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Abstract

We develop a model of interdependent value auctions in which two types of bidders compete: insiders, who are perfectly informed about their value, and outsiders, who are informed only about the private component of their value. Because of the mismatch of bidding strategies between insiders and outsiders, the second-price auction is inefficient. The English auction has an equilibrium in which the information outsiders infer from the history of drop-out prices enables them to bid toward attaining efficiency. The presence of insiders has positive impacts on the seller's revenue. A laboratory experiment confirms key theoretical predictions, despite evidence of naive bidding.

JEL Classification Numbers: *C92, D44, D82.*

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1 Introduction

Most auction literature assumes that bidders hold rather equally informative information about the value of the auctioned object, while each bidder's information is privately known

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to him.[1] However, there are real-world auctions in which there is an "information divide" among bidders in the sense that some bidders know more about the value of auction object than others. For instance, in art auctions, buyers with professional knowledge tend to more accurately appraise the potential value of an object than non-professional buyers (Ashenfelter and Graddy, 2003). In takeover auctions, buyers with existing shares of a target firm may have access to inside information unavailable to competitors.[2] In auctions for gas and oil leases (i.e., OCS auctions), firms owning neighboring tracts may have better information about the value of a lease—such as oil reserves or drilling conditions—than non-neighboring firms do (Hendrick and Porter, 1988). While these examples illustrate that informational asymmetry among bidders is commonly observed, they also highlight that the information held by a better-informed bidder helps other bidders, in particular less-informed ones, to evaluate the object.

We study an auction environment with insiders and outsiders in the interdependent value setup, which we refer to as *stratified information structure*. In this environment, bidders' values are composed of a private value component and a common value component. Our key assumption is that the set of bidders is partitioned into two types: insiders, who are perfectly informed of their values, and outsiders, who are informed only of the private value component and thus imperfectly informed of their values. Outsiders know the identity of insiders. We allow for any arbitrary number of insiders and outsiders and study the performance of two standard auction formats, second-price and English, in terms of their revenue and efficiency.

When only one insider and one outsider exist, the second-price and English auctions are equivalent and achieve an efficient allocation. With more than two bidders and at least one insider, however, the second-price auction does not admit an efficient equilibrium. This inefficiency arises from the mismatch between insiders' and outsiders' bidding strategy, because the former depends on both private and common components of values, whereas the latter depends on only the private component. Also, the one-shot nature of the second-price auction offers no means to overcome such information divide between outsiders and insiders.

In contrast, the English auction provides opportunities for outsiders to learn about others'—both insiders and outsiders'—private information through the history of prices at which bidders drop out. In an environment with standard information structure (i.e.,

[1] See, for instance, Myerson (1981) and Riley and Samuelson (1981) for the environment of private value auctions, and Milgrom and Weber (1982) for auctions with affiliated values.

[2] Sometimes, a current management team of the target firm participates in the bidding competition, which is a practice known as a management buyout (MBO). Shleifer and Vishny (1988) argue that the managers' special information about their company is one reason for an MBO.

with no insider), each bidder's private signal is fully revealed in equilibrium because his equilibrium drop-out price is monotonically related to his private signal. The inference problem becomes more complex in the presence of insiders who employ a (weakly) dominant strategy of dropping out at their values that reflect both private and common components. Our equilibrium construction for English auction overcomes this problem by extending the equilibrium constructions in Milgrom and Weber (1982) and in Krishna (2003) to our setup, which involves finding a system of equation that yields *break-even signals* at any given price. It turns out that the resulting allocation is efficient. The main driving force of the efficiency of English auction is that it enables the outsiders' signals to be fully revealed through their drop-out prices while guaranteeing that the insiders' signal, albeit not fully revealed, can be partially revealed to the extent that does not prevent allocation from being efficient. This is achieved through a continuous updating process by which active outsiders take into account the drop-out prices of inactive insiders (as well as those of inactive outsiders) to update their break-even signals as the price rises and new bidders drop out.

We also explore the revenue implications of the stratified information structure in the English auction. To investigate this, we consider two English auctions that differ by only one bidder who switches from an outsider in one auction to an insider in the other. Using our equilibrium construction for English auction, we show that for any realization of signal profile, the switching of an outsider to an insider (weakly) increases the seller's revenue. This result is based on two effects of turning an outsider into an insider on bidders' bidding behavior. First, the switched bidder drops out at a higher price. Second, the higher drop-out price of the switched bidder causes active outsiders to drop out at higher prices.

While our theory offers a benchmark for performance of auction formats in an environment with the stratified information structure, its predictions are based on the nontrivial inference process in the English auction. Whether human bidders can behave rationally up to the equilibrium analysis is an ultimately empirical question. To test the validity of the theory, we design an experiment by varying the auction format—between English and second-price—and the composition of insiders and outsiders. Specifically, we employ three-bidder auctions of each format with three outsiders, two outsiders and one insider, or one outsider and two insiders. Each combination of auction format and insider-outsider composition serves as a single treatment. In order to make the outsiders' inference problem as transparent as possible, we let the computer play the role of insider who follows the dominant strategy of dropping out at its own value. This was public information to all human subjects.

Our experiment presents several findings. First, the English auction achieves a higher level of efficiency than the second-price auction does when at least one insider is present.

Furthermore, there is no significant difference in efficiency between the two auctions when there is no insider. This finding on efficiency is consistent with our theory. Second, average revenues tend to deviate upward from the equilibrium benchmark, particularly in both auction formats with no insider. Despite this, the increase in the number of insiders has a positive impact on revenues in the English auction, as the theory predicts. We also find a similar pattern of the increase of revenues in the second-price auction with respect to the number of insiders. Third, there is evidence of naive bidding in both auction formats in the sense that subjects responded to their own signal less sensitively than predicted by the equilibrium theory, while those with low signals tend to overbid. This behavioral pattern is consistent with common findings of overbidding and the resulting winner's curse in the experimental literature. In addition, the degree of naive bidding in the data declines in the English auction as the number of insiders increases. We conjecture that the presence of insiders makes the outsider more cautious in bidding and creates a behavioral incentive for the outsider to hedge against an informational disadvantage held by insiders. This may work toward the alleviation of naive bidding and thus of the winner's curse in our setup.

Literature This paper contributes to the literature that studies an auction environment in which bidders hold different amounts of information. Engelbrecht-Wiggans et al. (1982) study a first-price common-value auction in which a single "insider" has proprietary information about the common value of the object while other bidders have public information. Hendrick and Porter (1988) and Hendrick et al. (1994) extend the analysis of Engelbrecht-Wiggans et al. (1982) and study oil and gas drainage lease auctions. In first-price common-value auctions, Campbell and Levin (2000) and Kim (2008) theoretically examine the effects of an insider on revenues, whereas Kagel and Levin (1999) experimentally study the effects of an insider on revenues and bidding behavior. Our paper differs from these in some important manners. First, we study the implications of insider information on efficiency as well as revenues in interdependent value auctions, whereas the existing literature focuses on revenue implications under the pure common value assumption. In addition, we study English auction and second-price auctions rather than first-price auction. Lastly, we provide an equilibrium analysis for any arbitrary number of insiders and offer the revenue implication of introducing an extra insider, whereas existing literature allows only a single insider.

The current paper also contributes to the literature that studies efficient auctions in the interdependent value environment, such as Dasgupta and Maskin (2000), Perry and Reny (2002), and Krishna (2003), to name a few. In particular, Krishna (2003) provides sufficient conditions under which English auction with asymmetric bidders admits an efficient equilibrium. Our efficiency result is established by adapting one such condition in Krishna (2003) to the stratified information structure.

Goeree and Offerman (2002,2003) study an interdependent values environment similar to ours but without insiders, in which bidders hold multidimensional information about their private value and common value components. This multidimensionality causes an inefficiency even in English auction, unlike our model in which the common value is deterministically related to the bidders' information (or signals) about their private values. In fact, such inefficiency persists in our setup if the insider's information is truly multidimensional (see Remark 1). In this light, the aim of the current paper is not to establish the efficiency of English auction in a multidimensional information setup, but to offer a conceivable information environment in which the information divide between bidders leads to a stark contrast in the efficiency performance of the sealed-bid and dynamic auctions.

Our experimental findings contribute to the experimental literature of auctions that investigates the effects of auction formats on outcomes and bidding behavior.[3] There are only a handful of experimental studies on performance of auction formats in the interdependent value environment (e.g., Goeree and Offerman (2002) for first-price auctions and Kirchkamp and Moldovanu (2004) for English and second-price auctions). Among them, Boone et al. (2009) is closely related to our paper in terms of the introduction of stratified information structure. Boone et al. (2009) study an auction environment with a restricted structure of interdependent values and a single insider in which both English and second-price auctions are inefficient, and report an experimental evidence that the English auction performs better in both efficiency and revenues than the second-price auction. We study the efficiency and revenue performance of the two standard auctions in a significantly more general environment than Boone et al. (2009).

The rest of the paper is organized as follows. Section 2 develops an interdependent value auction model with the stratified information structure and provides the theoretical results for the second-price and English auctions. Section 3 describes the experimental design and procedures. Section 4 summarizes experimental findings and Section 5 concludes. All theoretical proofs are contained in the Appendix.[4]

[3]See Kagel (1995) and Kagel and Levin (2011) for an extensive survey

[4]The paper is supplemented by four Online Appendices that provide sample instructions of the experiment and other technical results. These are available at https://sites.google.com/site/jikim72/home/working_papers.

2 Theory

2.1 Setup

A seller has a single, indivisible object to sell to one of n bidders. Let $N = \{1, \cdots, n\}$ denote the set of bidders. The value of the object to each bidder is determined by n-dimensional information $s = (s_1, \ldots, s_n) \in S = \times_{i \in N}[0, \bar{s}_i]$, which we call a signal profile. At this point, we do not specify who observes what signals, which is a central part of our stratified information structure and will be discussed shortly. However, we adopt the convention of referring to the i^{th} signal, s_i, as bidder i's signal. To denote signal profiles, we let $s_{-i} = (s_j)_{j \neq i}$, and let $s_B = (s_j)_{j \in B}$ for any subset of bidders $B \subseteq N$. It is assumed that the distribution of the signal profile has full support on S.

