*)$. Since profits at v_2 are equal it means that $p_{v_2}^*>p_{v_2}'^*$. By continuity there exists $\tilde{v}\in(v,v_2)$ such that $p_{\tilde{v}}^*=p_{\tilde{v}}'^*$. For the notational brevity denote $p_{\tilde{v}}^*$ as simply $\tilde{p}^*$. Since $\tilde{v}>v$ we know that $s(\mu,\tilde{p}^*)<s(\mu',\tilde{p}^*)$ and, therefore,

$$\frac{E_\mu v-\tilde{p}^*}{ELoss_\mu}<\frac{E_{\mu'}v-\tilde{p}^*}{ELoss_{\mu'}}. \tag{22}$$

From the FOCs (18) and (19) it follows that when the prices are equal then

$$ELoss_\mu\cdot\frac{\Phi\left(\dfrac{E_\mu v-\tilde{p}^*}{ELoss_\mu}\right)}{\phi\left(\dfrac{E_\mu v-\tilde{p}^*}{ELoss_\mu}\right)}=ELoss_{\mu'}\cdot\frac{\Phi\left(\dfrac{E_{\mu'}v-\tilde{p}^*}{ELoss_{\mu'}}\right)}{\phi\left(\dfrac{E_{\mu'}v-\tilde{p}^*}{ELoss_{\mu'}}\right)}. \tag{23}$$

By log-concavity of $\Phi(\cdot)$ term Φ/ϕ is an increasing function. Then from (22) and (23) follows that $ELoss_\mu>ELoss_{\mu'}$. We reached a contradiction. When $ELoss_\mu>ELoss_{\mu'}$ then equality (21) and inequality (22) cannot be satisfied simultaneously. This is because $\tilde{p}^*$ is an optimal price charged by type with a cost higher than c_v. Therefore, $\tilde{p}^*>c_v$. But then, given $ELoss_\mu>ELoss_{\mu'}$, the inequality in (22) must be reversed.

Finally, by applying exactly the same logic as above one can show that type v cannot strictly prefer m to m'. Thus we ruled out every possibility which means that there cannot be more than one type in $[\nu_1,\nu_2]$ that weakly prefers m' over m.

Now consider the second case where type $v_0\in(\nu_1,\nu_2)$ is unique. Then, $\pi(\mu,v_0)=\pi(\mu',v_0)$ and $\pi'(\mu,v_0)=\pi'(\mu',v_0)$. If the derivatives are not equal then in the neighborhood of v_0 there would exist a type that strictly prefers m', meaning that v^0 was not unique. By the envelope theorem, $\pi'(\mu,v_0)=\pi'(\mu',v_0)$ implies that $s(\mu,p_{v_0}^*)=s(\mu',p_{v_0}'^*)$, which combined with $\pi(\mu,v_0)=\pi(\mu',v_0)$ implies that $p_{v_0}^*=p_{v_0}'^*$. Then $p_{v_0}^*$ satisfies Equation (23) which would imply that $ELoss_\mu=ELoss_{\mu'}$. And then from $s(\mu,p_{v_0}^*)=s(\mu',p_{v_0}'^*)$ follows that $E_\mu v=E_{\mu'}v$, which is a contradiction.

The only thing left is to show that the number of intervals is at most countable. First, notice that in equilibrium at most one type can separate. If there are two types, $v'<v''$, that separate then they will set prices $p'=v'$ and $p''=v''$, and will serve the whole market. But then type v' could deviate and imitate type v'' and earn higher profit.[15] Let M be the set of on-equilibrium messages sent by types $v\in[\bar{v},v_H]$ excluding the separating message. By definition, for each $m\in M$ there are at least two types v' and v'' that send message m with positive probability. Thus any

[15] This is part of Proposition 3. We provide the proof here as well merely to show that the proof of the current Proposition does not depend on later results.

$v \in (v', v'')$ strictly prefers message m over any other message. Therefore, for any $m \in M$ there exists a positive measure of types that send message m. Thus, there can be at most countable number of messages sent in equilibrium and, therefore, at most countable number of intervals.