Each bidder i's value, denoted by $v_i(s)$, is assumed to be additively separable into two parts, a private value component $h_i : \mathbb{R} \to \mathbb{R}$ and a common value component $g : \mathbb{R}^n \to \mathbb{R}$: that is, $v_i(s) = h_i(s_i) + g(s)$. We also assume that h_i and g are twice continuously differentiable, that $h_i(0) = g(0) = 0$ for normalization, and that $\frac{dh_i}{ds_i} > 0$ and $\frac{\partial g}{\partial s_j} \geq 0$ for all $j \in N$. . According to this function, each bidder i's private value component depends only on his own signal s_i, whereas the common value component depends on the entire signal profile s.[5] We adopt this functional form in part because it provides a natural model of the stratified information structure in that some bidders often have superior information than other do about the common value aspect of the object.[6] We simplify the private value function h_i to be the identity function, i.e., $h_i(s_i) = s_i$, so

$$v_i(s) = s_i + g(s). \tag{1}$$

This is without loss of generality since, by change of variables, one can define $\tilde{s}_i \equiv h_i(s_i)$, $\tilde{s} \equiv (\tilde{s}_i)_{i \in N}$, and $\tilde{g}(\tilde{s}) \equiv g(h_1^{-1}(\tilde{s}_1), \ldots, h_n^{-1}(\tilde{s}_n)) = g(s)$. Note that with this simplification, the bidder i's signal s_i itself becomes his private value component. We say that bidders are *symmetric* when the signal distribution is symmetric (so that $\bar{s}_i = \bar{s}, \forall i \in N$) and $g(s)$ is invariant with respect to any permutation of s. It is easy to verify that the value function satisfies the *single crossing property*: For all s and $i \neq j$, $\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i}$. Moreover, for any s, $v_i(s) > v_j(s)$ if and only if $s_i > s_j$, that is, whoever has a higher private value component has a higher overall value. The allocation that assigns the object to a bidder with the

[5]Our efficiency and revenue results continue to hold in more general conditions for the value function. They are available upon request.

[6]Some previous studies have adopted this value function in the standard information setup. For instance, see Wilson (1998) and Hong and Shum (2003).

highest value—or the highest private component—for every realization of the signal profile in the support is called (ex-post) *efficient*.

The stratified information structure is modeled by partitioning N into I, a set of *insiders*, and O, a set of *outsiders*: Each outsider $i \in O$ knows only the private value component s_i, whereas each insider $i \in I$ knows both the private and common value components, s_i and $g(s)$, respectively. If $N = O$ (i.e., there is no insider), the model reduces to the *standard information case* where every bidder i is informed only of his own signal s_i. We make a parsimonious assumption on insiders' information by specifying only that they know the values of s_i and $g(s)$ for each s, and being silent on what signals precisely they are informed of.[7] From the perspective of Bayesian games, s_i is the bidder i's type if he is an outsider while $(s_i, g(s))$ is the bidder i's type if he is an insider.

We impose no restriction on the number of insiders or outsiders except that there are at least one insider and one outsider, that is, $|I| \geq 1$ and $|O| \geq 1$. The information structure described thus far is assumed to be common knowledge among bidders: in particular, outsiders know who insiders are. This assumption is reasonable in the examples mentioned in the introduction, because it is commonly known who owns neighboring tract in an OCS auction, who are existing shareholders or current management trying to buy the target firm in an takeover auction, and who are expert bidders in an artwork auction.

For the auction format, we focus on the sealed-bid second-price (SBSP) auction and the English auction.[8] In the second-price auction, the highest bidder wins the object and pays the second highest bid. For the English auction, we consider the Japanese format, where bidders drop out of the auction as the price continuously rises starting from zero until only one bidder remains and is awarded the object at the last drop-out price.[9] We assume that bidders observe who has dropped out at what price. Combined with the assumption that the identity of insiders is commonly known, this implies that bidders can distinguish whether it is an insider or an outsider who has dropped out, so the inference of his information will be made accordingly.

Despite the similarity in the pricing rules, the two auction formats are different in that bidders in the English auction, especially outsiders, are given an opportunity to observe others' drop-out prices and update their information while this does not occur in the second-price auction. In both auction formats, we assume that each insider, who knows his value

[7]In the first-price auction, however, what insiders know beyond their values can be important, because it provides useful information about their opponents' bids.

[8]We do not consider the first-price auction mainly because of its analytical intractability under the stratified information structure. In the case of one insider and one outsider, however, an inefficiency result can be established. Refer to Kim (2016) for this result.

[9]In both auction formates, ties are broken randomly.

precisely, employs the weakly dominant strategy of bidding (or dropping out at) his value.

Remark 1. As mention in the Introduction, our model differs from a multi-dimensional information setup in which private and common value components are determined independently. In such environment, there is little hope of achieving the efficient allocation even in a very simple setup with one insider and one outsider (in which the second-price and English auctions are strategically equivalent). To see it, suppose that each bidder's value is given as a sum of private component, which can be 0, 1, or 2, and common component, which can be 0 or 100, while there is no deterministic relationship between the two components. For efficiency, the outsider who has private value equal to 1 must bid higher than 100 so as to outbid the insider who has private value equal to 0 and thus bids 100 if the common value is 100. Given that the outsider's bid can *not* depend on the common value component, by bidding higher than 100, he would also outbid the insider who has private value equal to 2 and thus bids 2 if the common value is 0, resulting in an inefficiency. □

2.2 Second-Price Auction and Its Inefficiency

To begin, we provide the efficiency result with two bidders, one insider and one outsider. In this case, the second-price and English auctions are (strategically) equivalent because English auction ends as soon as one bidder drops out; thus, bidders have no chance to update their information as in the second-price auction. With two bidders, both auction formats yield the efficient allocation as shown by the following theorem. The proofs of this theorem and all subsequent results are contained in the Appendix, unless stated otherwise.

Theorem 1 (Efficiency with Two Bidders)**.** *With $n = 2$, there is an ex-post equilibrium of the second-price or English auction that achieves the efficient allocation.*

With more than two bidders, however, the second-price auction ceases to be efficient:

Theorem 2 (Inefficiency of Second-Price Auction)**.** *Suppose that $n \geq 3$ and $\frac{\partial g}{\partial s_i} > 0$ for all i. Suppose also that the efficient allocation requires insiders to obtain the object with a positive probability less than one. Then, there exists no efficient equilibrium for the second-price auction.*

The source of this inefficiency is that the bidding strategy of an outsider cannot efficiently adjust with that of an insider, since the former depends only on his own signal whereas the latter depends on the entire signal profile. To illustrate this, consider the case with three bidders. Recall that any bidder with the highest signal (i.e., the highest private value component) must win for efficiency. Suppose that bidder 1 is an outsider while bidder

2 is an insider and bids his value $v_2(s_1, s_2, s_3)$. The existence of an efficient equilibrium requires that any equilibrium bid $b_1(s_1)$ of bidder 1 with s_1 fixed at $\hat{s}_1$ satisfies

$$\max_{\{(s_2,s_3)|\hat{s}_1 \geq \max\{s_2,s_3\}\}} v_2(\hat{s}_1, s_2, s_3) \leq b_1(\hat{s}_1) \leq \min_{\{(s_2,s_3)|s_2 \geq \max\{\hat{s}_1,s_3\}\}} v_2(\hat{s}_1, s_2, s_3), \tag{2}$$

where the leftmost term is the maximal value for bidder 2 when bidder 1 should win (for efficiency) while the rightmost term is the minimal value for bidder 2 when bidder 2 should win. However, the inequality cannot hold since the left-hand side is equal to $v_2(\hat{s}_1, \hat{s}_1, \hat{s}_1)$ and thus greater than the right-hand side, which is equal to $v_2(\hat{s}_1, \hat{s}_1, 0)$, implying the lack of an efficient equilibrium. What causes this inefficiency here is that the outsider's bid does not respond to the signal of third party (i.e., bidder 3) while the insider's bid does.

While we are unable to fully analyze the equilibrium strategy for the second-price auction in the general setup, it is possible to pin down the equilibrium bid by outsider with the highest signal in the symmetric setup:

Proposition 1. *Suppose that $I \neq \emptyset$ and that bidders are symmetric. Then, in any undominated equilibrium where all outsiders employ the same strictly increasing bidding strategy, each outsider with the highest signal $\bar{s}$ must bid $v(\bar{\mathbf{s}})$, where $\bar{\mathbf{s}}$ is n-dimensional vector with every element being equal to $\bar{s}$.*

This result says that in any equilibrium where outsiders behave symmetrically, each outsider with the highest type (or signal) must bid the highest possible value, which is clearly higher than the equilibrium bid by the same type in the standard information setup. The intuition is simple: if an outsider with the highest signal can raise his bid (above the equilibrium level) and increase his winning probability, it must be against an insider, in which case the deviation is profitable since the outsider pays the insider's value that is lower than his. This means that such an outsider type must be bidding the highest possible value in any symmetric equilibrium. Thus, we can say that the presence of insider induces outsiders with high signals to bid more aggressively (provided that the equilibrium bidding strategy is continuous around $\bar{s}$). This observation will turn out to be useful for understanding a revenue implication of insiders in the second-price auction, later in Section 2.4 (in particular, Example 3). The following example offers a detailed description of equilibrium strategy for the second-price auction in a simple setup.

Example 1. Suppose that there are three bidders each of whom has a signal uniformly distributed on the interval $[0, 1]$. For each $i \in N = \{1, 2, 3\}$, $v_i(s) = 2s_i + \sum_{j \neq i} s_j$. We consider three information structures, $I = \emptyset$, $I = \{3\}$, and $I = \{2, 3\}$. Assuming that insiders use the dominant strategy of bidding their values, we aim to find symmetric

Bayesian Nash equilibrium bidding strategy for outsiders, denoted as $B : [0,1] \to \mathbb{R}_+$. In the case of $I = \emptyset$, Milgrom and Weber (1982) provides a symmetric equilibrium bidding strategy as follows:

$$B(s_i) = \mathbb{E}_{s_{-i}}[v_i(s_i, s_{-i}) | \max_{j \neq i} s_j = s_i] = \frac{5}{2} s_i.$$

In the case of $I = \{2,3\}$, the equilibrium bidding strategy for bidder 1, which is the best response to value-bidding by bidders 2 and 3, is given as

$$B(s_1) = \begin{cases} \frac{17}{5} s_1 & \text{if } s_1 \in [0, \frac{5}{6}] \\ 7s_1 - 3 & \text{otherwise.} \end{cases} \tag{3}$$

Detailed analysis for this result is provided in Online Appendix I, where we also provide a (numerical) analysis for the equilibrium strategy in the case $I = \{3\}$.