Proof of iii): First, take any two types, $v' < v''$, that belong to the same interval, say $[v^i, v^{i+1}]$. Both types face the same demand function and, by a standard argument, type v'' will charge a higher price because it has higher cost. Next compare prices within intervals $[v^i, v^{i+1}]$ and $[v^{i+1}, v^{i+2}]$. Take type v^{i+1}. It is indifferent between messages m_i and m_{i+1}. By part ii) types slightly higher than v^{i+1} strictly prefer m_{i+1}. Therefore, $\pi'(\mu(m_i), v^{i+1}) < \pi'(\mu(m_{i+1}), v^{i+1})$, which by the envelope theorem implies that $s(\mu(m_i), p^*_{v^{i+1}}(m_i)) > s(\mu(m_{i+1}), p^*_{v^{i+1}}(m_{i+1}))$. Since v^{i+1} is indifferent between m_i and m_{i+1} we then have that $p^*_{v^{i+1}}(m_i) < p^*_{v^{i+1}}(m_{i+1})$. This means that all types in $[v^i, v^{i+1}]$ will charge price less than $p^*_{v^{i+1}}(m_{i+1})$ and all types on $[v^{i+1}, v^{i+2}]$ will charge prices higher than $p^*_{v^{i+1}}(m_{i+1})$. By applying the argument inductively, one can establish that price is a strictly increasing function of quality on $[\bar{v}, v_H]$. ■

Proof of Proposition 3: Proof by contradiction. Assume that there are two types that separate: $v_1 < v_2$ with messages $m_1 \neq m_2$. Then these types will set prices $p_1 = v_1$ and $p_2 = v_2$ and will serve the whole market. But that cannot happen since type v_1 would deviate and send the message m_2. Thus at most one type separates. Label this type v_0, and let m_0 be its separating message. Given m_0 type v_0 will charge the price $p_0 = v_0$ and will serve the whole market. By Proposition 2, all types below v_0 will also serve the whole market.

Now we prove the second part. If v_0 separates and $v_0 = v_L$ then we are done as $v_L < Ev$. Otherwise, take type $\nu < v_0$. Let $m(\nu)$ be the message sent by type ν and μ_ν be corresponding beliefs about the quality. One can show that $E_{\mu_\nu} v \geq v_0$.

Consider, first, the case of loss-averse buyers. Since all sellers with lower quality, including seller ν, serve the whole market, they will charge price $p_\nu = E_{\mu_\nu} v - B \cdot ELoss_{\mu_\nu}$, which is the highest price that allows a seller to serve the whole market. Given that neither type v_0 nor type ν want to deviate and imitate each other, it must be the case that $p_\nu = v_0$, for any $\nu < v_0$. Thus, $E_{\mu_\nu} v - B \cdot ELoss_{\mu_\nu} = v_0$ and, therefore, $E_{\mu_\nu} v > v_0$. Consider now the case of risk-averse buyers. Any seller with $\nu < v_0$ serves the whole market. It will then choose the highest price for which the whole market is served: $E_{\mu_\nu} u_\Gamma(w + v - p_\nu) = u(w)$, where u_Γ is the Bernoully utility function of the most risk-averse buyer. Since type ν does not imitate type v_0, and vice versa, it has to be the case that $p_\nu = v_0$. But then by Jensen's inequality $Ev_{\mu_\nu} > v_0$.

The rest of the proof goes regardless of whether buyers are risk- or loss-averse. As established above, any $\nu < v_0$ will send an on-equilibrium message that will result in buyers having beliefs $Ev_{\mu_\nu} > v_0$. We will use it to show that $v_0 < Ev$. Let $\mathcal{M}$ be the set of messages sent by sellers with $\nu < v_0$. Let $\mathcal{V}$ be the set of all types that send messages in $\mathcal{M}$. As we established, $E_{\mu(m)} v > v_0$ for any $m \in \mathcal{M}$. Let $\mathcal{V}^c = [v_L, v_H]/(\mathcal{V} \cup \{v_0\})$ be the set of all types that are not in $\mathcal{V}$ or v_0, and $\mathcal{M}^c$ be the set of messages they send. Since all types in $\mathcal{V}^c$ have quality higher than v_0, we have that $E_{\mu(m)} v > v_0$ for any $m \in \mathcal{V}^c$. Thus any equilibrium message results in beliefs about expected quality that are either equal to v_0, which is v_0's separation message, or strictly greater than v_0.