Note first that consistent with Proposition 1, the outsider with the highest type bids $B(1) = 7 - 3 = 4$ (i.e., the highest possible value). A more detailed picture of the outsider's equilibrium strategies is provided in Figure 1, where k is the number of insiders. It shows that as the number of insiders increases, outsiders with high signals get to bid more aggressively while those with low signals get to bid slightly less aggressively.

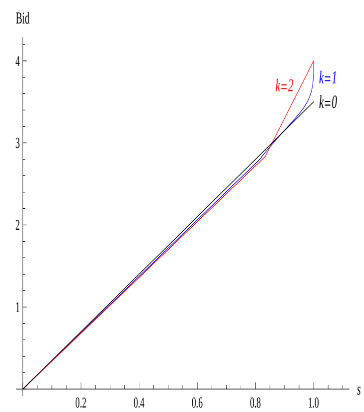

Figure 1: Outsider's equilibrium bidding strategy in the second-price auction

In Figure 2 below, we fix s_1 at $\hat{s}_1 = 0.7$ and illustrate how the equilibrium allocation is given depending on (s_2, s_3). The area E_i is where bidder i has the highest signal and should thus obtain the object in the efficient allocation. The dashed line corresponds to the signals (s_2, s_3) at which the equilibrium bid of bidder 1 with $\hat{s}_1 = 0.7$ is equal to $\max\{v_2(\hat{s}_1, s_2, s_3), v_3(\hat{s}_1, s_2, s_3)\}$, i.e., the higher of the two insiders' bids. Thus, bidder 1 is a winning (losing) bidder below (above) that line. This implies that in the shaded area A_i, the object is allocated to bidder i, although his value is not the highest. $\square$

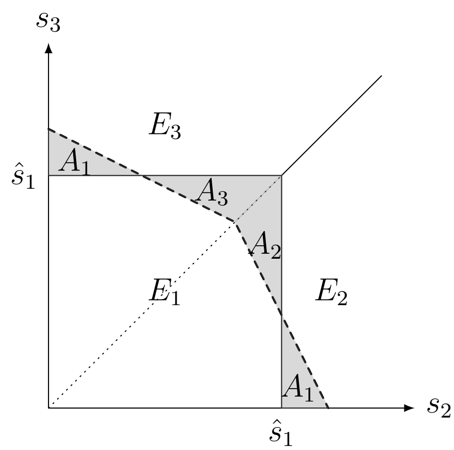

Figure 2: Inefficiency of equilibrium allocation in the second-price auction

2.3 Efficient Equilibrium of English Auction

We provide an equilibrium for English auction that yields the efficient allocation, by extending the equilibrium constructions in Milgrom and Weber (1982) and Krishna (2003) in our setup. A key feature of our equilibrium construction consists of describing how outsiders infer others' signals from their drop-out prices and how to use this information to determine their own drop-out prices. Assume for the moment that each outsider's drop-out price at any point in the auction is strictly increasing in his signal, so his signal is revealed as he drops out. Moreover, insiders' drop-out prices are equal to their values. Using this information, active outsiders who have yet to drop out calculate the *break-even signals* at each current price, which is defined as the signal profile that makes all active bidders break even if they acquire the object at the current price. Then, each active outsider stays in (resp. exits) the auction if his break-even signal at the current price is smaller (resp. greater) than his true signal. The assumed monotonicity of each outsider's drop-out price with respect to his signal is ensured if his break-even signal is strictly increasing as a function of the current price.

To formalize this idea, we introduce a few notations. Let A denote the set of active bidders. Then, $N \backslash A$, $O \backslash A$, and $I \backslash A$ denote the set of inactive bidders, inactive outsiders, and inactive insiders, respectively. With p_i denoting the drop-out price of bidder i, let $p_B = (p_i)_{i \in B}$ for a subset of bidders $B \in N$. Then, a price profile, $p_{N \backslash A}$, corresponds to the history of the game at the point where only bidders in A are active. Next, suppose that the current price is equal to p with a history $p_{N \backslash A}$. Suppose also that the signals of inactive outsiders have been revealed to be $s_{O \backslash A}$. Given these revealed signals, we define the *break-even signal profile*, denoted as $(s_A(p;p_{N\backslash A}), s_{I\backslash A}(p;p_{N\backslash A}))$, to solve the following system of equations:

$$v_i(s_{O\backslash A}, s_A(p;p_{N\backslash A}), s_{I\backslash A}(p;p_{N\backslash A})) = \begin{cases} p_i & \text{for } i \in I\backslash A \quad (4a) \\ p & \text{for } i \in A \quad (4b) \end{cases}$$

The equations in (4a) says that given the profile $(s_{O\backslash A}, s_A(p;p_{N\backslash A}), s_{I\backslash A}(p;p_{N\backslash A}))$, the value of each inactive insider i is equal to his drop-out price p_i, which is consistent with the insiders' value bidding strategy. The equations in (4b) says that if the active bidders acquire the object at the current price p, then they would break even, which is why the solution of (4) is called break-even signals. Then, the outsiders' equilibrium strategy is simple: After any history $p_{N\backslash A}$, each outsider i drops out (stays in) at price p if and only if $s_i \leq (>) s_i(p;p_{N\backslash A})$. Thus, if $s_i(p;p_{N\backslash A})$ is strictly increasing and continuous with p, an outsider i with signal s_i drops out at a (unique) price p at which $s_i = s_i(p;p_{N\backslash A})$. This means that each outsider's signal is revealed via his drop-out price, as we assumed in order to set up the system of equations (4).

The outsiders' strategy of dropping out when the break-even signals equal their true signals—combined with the insiders' value bidding strategy—implies that if an outsider (unilaterally) deviates to stay longer than prescribed by this strategy, then he would end up paying a price higher than his true value when becoming a winner after all other bidders have dropped out. This is because his break-even signal at the winning price is higher than his true signal and also calculated by setting all other bidders' signals or values at their true levels. Likewise, exiting the auction earlier would cause the bidder to forgo a chance to earn a positive payoff. Note that this observation depends crucially on the existence of monotonic break-even signals as solution to the system in (4). The proof of Theorem 3 below establishes this monotonicity and use it to show that the drop-out strategy described above constitutes an (ex-post) Nash equilibrium and also leads to the efficient allocation.[10]

Theorem 3 (Efficiency of English Auction)**.** *Consider the English auction with $n \geq 3$.*

(i) There exists an ex-post Nash equilibrium where each outsider $i \in O\backslash A$ drops out at price p after history $p_{N\backslash A}$ if and only if $s_i < s_i(p;p_{N\backslash A})$, where $s_i(p;p_{N\backslash A})$ solves (4).[11]

[10] To be more precise, we prove the strict monotonicity of the signals $S_A(p;p_{N\backslash A})$ by using the fact that the value function given in (1) satisfies a sufficient condition provided by Krishna (2003), called the *average dominant condition*.

[11] This result does not rule out the existence of other equilibria, in particular, where outsiders use alternative drop-out strategies. In fact, Bikhchandani and Riley (1991) show that English auction admits

(ii) The equilibrium strategy described in (i) leads to the efficient allocation.

An intuitive understanding of the efficiency result can be gained by comparing it with the inefficiency of the second-price auction. For this, let us recall from the three bidder example that the inefficiency of the second-price auction is caused by the mismatch between the bids of outsider and insider, that is, the insider's bid responds to a third party's signal while the outsider's bid does not. This problem disappears in the English auction. To see this, suppose that bidder 3 is the only insider, and that bidder 2 drops out first and reveals his true signal, which bidder 1 learns and subsequently reflects in his drop-out strategy. Then, there is no longer informational asymmetry between bidder 1 and 3 regarding s_2. Furthermore, from this point on, the bidding competition is reduced to the two bidder case with one outsider and one insider, as in Theorem 1, where efficiency is easily obtained. In the current example with three bidders and one insider, the argument for the efficiency result based on this intuition can be completed by observing that the outsiders' drop-out strategy implies the following: (i) they drop out before the price reaches their values; (ii) they drop out in order of their values. Because of (i), an insider with the highest value always becomes a winner. In case an outsider has the highest value, efficiency is also achieved because, given (ii), another outsider drops out and reveals his signal before the highest-value outsider does so. The proof of Theorem 3 generalizes this argument to cases with more insiders or outsiders.

The next example illustrates the construction of equilibrium strategy described above.

Example 2 (Equilibrium and Revenue in English Auction). Let us consider the same linear example as in Example 1. In the case of $I = \emptyset$ (i.e., no insider), the equilibrium strategy in Theorem 3 takes the same form as in MW: If no one has dropped out, the break-even signal for each bidder i is given as $\frac{p}{4}$, meaning that bidder i drops out at price equal to $4s_i$; if one bidder j has already dropped out at p_j and thereby revealed his signal s_j, the break-even signal for each remaining bidder i is given as $\frac{1}{3}(p - s_j)$, meaning that bidder i drops out at a price equal to $3s_i + s_j$.

Let us turn to the case where $I \neq \emptyset$. If no one has dropped out or one outsider has dropped out, an (active) outsider's drop-out strategy remains the same as before. After an insider $j \in I$ has dropped out at price p_j, the condition in (4) becomes

$$2s_i(p; p_j) + \sum_{k \neq i} s_k(p; p_j) = p \text{ for each } i \neq j$$

a large class of equilibria in the standard information structure. It is an interesting question how the equilibrium multiplicity is affected by the presence of insiders, which is beyond the scope of the current paper, though, and left for future research.

$$2s_j(p;p_j) + \sum_{k \neq j} s_j(p;p_j) = p_j,$$

which yields the break-even signal $s_i(p;p_j)$ for each $i \neq j$ that solves

$$3s_i(p;p_j) + \tfrac{1}{2}(p_j - 2s_i(p;p_j)) = p.$$

Given the equilibrium strategy that calls for each outsider i to drop out at price p satisfying $s_i(p;p_j) = s_i$, this equation implies that an outsider i drops out at a price equal to $3s_i + \frac{1}{2}(v_j(s) - 2s_i)$.

Focusing on the case of a single insider, Table 1 reports the drop-out prices that result from the equilibrium strategy in English auction with $I = \{3\}$ and $s_1 > s_2$. Observe first that a bidder with the highest signal never drops out earlier than others, implying that the allocation is efficient. To examine the drop-out orders more carefully, the outsider who has a higher signal, and thus a higher value, than the other outsider, always drops out later than the latter. Interestingly, however, the drop-out orders between outsider and insider need not be aligned with their values: for instance, in case (iii) of Table 1, bidder 2 drops out later than bidder 3, though the former has a lower value than the latter. The efficiency of resulting allocation is not affected despite this misalignment between the bidders' drop-out orders and their values.

The second drop-out price in the table corresponds to the sale price, which is (weakly) higher than the sale price with no insider in the rightmost column. Thus, the seller's ex-post revenue becomes (weakly) higher if we switch one outsider to an insider. In fact, this revenue result holds true generally in our setup, as will be shown in the next subsection. □

Table 1: Drop-out prices for the English auction with $I = \{3\}$ and $s_1 > s_2$

	1st drop-out price	2nd drop-out price (= sale price)	sale price with $I = \emptyset$
(i) $s_3 > s_1 > s_2$	$p_2 = 4s_2$	$p_1 = 3s_1 + s_2$	$3s_1 + s_2$
(ii) $s_1 > s_3 > s_2$	$p_2 = 4s_2$	$p_3 = v_3(s)$	$3s_3 + s_2$
(iii) $s_1 > s_2 > s_3$ and $4s_2 > v_3(s)$	$p_3 = v_3(s)$	$p_2 = 3s_2 + \frac{1}{2}(v_3(s) - 2s_2)$	$3s_2 + s_3$
(iv) $s_1 > s_2 > s_3$ and $4s_2 < v_3(s)$	$p_2 = 4s_2$	$p_3 = v_3(s)$	$3s_2 + s_3$

2.4 Revenue Implications of Insiders

Based on the equilibrium strategy given in Theorem 3, we study the impact of having more insiders on the seller's revenue. Our comparative statics exercise assumes *no* increase in the total number of bidders, only increasing the number of insiders by turning some outsiders into insiders. Though the source of such information enhancement is not modeled here, it may be interpreted as resulting from the seller's attempt to feed some selective bidders with precise information on an auctioned object or the bidders' information acquisition effort.

Let us consider two English auctions, E and E', which differ by only one bidder who switches from an outsider in E to an insider in E'. Given a signal profile s, let $P(s)$ and $P'(s)$ denote the seller's revenue in E and E', respectively, under the equilibrium described earlier.

Theorem 4. *For any signal profile $s \in \times_{i\in N}[0, \bar{s}_i]$, $P(s) \leq P'(s)$.*

This result follows from establishing two facts: (i') the switched insider drops out at a higher price in E' than in E; (ii') the higher drop-out price of the switched insider causes (active) outsiders to also drop out at higher prices in E'. To provide some intuition behind (ii'), we revisit the three bidder example. Suppose that bidder 1 is a winner and pays bidder 2's drop-out price p_2 after bidder 3, the only insider, first dropped out at p_3. Then, the break-even signals $s_1(p_2; p_3)$ and $s_3(p_2; p_3)$ satisfy

$$v_1(s_1(p_2; p_3), s_2, s_3(p_2; p_3)) = p_2 > p_3 = v_3(s_1, s_2, s_3). \tag{5}$$

Because bidder 1 is active at p_2, we have $s_1 > s_1(p_2; p_3)$, which implies $s_3 < s_3(p_2; p_3)$ by (5). The fact that $s_1 > s_1(p_2; p_3)$ means the break-even signal of the active outsider underestimates his true signal, which has the effect of lowering the selling price p_2. However, this effect is mitigated by the fact that $s_3 < s_3(p_2; p_3)$, that is, the insider's signal is overestimated. The underestimation of active outsiders' signals results from their attempt to avoid the winner's curse by bidding as if the currently unknown signals are just high enough to make them break even at the current price. This is why an outsider drops out before the price reaches his value, as (i') above suggests. Thus, an outsider becoming a better-informed insider alleviates the detrimental effect of the winner's curse on both his and other outsiders' drop-out prices.

Theorem 4 immediately implies the following corollary.

Corollary 1. *Suppose that the set of insiders expands from I to $I' \supseteq I$ (with the same set of bidders). Then, the seller's revenue (weakly) increases for each signal profile.*

Our revenue prediction is reminiscent of the linkage principle of Milgrom and Weber (1982) in that enhancing the bidders' information has a positive impact on the seller's revenue. In the linkage principle, the extra information is publicly disclosed to all bidders whereas, in our result, only a subset of bidders entertains better information. This difference, however, is somewhat diluted by the fact that, in English auction, the additional information of bidders who switch to insiders is partially disclosed to the other bidders during the bidding process. Another difference is that our revenue prediction holds *ex-post*—that is, for every realization of the signal profile—and thus does not depend on the assumption of affiliated signals.

One may ask how the presence of more insiders affects the revenue of the second-price auction. Also, it will be interesting to see how the linkage principle, applied to the revenue comparison between the second-price and English auctions, is affected by the presence of insiders. Giving general answers to these questions is beyond the scope of the current paper due to the lack of equilibrium analysis for the second-price auction in the general setup. Instead, we rely on the analysis of the second-price auction in Example 1 to shed some light on the above questions.

Example 3 (Revenue Comparison for English and Second-Price Auctions). Let us continue with the setup in Example 1 and 2. Table 2 below reports the seller's expected revenues from the second-price and English auctions that are (numerically) calculated using the equilibrium strategies in Example 1 and 2: Consistent with Theorem 4, the table shows

Table 2: Seller's expected revenues from the second-price and English auctions

Number of insiders	Second-Price Auction	English Auction
$k = 0$	1.75	1.75
$k = 1$	1.92	1.83
$k = 2$	1.95	1.97
$k = 3$	2	2

that the expected revenue of English auction increases with the number of insiders (since its ex-post revenue increases). The same is true for the second-price auction, while the increase mostly occurs as k increases from 0 to 1. First, this is due to a more aggressive bidding by the bidder who switches to an insider. Also, as we have observed from Proposition 1 and Example 1, the presence of (more) insiders induces outsiders with high signals to bid more aggressively, though it may make those with low signals slightly less aggressive. According to the above table, however, the impact of having more than one insider on the revenue of the second-price auction is only marginal, relative to its impact on English auction. Also, the insider's positive impact on the revenue of the second-price auction does not occur in the ex-post sense, unlike English auction.[12] It is thus unclear whether the insider's revenue effect in the second-price auction is robust beyond the uniform distribution case.

Lastly, the above table shows that the revenue comparison between the two auction formats is ambiguous: the expected revenue of the second-price auction is higher with one insider and lower with two insiders than that of English auction. This suggests that the linkage principle breaks down in our setup, provided that the revenues change continuously as the signal distribution is slightly perturbed to allow for some affiliation. □

3 Experimental Design and Procedures

The experiment was run at the Experimental Laboratory of the Centre for Economic Learning and Social Evolution (ELSE) at University College London (UCL) between December 2011 and March 2012. The subjects in this experiment were recruited from an ELSE pool of UCL undergraduate students across all disciplines. Each subject participated in only one of the experimental sessions. After subjects read the instructions, the instructions were read aloud by an experimental administrator. Each experimental session lasted around 2 hours. The experiment was computerized and conducted using experimental software z-Tree developed by Fischbacher (2007). Sample instructions are reported in Online Appendix II.

In the design, we use a variety of auction games with three bidders, $i = 1, 2, 3$. Each bidder i receives a private signal, s_i, which is randomly drawn from the uniform distribution over the set of integer numbers, $\{0, 1, 2, ..., 100\}$. Given a realization of signal profile $s = (s_1, s_2, s_3)$, the object valuation for each bidder i is given by equation 1 where $g(s) = \sum_j s_j$.

We have in total six treatments in terms of the auction formats—the SBSP auction and the English auction—and the number of insiders varying from zero to two, $k = 0, 1, 2$. A single treatment consisting of one auction format and a single value of k is used for each session. We conduct 12 sessions in total with two sessions for each auction game treatment. Each session consists of 17 independent rounds of auction games, where the first two rounds were practice rounds in which auction outcomes were not counted for actual payoffs. Throughout the paper, we use data generated only after the first two rounds. The following Table 3 summarizes the experimental design and the amount of experimental data. The first number in each cell is the number of subjects and the second is the number of group observations in each treatment. In total, 233 subjects participated

[12]For instance, an outsider whose signal is equal to 1 and bids $B(1) = 4$ (almost) always reduces his bid below 4 as he switches to an insider, decreasing the seller's ex-post revenue.

in the experiment.

Table 3: Experimental sessions and observations

Auction format	# of insiders (k)	Session 1	Session 2	Total
English	0	21 / 105	21 / 105	42 / 210
	1	18 / 135	20 / 150	38 / 285
	2	23 / 345	19 / 285	42 / 630
SBSP	0	21 / 105	21 / 105	42 / 210
	1	20 / 150	16 / 120	36 / 270
	2	17 / 255	16 / 240	33 / 495

We use an irrevocable-exit, ascending clock version of the English and the SBSP auctions (Kagel et al., 1987; Kirchkamp and Moldovanu, 2004). In the English auction treatments, each subject sees three digital clocks representing the bidding process on his or her computer screen, one for each bidder in the group. The computer screen indicates which clock belongs to whom and which belongs to an insider, if any. After one bidder stops his or her clock, the remaining bidders observe that the bidder's clock has stopped. If one more participant drops out, the auction is over. The remaining bidder wins the auction and pays the price at which the second bidder drops out. In the second-price auction treatments, there is only one clock presented to each subject. The subject drops out by stopping his or her clock. If the other two participants have not yet dropped out, their clocks continue to ascend. Once all three bidders have stopped their own clocks, the auction is over. The remaining bidder who chooses the highest price wins the auction and pays the price at which the second bidder drops out. Ties are broken (uniform) randomly. Once subjects have dropped out, they are not allowed to re-enter the auction.

In treatments with insider(s), each insider bidder is played by the computer, whereas outside bidders are played by human subjects. The groups formed in each round depend solely upon chance and are independent of the groups formed in any of the other rounds. Once assigned to a three-bidder group, each subject observes his or her private signal and the valuation formula. Other bidders' signals in the formula are hidden.