Given that on-equilibrium beliefs are correct, it has to be the case that $v_0 < Ev$. ■

Proof of Proposition 5: Let y denote $e^{-\gamma^0}$ so that (9) is

$$y^{p_H - p_L} - q_{LH} - (1 - q_{LH})y^{v_H - v_L} = 0. \tag{24}$$

As γ^0 varies between 0 and $+\infty$, variable y varies between 1 and 0. There is one solution $y = 1$, which corresponds to $\gamma = 0$. In order to prove the proposition, we need to show that on interval $0 \leq y < 1$ there is at most one solution. Given the root $y = 1$, it is equivalent to showing that there are at most two solutions of (24) on $0 \leq y \leq 1$.

Assume not. If function (24) has three or more roots on $[0, 1]$, then its derivative should have two or more roots on $[0, 1]$. Taking derivative of the LHS of (24) with respect to y and setting it equal to zero we get:

$$(p_H - p_L)y^{p_H - p_L - 1} - (1 - q_{LH})(v_H - v_L)y^{v_H - v_L - 1} = 0,$$

so that

$$1 - (1 - q_{LH})\frac{v_H - v_L}{p_H - p_L} y^{(v_H - v_L) - (p_H - p_L)} = 0.$$

The equation above has at most one solution on interval $[0, 1]$ and, therefore, equation (24) has at most two solutions on $[0, 1]$. As one solution is $y = 1$, it implies that there is at most one solution when $0 < y < 1$.

To prove the second part of the proposition, we observe that buyers prefer the L-product if

$$1 - e^{-\gamma(v_L - p_L)} \geq q_{LH}(1 - e^{-\gamma(v_L - p_H)}) + (1 - q_{LH})(1 - e^{-\gamma(v_H - p_H)}),$$

which is equivalent to

$$e^{-\gamma(p_H - p_L)} \leq q_{LH} + (1 - q_{LH})e^{-\gamma(v_H - v_L)}. \tag{25}$$

When $\gamma = +\infty$ then (25) is satisfied as the LHS is zero, and the RHS is positive. In other words, extremely risk-averse buyers will always purchase the L-product. Thus if $\gamma^0 > 0$ is the solution to (9), then all types with $\gamma > \gamma^0$ will purchase the L-product. Customers with $\gamma < \gamma^0$ will purchase the H-product. This is because, as established earlier, the indifference condition holds with equality when $\gamma = \gamma^0$ and $\gamma = 0$. Were it the case that customers with $0 < \gamma < \gamma^0$ prefer the L-product then it would mean that derivatives of the LHS and the RHS of (25) are equal to each other twice: once at γ^0 and once at some point in $(0, \gamma^0)$. But that contradicts the earlier established fact that the derivatives of LHS and RHS can be equal to each other for at most one value of γ. ■

Proof of Proposition 6: The proof of the proposition consists of two parts. In the first part we reduce the equilibrium system (13) to one equation with one unknown, γ^0. In the second part, we analyze that equation and develop sufficient conditions on $\Phi(\gamma)$ stated in Proposition 6. We begin the first part by proving Lemma 2, which says that for the purpose of proving the existence one can ignore the last equation in (13) and variable λ. We will refer to system (13) without the last equation as the *reduced system*.