The (computer-generated) insider always drops out at a price equal to its own valuation. This information is common knowledge to subjects. Using computer-generated insiders has the benefit of keeping our experiment close to the theory. Notice that if we use human insiders instead, and they deviate from bidding their valuations, it will be more difficult to interpret outsiders' behavior within our theoretical framework.

When an auction round ends, the computer informs each subject of the results of that round, which include bids at which bidders dropped out, signals bidders received, auction object values, payments and earnings in the round. The next round starts with the computer randomly forming new groups of three bidders and selecting signals for them.

Earnings were calculated in terms of tokens and exchanged into pounds at the rate of 40 tokens to £1. In order to avoid potentially adverse impacts of limited liability for losses on bidding behavior,[13] we provided each subject a relatively large amount of money, £10, as the initial balance in the experiment. None of the subjects experienced bankruptcy during the experiment. The total payment to a subject was the sum of his or her earnings over rounds, plus the initial balance £10 and an extra £5 participation fee. The average payment was about £18.32 (min *£* 11.5, max *£*26.8). Subjects received their payments privately at the end of the session.

4 Experimental Findings

4.1 Efficiency

Table 4 reports the frequency of efficient allocation as well as the average efficiency ratio by measuring the economic magnitude of inefficient outcomes across auction treatments. The efficiency ratio is defined as the actual surplus improvement over random allocation as a percentage of the first-best surplus improvement over random assignment.[14] This ratio equals one if the allocation is efficient, and less than one otherwise. For auction treatments with one insider (resp. two insiders), we group the data with respect to the ranking of the insider's value (resp. the outsider's).

- Table 4 here -

The frequencies of efficient allocation are high in all treatments and range from 76% (in the SBSP auction with $k = 1$) to almost 90% (in the English auction with $k = 2$). The efficiency ratio shows similar patterns. In the symmetric information structure with

[13] Hansen and Lott (1991) argued that aggressive bidding behavior in a common value auction experiment conducted by Kagel and Levin (1986) may be a rational response to limited liability rather than a result of the winner's curse. Lind and Plott (1991) designed an experiment eliminating the limited-liability problem and found that this problem does not account for the aggressive bidding patterns in the experiment of Kagel and Levin.

[14] Our measure of the efficiency ratio normalizes the realized surplus by both the best-case scenario (efficiency) and the worst-case one (random assignment). This double-normalization renders a more robust measure against the rescaling of the value support than an alternative measure such as the percentage of the first-best surplus realized.

no insider ($k = 0$), there is no significant difference in efficiency between the two auction formats. In the presence of an insider, however, the efficiency outcomes are significantly higher in the English auction than in the SBSP auction. These results are qualitatively consistent with the theoretical predictions that when at least one insider is present in auction, an efficient equilibrium exists in the English auction but not in the SBSP auction. We further divide the data with respect to the value ranking of bidders. In the auctions with one or two insiders ($k \in \{1, 2\}$), the efficiency ratio of the English auction is significantly higher than that of the SBSP auction, regardless of the value ranking (except for the case where the outsider has the second-highest value and $k = 2$).

To further examine the English auction's superior efficiency performance, we divide the English auction treatment data into two subsamples with respect to whether the SBSP auction, if it had been used, would have produced an efficient or inefficient allocation according to the theoretical prediction. Because of the small sample size of the inefficient equilibrium allocations, we allow for a five-token margin to classify the case of inefficiency such that a single data point is treated as *inefficient* if the equilibrium of the SBSP auction is either inefficient or efficient but the difference between its two high bids is less than five tokens.[15] We then check how often subjects are able to achieve an efficient allocation in each of these data points. We conduct the same analysis for the SBSP auction treatments. Table 5 presents the frequencies of efficient allocation in each case in the treatments with at least one insider with the number of observations in parentheses.

- *Table 5 here* -

In the case where the SBSP auction format predicts an efficient allocation, actual efficiency frequencies are quite high, ranging from 81% (in the SBSP auction with $k = 1$) to 92% (in the English auction with $k = 2$). In contrast, the level of efficiency becomes much lower in the case where the equilibrium allocation under the SBSP auction format is inefficient: the frequencies of efficiency range between 33% (in the SBSP auction with $k = 1$) and 66% (in the English auction with $k = 2$). In each treatment, the frequency difference of efficient outcomes between these two cases is statistically significant at usual significance levels. Thus, when it is predicted that the bidding mismatch problem between outsiders and insiders would arise, subjects are more likely to fail to attain an efficient allocation. Furthermore, in the case where the equilibrium of the SBSP auction predicts an inefficient allocation, the English action performs significantly better than the second-price auction

[15]The results remain basically the same either whether we use no margin or small one. Also, whether we perform a round by round analysis.

when there is a single insider. In the case of two insiders, we find no difference between the two auction formats. We summarize the efficiency outcomes as follows.

Finding 1 (efficiency) *The English auction exhibits higher efficiency performance than the second-price auction does in the presence of insiders, as theory predicts. In the symmetric information structure where there is no insider, there is no difference of efficiency performance between the two auction formats.*

4.2 Revenue

Next, we compare revenue performance across treatments. Table 6 presents the average percentage deviations of observed revenues from their theoretically predicted values across treatments. For auction treatments with one insider (resp. two insiders), we divide the data with respect to the ranking of the value of the insider (resp. the outsider).

- *Table 6 here* -

When all bidders are outsiders, observed revenues are significantly higher than theoretically predicted ones. This tendency becomes weaker in auctions with at least one insider. This may not be unexpected, because insiders in the experiment are computer-generated and play the equilibrium strategy of bidding their own values. Despite this consideration, observed revenues in the SBSP auction with $k = 1$ are significantly above the theoretical prediction. In auctions with $k = 1$, the tendency of revenues to be higher than theoretically predicted strengthens when both outsiders have lower or higher values than the insider. This apparently results from overbidding, relative to the BNE, by outsiders (see next subsection). In auctions with $k = 2$, the magnitude of the departures of observed revenues from theoretical ones is not large, although some of the departures remain significant.

Our theory establishes the linkage principle of the English auction that for any signal profile, the switch of an outsider to an insider weakly increases revenue. We thus run regressions of observed revenues (resp. theoretical revenues) on signal profiles and dummies for the number of insiders in each auction format. The results are reported in Table 7.[16]

- *Table 7 here* -

[16] As a robustness check of Table 7, we conduct regression analysis with more flexible functional specifications of quadratic forms of signals or dummies for insider/outsider and their interactions with signals. These results are reported in Online Appendix III. In essence, the empirical findings on the linkage principle remain unchanged. These findings do not change either whether we include round fixed effects.

Controlling for the signal profile, switching an additional outsider to an insider improves revenues significantly in the English auction. The magnitudes of revenue improving with an extra insider in the data are also consistent with those predicted by theory as shown in regressions with theoretical revenues. Analogously, we observe the revenue-improving outcomes in the SBSP auction data, which is qualitatively consistent with the prediction in Table 2 of Example 3.

We summarize our revenue findings as follows.

Finding 2 (revenue) *Revenues in the data tend to deviate above theoretically predicted values, in particular in auctions with no insider. Despite this tendency, the increase in the number of insiders has a positive impact on revenues in the English auction, consistent with the linkage principle of the English auction. We find similar revenue-improving patterns with extra insiders in the SBSP auction.*

4.3 Bidding Behavior

In this section, we examine subjects' bidding behavior and quantitative departures from the BNE predictions. We begin this analysis with the SBSP auction.

SBSP Auction We run linear regressions of subjects' bids on their private signals with robust standard errors clustered at the individual level. Our theory predicts that the BNE strategy has a kink at the signal value equal to 82.109 when $k = 1$ and that equal to 500/6 when $k = 2$. We thus use the regression specifications with and without these kinks.[17] Table 8 reports the results of the regressions presenting p-values of the F-test for the null hypothesis that observed bids follow the equilibrium strategy.

- Table 8 here -

The regression analysis indicates that subjects respond less sensitively to their own private signals than theory predicts. The estimated coefficients on signals are significantly lower than the theoretical prediction of approximately 3.5 in each insider treatment. Furthermore, the constant term in the regression is significantly positive in each treatment also suggesting that subjects tend to overbid relative to the equilibrium when signals are low. The joint test based on the F-statistic indicates that subjects' behavior differ significantly from the equilibrium strategy. Overall, the overbidding pattern in our data is consistent with previous findings in the literature (see Kagel, 1995; Kagel and Levin, 2002).

[17]For the sake of visual inspection, in Online Appendix V we present a set of scatter plots between subjects bids and independent variables used in the regression analysis, such as their private signals, in the SBSP and English auction treatments.

English Auction We run censored regressions of the first drop-out and second drop-out prices with the sample of outsiders for the English auction treatments. The censored regression method is required because we observe only the first drop-out price for the lowest bidder and the bids of two remaining bidders are right-censored, and because the second drop-out price is left-censored by the first drop-out price. In the treatment with one insider ($k = 1$), as the BNE strategy predicts, the regression specification for the second drop-out price interacts with the signal of the second drop-out bidder; furthermore, the first drop-out price interacts with the dummy indicating whether it is an insider who drops out first. We further include an alternative specification by adding this dummy in the regression equation to capture the potential empirical impact of this dummy on the constant term. Table 9 reports the regression results and statistical tests for the joint null hypothesis that observed bids follow the equilibrium strategy.[18]

- Table 9 here -

The regression analysis of the first drop-out prices reveals that the subjects respond less sensitively to their private signal than the equilibrium strategy across insider treatments predicts. Whereas the equilibrium behavior responds to the private signal by a factor of 4, regardless of the number of insiders, the estimated coefficients are significantly lower than this theoretical prediction. We also found that the constant term is significantly positive in all insider treatments. Given these results, the null hypothesis that the first drop-out prices follow the BNE strategy is rejected at usual significance levels. The estimated coefficient on the private signal increases and the constant term declines as the number of insiders increases. Thus, the presence of an insider who knows the value of the object may make outsiders more wary of hedging against informational asymmetry between insider and outsider and may help alleviate the winner's curse.