Lemma 2 *If for a given $q_{LH} \in [0,1]$ the solution $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ to the reduced system exists, then there exists $q \in [0,1]$ and $\lambda \in [0,1]$ such that $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H, \lambda)$ is a solution to the equilibrium system (13).*

Proof. Plug values $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ into the last equation of (13) to recover $\lambda(1-q)$. Notice that for any $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ one can find $\lambda(1-q)$ such that it is between 0 and 1 and the last equation is satisfied. One can then uniquely recover q and λ: $q = (1-q_{LH})(1-\lambda(1-q))$ and $\lambda = \lambda(1-q)/(1-q)$. The only thing one has to check is that $\lambda, q \in [0,1]$ and that $q_{LH} \leq 1-q$. The last inequality states that the share of low-quality sellers who announce high quality, q_{LH}, cannot be higher than the total share of low-quality sellers, $1-q$. This is straightforward. That $q_{LH} \leq 1-q$ is trivial:

$$q_{LH} \leq 1-q = 1-(1-q_{LH})(1-\lambda(1-q)) = q_{LH} + \lambda(1-q)(1-q_{LH}).$$

Given that $q = (1-q_{LH})(1-\lambda(1-q))$, it is between 0 and 1. Finally, $\lambda < 1$ is equivalent to $\lambda(1-q) < 1-q$, which is also true.

$$\lambda(1-q) \leq 1-q = 1-(1-q_{LH})(1-\lambda(1-q)) = \lambda(1-q) + q_{LH}(1-\lambda(1-q)).$$

This completes the proof of the lemma. ■

The reduced system has three equations and three unknowns (γ^0, p_L, p_H), and it treats q_{LH} as a given parameter. The three equations of the reduced system are the indifference condition

$$e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}, \tag{26}$$

and two FOCs that determine prices:

$$\begin{cases} -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \end{cases} \tag{27}$$

Take the second equation of (27) and subtract from it the first equation of (27). We get

$$\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)} = \frac{\partial\gamma^0(p_L,p_H)}{\partial p_L}(p_H-p_L). \tag{28}$$

It will be convenient to denote $\dfrac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \dfrac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ as $A(\gamma^0)$. From (26) we get that

$$\begin{aligned} \frac{\partial\gamma^0}{\partial p_L} = -\frac{\partial\gamma^0}{\partial p_H} &= -\frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{-(p_H-p_L)e^{-\gamma^0(p_H-p_L)} + (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}} = \\ &= \frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{(p_H-p_L)e^{-\gamma^0(p_H-p_L)} - (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}}. \end{aligned}$$

Let $\Delta p = p_H - p_L$ and $\Delta v = v_H - v_L$. Thus the reduced system becomes a system of two equations and two unknowns:

$$\begin{cases} e^{-\gamma^0\Delta p} &= q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v} \\ A(\gamma^0) &= \dfrac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p \end{cases} \tag{29}$$

We can now eliminate Δp from (29) and reduce to one equation. First, we re-write the second equation of (29) as

$$\begin{aligned} A(\gamma^0)\Delta p e^{-\gamma^0 \Delta p} - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0 \Delta v} &= \gamma^0 \Delta p e^{-\gamma^0 \Delta p} \\ A(\gamma^0)\Delta p - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0 \Delta v} e^{\gamma^0 \Delta p} &= \gamma^0 \Delta p \\ A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0 \Delta v} \frac{e^{\gamma^0 \Delta p}}{\Delta p} &= A(\gamma^0) - \gamma^0 \\ \frac{\gamma^0 A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} &= \gamma^0 \Delta p e^{-\gamma^0 \Delta p}. \end{aligned} \tag{30}$$

Second, from the indifference condition we get that

$$\gamma^0 \Delta p = -\ln(q_{LH} + (1-q_{LH})e^{-\gamma^0 \Delta v}),$$

and then

$$\gamma^0 \Delta p e^{-\gamma^0 \Delta p} = -\ln(q_{LH} + (1-q_{LH})e^{-\gamma^0 \Delta v})(q_{LH} + (1-q_{LH})e^{-\gamma^0 \Delta v}).$$

Plugging it into (30) we get that the equilibrium value of γ^0 is determined by

$$-\frac{\gamma^0 A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} = (q_{LH} + (1-q_{LH})e^{-\gamma^0 \Delta v}) \ln(q_{LH} + (1-q_{LH})e^{-\gamma^0 \Delta v}). \tag{31}$$

This completes the first part of the proof. In the second part of the proof, we will analyze equation (31) and derive condition that determines whether the solution exists or not. In what follows we will refer to the RHS and LHS of (31) simply as RHS and LHS without referring to the equation's number.