We turn to the regression analysis of the second drop-out prices. Similar to the first drop-out bidders, the second drop-out bidders respond less sensitively to their own private signal than the equilibrium analysis predicts. However, they respond excessively to the first drop-out prices. Despite the excessive responsiveness to the first drop-out price, the combined behavioral response of the private signal and first drop-out price appears less sensitive than the equilibrium strategy dictates. The joint null hypothesis that the experimental behavior is equivalent to the equilibrium strategy is rejected at the usual significance levels in each treatment. Finally, in the auction treatment with one insider ($k = 1$), the subjects differentially responded to the identity of the first drop-out bidder:

[18] The censored regression approach, using the maximum likelihood estimation method, is given in detail in Online Appendix IV. The analysis is robust to the inclusion of round fixed effects.

they tend to place more weight on the first drop-out price and less on their private signal when the insider drops out first. This is qualitatively consistent with the BNE prediction.

Quantifying naive bidding The overbidding pattern in our interdependent value environment with insider information is closely related to the findings of the winner's curse in the experimental literature of common value auctions (see Kagel and Levin, 2002). Winning against other bidders implies that the outsider's value estimate happens to be the highest among outsiders *as well as* higher than each insider's value. Thus, failure to account for this adverse selection problem results in overbidding and can make the outsider fall prey to the winner's curse. Indeed, the winners in our data ended up with negative surplus more frequently than the theory predicts.[19]

We employ a simple strategy of quantifying the extent to which subjects in our experiment fail to account properly for the adverse selection problem and thus bid naively. We define naive bidding as bidding based on the unconditional expected value (by completely ignoring the adverse selection problem). In both auction formats, the naive bidding strategy takes the form $b^{naive}(s_i) = 2s_i + 100$. For the English auction, it means that the naive bidder ignores any information from the first drop-out price. We then consider a convex combination of the naive bidding strategy and the BNE one. For all bidders in the second-price auction and the first drop-out bidders in the English auction, this combined bidding strategy is represented by

$$b(s_i; \alpha) = \alpha \times b^{naive}(s_i) + (1-\alpha)\, b^{BNE}(s_i).$$

The equation for the second drop-out bidders in the English auction with first drop-out price p_j can be written as

$$b(s_i, p_j; \alpha) = \alpha \times b^{naive}(s_i) + (1-\alpha)\, b^{BNE}(s_i, p_j).$$

We estimate α by matching this form to the data for each treatment. α measures the degree to which the subjects' behavior departs from the BNE and is close to the naive bidding strategy. In our setup, α is well identified. For instance, for the first drop-out bidder in the English auction with each k, this convex combination can be rewritten as $b(s_i; \alpha) = 100 \times \alpha + (2\alpha + 4(1-\alpha))\, s_i$; then α is identified by matching the constant term and the slope of this equation to the data. We pool samples of first drop-out and second

[19] Online Appendix V reports empirical and theoretical frequencies of the winner getting negative surplus and average surplus across treatments.

drop-out bids to estimate a single α for each treatment of the English auction.

We note that auctions with a single outsider in our experiment are a non-strategic single-agent problem and that any deviation from the equilibrium strategy is driven by failure in optimization. In auctions with more than one outsider, deviations from the BNE can be resulted from the subject's strategic response to others' deviations on top of the subject's own deviation from optimization. Instead of developing fully a bounded rational model to decompose these, we estimate the aforementioned specification of quantifying the extent of naive bidding and compare them across treatments.

Table 10 reports the regression results of α estimates across auction treatments.

- Table 10 here -

There is substantial evidence of naive bidding in all treatments: an estimated parameter α is statistically significant at the usual significance level in all auction treatments. Moreover, in the English auction the degree of naive bidding significantly decreases in the number of insiders: estimated $\widehat{\alpha}$ are 0.67 when $k = 0$, 0.51 when $k = 1$, and 0.28 when $k = 2$. The degree of naive bidding when there is a single outsider is significantly lower than those in the other cases where there are more than one outsider. We conjecture that the presence of insiders makes the outsider more wary about information asymmetry and thus motivates the outsider to hedge against such asymmetry. This may work toward the correction of naive bidding and thus the winner's curse in our setup. On the other hand, we do not find similar monotonic patterns of $\widehat{\alpha}$ in the SBSP auction.

We summarize the bidding behavior in the experiment as follows.

Finding 3 (bidding behavior) (i) *There is evidence of overbidding relative to the equilibrium prediction in both the second-price auction and the English auction.* (ii) *The degree of this naive bidding declines significantly as the number of insiders in the English auction increases.*

5 Conclusion

We have proposed a model of interdependent value auctions with the stratified information structure and examined key predictions of the model via a laboratory experiment. Our model is distinct from the existing auction literature with insider information in a couple of important respects. First, unlike the literature where a common value is typically assumed, we adopt the interdependent value setup and study the effects of insider information in standard auctions—the SBSP and English auctions—on their efficiency and revenue. The English auction has an efficient equilibrium, whereas the second-price auction suffers inefficiency caused by the presence of the insider. Second, we allow for any number of insiders, in contrast to most studies where there is only one insider. This enables us to study the effects of varying the number of insiders on the performance of the two auction formats.

The experimental evidence supports the theoretical predictions on efficiency and revenues. As the theory predicts, the English auction achieves efficiency more frequently than the second-price auction when insider information is present, and revenues of the English auction increase in the number of insiders after controlling for realized signals. Despite the accordance of experimental data with the theory, there is substantial evidence of naive bidding, as common in the experimental literature (see Kagel, 1995; Kagel and Levin, 2011). However, the degree of naive bidding declines as more insiders are in the English auction. This offers an interesting phenomenon that warrants further theoretical investigation such as developing an alternative, behavioral model.

Appendix

Proof of Theorem 1: Suppose that bidder 1 is an insider and employs the undominated strategy of bidding $v_1(s)$ for each s. Given this, the optimal bid $b_2(s_2)$ of bidder 2 as an outsider must satisfy

$$\begin{aligned} b_2(s_2) &< v_1(0,s_2) \;\text{ if }\; v_1(0,s_2) > v_2(0,s_2) \\ &= v_2(\alpha,s_2) \;\text{ if }\; v_1(\alpha,s_2) = v_2(\alpha,s_2) \text{ for some } \alpha \\ &> v_1(\bar{s}_1,s_2) \;\text{ if }\; v_1(\bar{s}_1,s_2) < v_2(\bar{s}_1,s_2). \end{aligned} \tag{6}$$

Assume first that $v_1(0,s_2) > v_2(0,s_2)$, which implies by the single crossing property that $v_1(s_1,s_2) > v_2(s_1,s_2)$ for every $s_1 \in [0,\bar{s}_1]$. Thus, it is efficient for bidder 1 to obtain the object regardless of s_1. Because bidder 1 bids $v_1(s_1,s_2)$, bidder 2 would incur a loss by winning. Bidder 2 could avoid this loss by bidding any $b < v_1(0,s_2) \le v_1(s_1,s_2)$ and losing. A similar argument can be used to establish that an optimal bid must be at least $v_2(\bar{s}_1,s_2)$ if $v_1(\bar{s}_1,s_2) < v_2(\bar{s}_1,s_2)$, which leads to the efficient allocation. Lastly, assume that $v_1(\alpha,s_2) = v_2(\alpha,s_2)$ for some $\alpha \in [0,\bar{s}_1]$. Such α is unique because of the single crossing. Bidder 2's optimal bid b has to lie in the interval $[v_1(0,s_2), v_1(\bar{s}_1,s_2)]$ such that there exists $\phi_1(b,s_2) \in [0,1]$ satisfying $v_1(\phi_1(b,s_2),s_2) = b$. Letting $F_{S_1|S_2}(\cdot|s_2)$ denote the distribution of s_1 conditional on s_2, the expected payoff of bidder 2 with s_2 is

$$\int_0^{\phi_1(b,s_2)} (v_2(s_1,s_2) - v_1(s_1,s_2)) dF_{S_1|S_2} s_1 s_2.$$

The integrand is positive if and only if $s_1 < \alpha$, and thus the expression is maximized by setting $b = v_1(\alpha, s_2) = v_2(\alpha, s_2)$. Hence, bidder 2 wins if and only if $s_1 < \alpha$ or $v_1(s_1, s_2) < v_2(s_1, s_2)$ because of the single crossing, meaning that the resulting allocation is efficient. □

Proof of Theorem 2. Suppose to the contrary that there exists an efficient equilibrium of the second-price auction. For a given bidder i, we define $E_i := \{s \in \times_{i\in N}[0, \bar{s}_i] \mid v_i(s) \geq v_j(s) \text{ for all } j \neq i\}$, that is the set of signals for which bidder i wins the object at the efficient equilibrium. Because of the assumption that insiders obtain the good with some positive probability less than one, there must exist an outsider i, an insider j, and a signal profile s in the interior such that $v_i(s) = v_j(s) > \max_{k\neq i,j} v_k(s)$ (or $h_i(s_i) = h_j(s_j) > \max_{k\neq i,j} h_k(s_k)$). Fix any such profile s and let $E_{ij}(s_i) := \{s' | s'_i = s_i \text{ and } s' \in E_i \cap E_j\}$. Then, we can find another profile $\tilde{s} \in E_{ij}(s_i)$ such that $\tilde{s}_i = s_i$, $\tilde{s}_j = s_j$, and $\tilde{s}_k < s_k, \forall k \neq i, j$.