First, we establish that in equilibrium $\Delta p > 0$. Indeed, consider the indifference condition

$$e^{-\gamma^0 (p_H - p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0 (v_H - v_L)},$$

and take natural logarithm of both sides:

$$-\gamma^0 (p_H - p_L) = \ln(q_{LH} + (1-q_{LH})e^{-\gamma^0 (v_H - v_L)}).$$

The expression inside the logarithm is less than one. Then $\ln(q_{LH} + (1-q_{LH})e^{-\gamma^0 (v_H - v_L)}) < 0$, which implies that $\Delta p > 0$.

Next, we establish that in equilibrium $A(\gamma^0) > \gamma^0$. Since $\Delta p > 0$, it follows from (28) that $\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} > \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ and, therefore, in equilibrium $A(\gamma^0) > 0$. Furthermore, since $\Delta p > 0$, the LHS of (30) must be positive. Since, in equilibrium, $A(\gamma^0) > 0$, it then implies that $A(\gamma^0) > \gamma^0$. If such γ^0 does not exist, then (30) cannot be satisfied and no equilibrium with the disclosure of negative information can exist.

<u>Proof of i and ii:</u> As we have established earlier, if $A(\gamma) < \gamma$ for every γ, then the solution to (30) and, therefore, to (31) does not exist. We will show that this is the case for uniform and convex

distributions. Let the support of $\Phi(\gamma)$ be $[0,\Gamma]$. For uniform and convex distributions it is finite. Inequality $A(\gamma)<\gamma$ is equivalent to $2\Phi(\gamma)-1<\gamma\phi(\gamma)$. We will write it as $\Phi(\gamma)-1<\gamma\phi(\gamma)-\Phi(\gamma)$. Function $\Phi(\gamma)$ is a weakly convex function such that $\Phi(0)=0$. By a standard property of convex functions $\gamma\phi(\gamma)\geq\Phi(\gamma)-\Phi(0)$ and since $\Phi(0)=0$ we have $\gamma\phi(\gamma)\geq\Phi(\gamma)$. Thus in the inequality $\Phi(\gamma)-1<\gamma\phi(\gamma)-\Phi(\gamma)$ the left-hand side is negative and the right-hand side is non-negative, which means that it is satisfied for any $\gamma\in[0,\Gamma)$. When $\gamma=\Gamma$ and $\Phi(\gamma)$ is linear, then $A(\Gamma)=\Gamma$, and in all other cases, $A(\Gamma)<\Gamma$. Thus, for the case of convex and uniform distribution functions there is no equilibrium where both firms split the market.

Proof of iii: The RHS is a continuous function of γ. It is negative for any $\gamma>0$. When $\gamma=0$ it is equal to zero. When $\gamma\to\infty$, its limit is equal to $q_{LH}\cdot\ln(q_{LH})<0$.

The LHS is discontinuous when $A(\gamma)=\gamma$. Let $\hat{\gamma}$ denote the largest root such that $A(\gamma)=\gamma$. We can show that it exists. First, $A(0)=-1/\phi(0)<0$. Second, $\lim_{\gamma\to\infty}\gamma\phi(\gamma)=0$. If the limit is positive, say $z>0$, then it means that for all sufficiently large γ^0, say for all $\gamma>\Gamma^0$, it has to be the case that $\phi(\gamma)>\frac{1}{2}\frac{z}{\gamma}$. But then

$$\int_{\Gamma^0}^{\infty}\phi(s)ds>\frac{1}{2}\int_{\Gamma^0}^{\infty}\frac{z}{\gamma}d\gamma=\infty,$$

which is a contradiction since it has to be less than or equal to 1. Third,

$$\lim_{\gamma\to\infty}(A(\gamma)-\gamma)=\lim_{\gamma\to\infty}\frac{2\Phi(\gamma)-1-\gamma\phi(\gamma)}{\phi(\gamma)}=\frac{1}{0}=\infty.$$