Next, given the efficient allocation and value bidding of bidder j, bid $b_i(s_i)$ of bidder i with s_i has to satisfy

$$\max_{\{s'\in E_i | s'_i = s_i\}} v_j(s') \leq b_i(s_i) \leq \min_{\{s'\in E_j | s'_i = s_i\}} v_j(s'). \tag{7}$$

If the first inequality were violated, bidder i with signal s_i would lose to bidder j when the former has a higher value. If the second inequality were violated, bidder j would lose to bidder i with signal s'_i when the former has a higher value. From (7),

$$\max_{s'\in E_{ij}(s_i)} v_j(s') \leq \max_{\{s'\in E_i | s'_i = s_i\}} v_j(s') \leq b_i(s_i) \leq \min_{\{s'\in E_j | s'_i = s_i\}} v_j(s') \leq \min_{s'\in E_{ij}(s_i)} v_j(s').$$

Thus, $v_j(\cdot)$ has to be constant on $E_{ij}(s_i)$. This implies that for some constant k, $v_j(s') = k, \forall s' \in E_{ij}(s_i)$, which in turn implies that $v_i(s') = k, \forall s' \in E_{ij}(s_i)$ because $v_i(s') = v_j(s')$ for any $s' \in E_{ij}(s_i)$. Thus, for any $s' \in E_{ij}(s_i)$, we must have $h_i(s_i) = h_i(s'_i) = k - g(s')$; this cannot be true, because given our assumption, we have $g(s) > g(\tilde{s})$ even though $s, \tilde{s} \in E_{ij}(s_i)$. □

We provide the proofs of Theorem 3 and 4 in Section 2.3. To simplify the notation, we let $s(p) := (s_{O\setminus A}, s_A(p; p_{N\setminus A}), s_{I\setminus A}(p; p_{N\setminus A}))$ and $s_i(p) = s_i(p; p_{N\setminus A})$ for $i \in I \cup A$, by omitting the price history $p_{N\setminus A}$. We first establish a couple of preliminary results in Lemma 2 and 3. To do so, let $\overline{v} = \max_{i\in N} \max_{s\in \times_{i\in N}[0,\bar{s}_i]} v_i(s)$. Then, we can find some $M_i \in \mathbb{R}_+$ for each i such that $v_i(M_i, s_{-i}) \geq \overline{v}$ for any $s_{-i} \in \times_{j\neq i}[0, \bar{s}_j]$.

The following result from Krishna (2003) is used to prove the existence of solution of (4):

Lemma 1. *Suppose that $R = (r_{ij})$ is an $m \times m$ matrix that satisfies the dominant average condition:*

$$\frac{1}{m}\sum_{k=1}^{m} r_{kj} > \quad r_{ij}, \forall i \neq j \quad \text{and} \quad \sum_{k=1}^{m} r_{kj} > 0, \forall j. \tag{8}$$

Then, A is invertible. Also, there exists a unique $x \gg 0$ such that $Ax = 1$, where 1 is a column vector of m 1's.

Lemma 2. *For any $s_{O\setminus A}$ and $p_{N\setminus A}$, there exists a solution $(s_A, s_{I\setminus A}) : [\max_{i\in N\setminus A} p_i, \overline{v}] \to \times_{i\in I\cup A}[0, M_i]$ of (4) such that for each $i \in A$, $s_i(\cdot)$ is strictly increasing.*

Proof. Let $v'_{A\cdot B}(s)$ denote a $|A| \times |B|$ matrix, where its ij element is $\frac{\partial v_i}{\partial s_j}(s)$ for $i \in A$ and $j \in B$. We denote $v'_{A\cdot B}$ for convenience. Let 0_A and 1_A respectively denote column vectors of 0's and 1's with dimension $|A|$.

To obtain a solution to (4) recursively, suppose that the set of active bidders is A and that the unique solution of (4) exists up to price $\overline{p} = \max_{k\in N\setminus A} p_k$. Let $(\underline{s}_A, \underline{s}_{I\setminus A})$ denote this solution at $\overline{p}$. We extend the solution beyond $\overline{p}$ to all $p \in [\overline{p}, \overline{v}]$. To do so, we differentiate both sides of (4) with p to obtain the following differential equation:

$$\begin{pmatrix} v'_{A\cdot A} & v'_{A\cdot I\setminus A} \\ v'_{I\setminus A\cdot A} & v'_{I\setminus A\cdot I\setminus A} \end{pmatrix} \begin{pmatrix} s'_A(p) \\ s'_{I\setminus A}(p) \end{pmatrix} = \begin{pmatrix} 1_A \\ 0_{I\setminus A} \end{pmatrix} \tag{9}$$
$$(s_A(\overline{p}), s_{I\setminus A}(\overline{p})) = (\underline{s}_A, \underline{s}_{I\setminus A}).$$

The first matrix on the left-hand side can be written as $v'_{I\cup A\cdot I\cup A}$. Assume for the moment that $v'_{N\cdot N}$ is invertible, which implies that its principal minors $v'_{I\cup A\cdot I\cup A}$ and $v'_{I\setminus A\cdot I\setminus A}$ are also invertible. Then, by Peano's theorem, a unique solution of (9) exists since the value functions are twice continuously differentiable. We next show that $v'_{N\cdot N}$ is invertible and that $s_A(p)' \gg 0$. To do so, we rewrite the last $|I\setminus A|$ lines of (9) as $s'_{I\setminus A} = -(v'_{I\setminus A\cdot I\setminus A})^{-1} v'_{I\setminus A\cdot A} s'_A$. Substituting this into the first $|A|$ lines of (9) yields $V s'_A = 1_A$ after rearrangement, where

$$V := v'_{A\cdot A} - v'_{A\cdot I\setminus A}(v'_{I\setminus A\cdot I\setminus A})^{-1} v'_{I\setminus A\cdot A}.$$

If V satisfies the dominant average condition for any A, then, with $A = N$, $V = v'_{N\cdot N}$ is invertible by Lemma 1. Moreover, by Lemma 1, $s'_A(p) \gg 0$.

To prove that V satisfies the dominant average condition, let $g'_k = \frac{\partial g}{\partial s_k}$ and $g'_A = (g'_k)_{k\in A}$, where g'_A is considered a column vector. Let D_A denote the $|A|\times|A|$ identity matrix. Then, for any $A, B \subset N$,

$$v'_{A\cdot B} = \begin{cases} D_A + 1_A(g'_A)^t & \text{if } A = B \\ 1_A(g'_B)^t & \text{if } A \cap B = \emptyset, \end{cases}$$

where $(\cdot)^t$ denotes the transpose of the matrix. Using this, we can rewrite V as

$$\begin{aligned} V &= D_A + 1_A(g'_A)^t - 1_A(g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A}(g'_A)^t \\ &= D_A + (1-x)1_A(g'_A)^t, \end{aligned} \tag{10}$$

where $x = (g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A}$. Because all entries in any given column of the matrix $1_A(g'_A)^t$ are identical and because the diagonal entries of D_A are positive, the first inequality of (8) is easily verified. The proof is complete if the second inequality of (8) is shown to hold, for which it suffices to show $x < 1$:

$$\begin{aligned} x &= (g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A} \\ &= (g'_{I\setminus A})^t \left(D^{-1}_{I\setminus A} - \left(\frac{1}{1 + (g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}}\right) D^{-1}_{I\setminus A} 1_{I\setminus A}(g'_{I\setminus A})^t D^{-1}_{I\setminus A}\right) 1_{I\setminus A} \\ &= (g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A} - \frac{\left((g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}\right)^2}{1 + (g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}} \\ &= \frac{(g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}}{1 + (g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}} = \frac{\sum_{k\in I\setminus A} g'_k}{1 + \sum_{k\in I\setminus A} g'_k} < 1, \end{aligned}$$

where the second equality is derived using the formula for an inverse matrix,

$$\left(A + bc^t\right)^{-1} = A^{-1} - \left(\frac{1}{1 + c^t A^{-1} b}\right) A^{-1} b c^t A^{-1},$$

with $A = D_{I\setminus A}$, $b = 1_{I\setminus A}$, and $c = g'_{I\setminus A}$. □

Given the break-even signals obtained in Lemma 2, we consider each outsider i's strategy of dropping out (staying in) at p if and only if $s_i < s_i(p)$ after any history $p_{N\setminus A}$. Along with the insiders' value-bidding strategy, we refer to this strategy profile as β^*.

Lemma 3. *Given the strategy profile β^*, for any signal profile $s \in S$, (i) outsiders drop out in order of their values; (ii) for each outsider i, $p_i \leq v_i(s)$; and (iii) at any outsider i's* *drop-out price p_i, $s_j(p_i) \geq s_j$ for each insider j who is inactive at p_i.*

Proof. Consider two outsiders i and j with $p_i \leq p_j$. Then, at price p_i at which bidder i drops out, we have

$$s_i = s_i(p_i) = p_i - g(s(p_i)) = s_j(p_i) \leq s_j,$$

where the first equality and the inequality follow from the drop-out strategy of outsiders i and j, respectively, whereas the second and third equalities follow from the break-even condition at p_i. This proves (i) because $s_i \leq s_j$ implies that $v_i(s) \leq v_j(s)$.

To prove (ii), suppose to the contrary that $p_i > v_i(s)$. Because $s_i(p_i) = s_i$, this implies that $s_i + g(s(p_i)) = p_i > s_i + g(s)$, so $g(s(p_i)) > g(s)$. Given this, for each insider $j \in I$ (whether active or not), we must have

$$s_j - s_j(p_i) \geq g(s(p_i)) - g(s) > 0, \tag{11}$$

where the first inequality holds because the break-even condition implies that for an inactive insider j, $s_j + g(s) = v_j(s) = p_j = v_j(s(p_i)) = s_j(p_i) + g(s(p_i))$ and because, for an active insider j, $s_j + g(s) = v_j(s) \geq p_i = v_j(s(p_i)) = s_j(p_i) + g(s(p_i))$. The inequality (11) implies that $s_j > s_j(p_i)$ for each insider j. Furthermore, for each active outsider $j \in O \cap A$, we have $s_j \geq s_j(p_i)$. Thus, $s \geq s(p_i)$, and thus $v_i(s) \geq v_i(s(p_i)) = p_i$, which a contradiction.

To prove (iii), note first that we have $s_i + g(s) = v_i(s) \geq p_i = s_i(p_i) + g(s(p_i)) = s_i + g(s(p_i))$ because of (ii) and the fact that $s_i(p_i) = s_i$. This inequality implies that $g(s) \geq g(s(p_i))$. If an insider j is inactive at p_i, we must have $v_j(s) = p_j = v_j(s(p_i))$ or $s_j(p_i) - s_j = g(s) - g(s(p_i)) \geq 0$, which yields $s_j(p_i) \geq s_j$. □

Proof of Theorem 3. We first prove Part (ii) by showing that the strategy profile β^*, if followed by all bidders, leads to the efficient allocation. Then, we show that it constitutes an ex-post equilibrium.