Given that $A(\gamma)-\gamma$ is continuous, we can conclude now that it has roots and that there exists the largest root. In other words, there exists $\hat{\gamma}$ such that $A(\hat{\gamma})=\hat{\gamma}$ and $A(\gamma)>\gamma$ for every $\gamma>\hat{\gamma}$. Therefore, the LHS is a continuous function for any $\gamma>\hat{\gamma}$.

We can now prove the equilibrium existence. Since $\hat{\gamma}$ is the largest root, it means that for any $\gamma>\hat{\gamma}$ it must be the case that $A(\gamma)>\gamma$, and in a sufficiently small right neighborhood of $\hat{\gamma}$ fraction $A(\gamma)/(A(\gamma)-\gamma)$ is close to plus infinity. Then the LHS is close to $-\infty$ and, therefore, is less than the RHS. When γ is close to infinity, the LHS gets arbitrarily close to zero. This is because all terms of the LHS, including $A(\gamma)/(A(\gamma)-\gamma)$, are bounded and the term $e^{-\gamma\Delta v}$ converges to zero. That $A(\gamma)/(A(\gamma)-\gamma)$ is bounded follows from

$$\lim_{\gamma\to\infty}\frac{A(\gamma)}{A(\gamma)-\gamma}=\frac{2\Phi(\gamma)-1}{2\Phi(\gamma)-1-\gamma\phi(\gamma)}=1.$$

Therefore, for sufficiently large γ the LHS of (31) is less than the RHS. By continuity, the solution to (31) exists.

Proof of iv: Let support of $\Phi(\gamma)$ be $[0,\Gamma]$ where $\Gamma<\infty$. Then $A(\Gamma)>\Gamma$. Indeed, $A(\Gamma)>\Gamma$ is equivalent to $1>\Gamma\phi(\Gamma)$. Assume that it is not satisfied so that $\phi(\Gamma)>1/\Gamma$. Then, since $\phi(\gamma)$ is strictly decreasing we have

$$1=\int_0^{\Gamma}\phi(s)ds>\int_0^{\Gamma}\frac{1}{\Gamma}ds=1,$$

which is a contradiction. Also, one can show that $A(\gamma) < \gamma$ when γ is sufficiently close to zero, or more precisely for any γ such that $\Phi(\gamma) < 1/2$. Thus there exists γ such that $A(\gamma) = \gamma$ and let $\hat{\gamma}$ be the largest such γ. Then the LHS of (31) is continuous when $\gamma \in (\hat{\gamma}, \Gamma]$. As in case iii, one could try to use continuity to establish that the solution to (31) exists. However, it might not work with the original distribution because unless Γ is sufficiently large, the LHS will not be close enough to zero to guarantee that the solution exists.

Consider now a cdf function Φ_α defined as $\Phi(\alpha\gamma)$. It is a concave function with support $[0, \Gamma/\alpha]$. Now the largest value of γ is Γ/α. By taking α sufficiently small we can make the support $[0, \Gamma/\alpha]$ large enough so that $\gamma e^{-\gamma\Delta v}$ can be made sufficiently close to zero within the support.

Term $A(\gamma)/(A(\gamma) - \gamma)$ on the other hand will not change. Let $A_\alpha(\gamma)$ be defined similarly to $A(\gamma)$ but with a cdf Φ_α. Then for any $\gamma \in [0, \Gamma]$,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Indeed,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{2\Phi_\alpha(\gamma/\alpha) - 1}{2\Phi_\alpha(\gamma/\alpha) - 1 - (\gamma/\alpha)\phi_\alpha(\gamma/\alpha)} = \frac{2\Phi(\gamma) - 1}{2\Phi(\gamma) - 1 - (\gamma/\alpha)\alpha\phi(\gamma)} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Thus, when α is sufficiently small we can apply the reasoning of case iii) to function $\Phi_\alpha(\gamma)$ to show that the solution exists. ■

7 Appendix B (not for publication). Prices used as signals.

In this Appendix, we discuss what happens when buyers use prices to update their beliefs. We will focus on the case of loss-averse buyers. Except for prices determining buyers' beliefs all other assumption are the same as in the main model. As a solution concept we use a Sender-preferred subgame perfect equilibrium which, in addition to standard requirements assumes that if a Receiver is indifferent between some actions at a given beliefs, he takes an action that maximizes Sender's expected utility (see e.g. Kamenica and Gentzkow, 2011). In our model, it implies that whenever buyers are indifferent between purchasing and not they purchase the product.

Definition 3 *An equilibrium is a triple $(p(v), \mu(p), s(\mu, p))$, where $p(v)$ is the seller's pricing strategy, $\mu(p)$ is the buyers' beliefs about the quality distribution given p, and $s(\mu, p)$ is the share of buyers who purchase the product, such that the following conditions hold:*

a) the seller of type v chooses price p that maximizes his profit given buyers' beliefs $\mu(p)$ and buyers' purchasing decision:

$$\max_p (p - c_v) s(\mu(p), p);$$

b) buyers' purchasing decision is optimal:

$$s(\mu, p) = \Phi(\{b | U_b(p, \mu) \geq 0\});$$

c) if a given price p is chosen with positive probability, buyers' beliefs $\mu(p)$ are derived from $p(v)$ by Bayes' rule.

First, we prove the analogue of Proposition 1 which is trivial in this setting.

Proposition 7 *If all buyers are risk-neutral then the trade can occur at one on-equilibrium price only.*

Proof. Assume there are at least two on-equilibrium prices: $p_1 > p_2$. Since buyers are risk-neutral the sales are equal to 1 for each price. Then types charging price p_2 have a profitable deviation of charging price p_1. ■

The next Proposition is crucial in establishing the interval structure of the equilibrium and which types can separate.

Proposition 8 *If there are two different types $v_1 < v_2$ that on-equilibrium choose the same price p with a positive probability then all types in (v_1, v_2) will strictly prefer p over any other price.*[16]

Proof. The proof is based on the following Lemma.

Lemma 3 *Assume that price p results in beliefs $\mu(p)$ and sales s, while price p' results in beliefs μ' and sales s'. Assume type v weakly prefers price p to price p'. Then*

i) if $s < s'$ then all higher-quality types ($\nu > v$) will strictly prefer p to p';

ii) if $s > s'$ then all lower-quality types ($\nu < v$) will strictly prefer p to p'.

[16] Since price p is an on-equilibrium price it means that all types in (v_1, v_2) will choose p with probability 1.

Proof. Prove part i) first. Let $\nu > v$. Since type v weakly prefers p to p' it follows that

$$\begin{aligned}
(p - c_v)s &\geq (p' - c_v)s' \\
(p - c_\nu)s + (c_\nu - c_v)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)s' \\
(p - c_\nu)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)(s' - s) > (p' - c_\nu)s',
\end{aligned}$$

where the last inequality follows from the fact that $c_\nu > c_v$.

Part ii) is similar. Let $\nu < v$ and then

$$\begin{aligned}
(p - c_v)s &\geq (p' - c_v)s' \\
(p - c_\nu)s + (c_\nu - c_v)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)s' \\
(p - c_\nu)q(p) &> (p - c_\nu)q(p) + (c_v - c_\nu)(s' - q(p)) \geq (p' - c_\nu)s'.
\end{aligned}$$

It completes the proof. ■

Now we can prove the proposition statement by contradiction. Assume type $v' \in (v_1, v_2)$ prefers price p'. If price p' results in higher sales then by the first part of Lemma 3 then v_1 would strictly prefer p' to p and, therefore, would choose p with positive probability. Similarly, if price p' results in lower sales than by the second part of Lemma 3 type v_2 would strictly prefer p' to p and, therefore would not choose p with positive probability. Finally, consider the case when price p' results in the same sales as p. Then if $p' > p$ both v_1 and v_2 would strictly prefer p' over p and would not choose p with positive probability. Finally, if $p' < p$ then no type would choose price p', including types in (v_1, v_2). ■

Now we can prove the analogue of Proposition 3. The statement, however, is different, since now the only type that can separate is v_L. The intuition is simple. Separation is valuable as it reduces the uncertainty about the quality. If type v separates then it will serve the whole market. But then, for all lower types the optimal strategy is to mimic type v, which contradicts the fact that type v separates.

Proposition 9 *In any equilibrium that satisfies the indifference condition if there is a type that separates, it is v_L.*

Proof. Proof by contradiction. Assume type v separates with price p_v. This type serves the whole market. If $p_v < v$ all buyers purchase the product; if $p_v = v$ buyers are indifferent and given our equilibrium concept they will purchase the product; if $p_v > v$ then no one will purchase the product but then type v would not choose this price.

Since p_v is an on-equilibrium price type v weakly prefers it to any other price. Furthermore, since price p_v results in the highest possible demand, $s = 1$, by Lemma 3 all types with $\nu < v$ will strictly prefer p_v to any price that results in serving less than full market. It means that in equilibrium all types with $\nu < v$ sever the whole market as well. Since type v separates it means that lower types choose different price. Consider type $\nu < v$ that chose price $p_\nu \neq p$ and serves the whole market. If $p_\nu > p_v$ then type v will prefer to deviate to p_ν. If $p_\nu < p_v$ then type ν will strictly gain from deviating to p_v. Contradiction. Thus, except for the type with the lowest quality separation is impossible. ■

Finally, we prove the equivalent of Proposition 2.

Proposition 10 *Equilibrium has there following structure:*

i) there exist $v^1, \ldots, v^{N+1}$ *such that* $v_L = v^1 \leq v^2 < \cdots < v^N < v^{N+1} = v_H$*, where* N *can be infinite; all sellers with* $v \in (v^i, v^{i+1})$ *choose the same price, which we denote as* p_i*, with probability 1. If type distribution is continuous and* $1 < i < N+1$ *then type* $v = v^i$ *is indifferent between between* p_i *and* p_{i-1}*;*

ii) In equilibrium $p_1 < \cdots < p_N$*. Let* s_i *be the equilibrium sales of sellers with* $v \in [v^i, v^{i+1}]$*. Then* $s_1 > \cdots > s_N$*.*

Here we took into account that the only type that can separate itself is v_L, which is why $v^1 \leq v^2$ but all other inequalities are strict.

Proof. The first part almost immediately from results established earlier. To show indifference consider type v^i where $1 < i < N+1$. If type v^i strictly prefers message m_i to m_{i-1} then all types in a sufficiently small neighborhood of v^i also strictly prefer m_i. Similarly if type v^i strictly prefers m_{i-1} to m_i then all all types in a sufficiently small neighborhood of v^i will strictly prefer m_{i-1}. Contradiction.

To show that the cardinality of on-equilibrium prices is finite or countable, i.e. N in the statement of the proposition exists, we use the fact that $p(v)$ is a monotone step function and thus can have only countable number of jumps. Consider type v^i, where $1 < i < N$. This type is indifferent between (p_{i+1}, s_{i+1}) and (p_i, s_i). It can't be the case that $s_i = s_{i+1}$ because $p_i \neq p_{i+1}$. It cannot be that $s_i > s_{i+1}$ either. Were that the case then by Lemma 3 all types with $v > v^i$ would strictly prefer p_{i+1} to p_i, which contradicts the definition of v^i. Thus, $s_i < s_{i+1}$. But then $p_i > p_{i+1}$ because were it not the case all the types in $[v^i, v^{i+1}]$ would strictly prefer to send message p_{i+1} and enjoy both higher price and higher sales. ■

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