Given that outsiders drop out in order of their values (according to (i) of Lemma 3), the efficiency result follows if an outsider i with the highest value among outsiders drops out before (after) an insider j with the highest value among insiders if and only if $v_i(s) < (>) v_j(s)$. In case $v_i(s) < v_j(s)$, outsider i dropping out at some $p_i \leq v_i(s)$ (from (ii) of Lemma 3) means that the insider j is a winner, because $p_i \leq v_i(s) < v_j(s)$ is lower than insider j's drop-out price $v_j(s)$. Assume now that $v_i(s) > v_j(s)$ and suppose to the contrary that the outsider i drops out at some price $p_i < v_j(s)$ at which only insiders, including j, are active.[20] Then, the break-even condition at p_i implies that $s_i = p_i - g(s(p_i)) = s_k(p_i)$

[20]This holds because bidder i is the last to drop out among outsiders, according to (i) of Lemma 3.

for each $k \in I \cap A$. Because $s_i > s_k$ for all those k, this means that $s_k(p_i) > s_k$. Thus, due to (iii) of Lemma 3, we have $s(p_i) = (s_O, s_{I\cap A}(p_i), s_{I\setminus A}(p_i)) \geq s$ with $s_k(p_i) > s_k$, which implies that $p_i = v_k(s(p_i)) > v_k(s)$ for all $k \in I \cap A$. This contradicts the value bidding strategy of insiders.

Turning to the proof of Part (i), we show that β^* constitutes an ex-post equilibrium, by focusing on an arbitrary outsider i. If i has the highest value and follows the equilibrium strategy to become a winner, his payoff is $v_i(s) - \max_{k\neq i} p_k \geq v_i(s) - \max_{k\neq i} v_k(s) \geq 0$.[21] Then, any nontrivial deviation by i cannot be profitable, because it results in losing and earning a zero payoff. Suppose now that there is some j with $v_j(s) > v_i(s)$. If j is an insider, any nontrivial deviation by i to become a winner makes him pay at least $v_j(s)$, that is, more than his value. Let us therefore focus on the case where j is an outsider with the highest value. Any nontrivial deviation by i would require him to wait beyond some price p such that $s_i(p) = s_i$, and then become a winner after j drops out last at some $p_j > p$.[22] Then, we must have $s_i(p_j) > s_i$ and $s_j(p_j) = s_j$. Combining this with $s_I(p_j) \geq s_I$ (from (iii) of Lemma 3), we have $s(p_j) = (s_{O\setminus\{i\}}, s_i(p_j), s_I(p_j)) \geq s$ with $s_i(p_j) > s_i$, so $v_i(s(p_j)) = p_j > v_i(s)$, implying that the deviation incurs a loss to i. □

Proof of Theorem 4: Throughout the proof, for any variable x in E, we let x' denote its counterpart in E'. For instance, p'_k denotes the drop-out price of bidder k in E'. Let $O = \{1, 2, \cdots, l\}$ and thus $I = \{l+1, \cdots, n\}$, and assume that $v_1(s) \leq v_2(s) \leq \cdots \leq v_l(s)$, without loss of generality. Then, $O' = O\setminus\{i\}$ and $I' = I \cup \{i\}$.

First, according to (ii) of Lemma 3, switched insider i drops out at a (weakly) higher price in E' than in E. The proof is complete if we show that all other outsiders drop out at (weakly) higher prices in E' as well. Next, suppose by contradiction that some outsider drops out at a lower price in E' than in E. For all outsiders $k < i$, we have $p'_k = p_k$, because the history of drop-out prices is the same across E and E' until p_i is reached. Using this and our assumption, we define $j = \min\{k \,|\, p'_k < p_k, \text{ and } i < k \leq l\}$. We first make a few of observations: (i) a signal s_k for each $k < j$ with $k \neq i$ is revealed in both E and E' by the time the price clock reaches p'_j, because $p_k \leq p'_k \leq p'_j$ for all such k[23]; (ii) $s'_j(p'_j) = s_j = s_j(p_j) > s_j(p'_j)$, because $p_j > p'_j$; and (iii) $s'_i(p'_j) \geq s_i$. (iii) here follows from (iii) of Lemma 3 if i is inactive at p'_j in E'. If i is active at p'_j, the monotonicity of $s'_i(\cdot)$

[21] The first inequality holds because each insider drops out at his value and each outsider drops out below his value according to (ii) of Lemma 3.

[22] An argument similar to that in the proof of the efficiency can be used to show that because j has the highest value, j drops out last (except for i) even under deviation by i.

[23] The second inequality here holds, because outsiders drop out in order of their values in E'.

implies that $s_i'(p_j') \geq s_i'(p_i) = s_i(p_i) = s_i$, because $p_j' \geq p_i$.[24] We next show that

$$s_k'(p_j') \geq s_k(p_j') \text{ for all } k \in \{j+1, \cdots, n\}, \tag{12}$$

which, given (i), (ii), and (iii) above, implies that $s'(p_j') \geq s(p_j')$[25] with $s_j'(p_j') > s_j(p_j')$, so $p_j' = v_j(s'(p_j')) > v_j(s(p_j')) = p_j'$, yielding the desired contradiction. To prove (12), observe first that the break-even conditions at price p_j' in E and E' yield

$$g(s(p_j')) = p_j' - s_j(p_j') > p_j' - s_j'(p_j') = g(s'(p_j')),$$

where the inequality holds because of (ii). We then prove (12) by considering two cases depending on whether $k \in \{j+1, \cdots, n\}$ or not is active at p_j' in E'. Because each outsider $k \in \{j+1, \cdots, l\}$ is active at p_j' in E', an inactive bidder $k \in \{j+1, \cdots, n\}$ must be an insider. For such k, we obtain (12) because the break-even conditions at price p_j' in E and E' yield

$$s_k'(p_j') = p_k - g(s'(p_j')) > p_k - g(s(p_j')) = s_k(p_j'), \tag{13}$$

where the inequality follows from (13).Turning to the case in which bidder $k \in \{j+1, \cdots, n\}$ is active at p_j' in E', he must also be active at p_j' in E. This is because if k is an outsider, $p_j' < p_j$ and he drops out no sooner than j in E (because of (i) of Lemma 3) and because if k is an insider, he drops out at the same price (i.e., his value) in E and E'. Thus, we obtain (12), because the break-even conditions at p_j' in E and E' yield

$$s_k'(p_j') = p_j' - g(s'(p_j')) > p_j' - g(s(p_j')) = s_k(p_j'),$$

where the inequality follows again from (13). □

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[24] The equality $s_i'(p_i) = s_i(p_i)$ follows from the fact that the price history is the same across E and E' until p_i is reached.

[25] The i-th component of $s(p_j)$ is equal to s_i, so this inequality follows from (iii).

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Table 4. Frequencies and ratios of efficient allocation

# of insiders	Ranking of values (*I* or *O*)	Second-price auction (1) Freq.	(2) Ratio	English auction (3) Freq.	(4) Ratio	H_0: (1) = (3)	H_0: (2) = (4)
0	All	0.78	0.80	0.83	0.84	0.17	0.39
		(210)		(210)			
1	All	0.76	0.74	0.83	0.89	0.05	0.00
		(270)		(285)			
	I = (highest-value)	0.70	0.65	0.81	0.83	0.08	0.03
		(83)		(100)			
	I = (second highest-value)	0.73	0.79	0.88	0.93	0.02	0.01
		(93)		(82)			
	I = (lowest-value)	0.83	0.75	0.80	0.90	0.55	0.03
		(94)		(103)			
2	All	0.83	0.86	0.89	0.93	0.01	0.00
		(495)		(630)			
	O = (highest-value)	0.73	0.80	0.87	0.92	0.00	0.00
		(169)		(206)			
	O = (second highest-value)	0.83	0.91	0.79	0.90	0.37	0.58
		(168)		(205)			
	O = (lowest-value)	0.94	0.87	0.99	0.97	0.01	0.02
		(158)		(219)			

Note. The efficiency ratio is defined as (realized surplus minus random surplus) divided by (first-best surplus minus random surplus). The two columns on the right side report the *p*-value of the *t*-test for the null hypothesis that outcomes between the second-price auction and the English auction are equivalent. *I* denotes an insider and *O* an outsider. The number of observations is in parentheses

Table 5. The decomposition of efficiency outcomes against theoretical predictions of the second-price auction

A. (# of insiders) = 1

Theoretical allocation Second-price auction	Second-price auction: Observed Allocation (1) Inefficient	Second-price auction: Observed Allocation (1) Efficient	Theoretical allocation Second-price auction	English auction: Observed Allocation (2) Inefficient	English auction: Observed Allocation (2) Efficient	H_0: (1) ≥ (2)
Inefficient	0.67 (20)	0.33 (10)	Inefficient	0.39 (10)	0.62 (16)	0.017
Efficient	0.19 (46)	0.81 (194)	Efficient	0.15 (40)	0.85 (219)	0.136
Total	0.24 (66)	0.76 (204)	Total	0.18 (50)	0.82 (235)	
	Ho: Ineff-Eff=0	0.000		Ho: Ineff-Eff=0	0.003	

B. (# of insiders) = 2

Theoretical allocation Second-price auction	Second-price auction: Observed Allocation (1) Inefficient	Second-price auction: Observed Allocation (1) Efficient	Theoretical allocation Second-price auction	English auction: Observed Allocation (2) Inefficient	English auction: Observed Allocation (2) Efficient	H_0: (1) ≥ (2)
Inefficient	0.36 (21)	0.64 (37)	Inefficient	0.34 (25)	0.66 (48)	0.408
Efficient	0.15 (64)	0.85 (373)	Efficient	0.08 (47)	0.92 (510)	0.001
Total	0.17 (85)	0.83 (410)	Total	0.11 (72)	0.89 (558)	
	Ho: Ineff-Eff=0	0.000		Ho: Ineff-Eff=0	0.000	

Note. In discerning if the second-price auction format predicts an inefficient allocation for each sample, we allow a 5-token margin with which a sample is treated inefficient if the equilibrium of the second-price auction is either inefficient or efficient but the difference of its two high bids is less than 5 tokens.The last column on the right side reports the *p* -value from the *t* -test for the null hypothesis that the efficiency outcome of the second-price auction is no lower than that of the English auction. The bottom row reports the p-value from the t-test for the nully hypothesis that the frequencies of efficient allocation is the same between when the second-price auction format predicts an efficient allocation and when not. The number of observations is in parentheses.

Table 6. Average percentage deviations of observed revenues from theoretically predicted revenues

# of insiders	Ranking of values (I or O)	English auction	SBSP auction
0	All	0.22 (0.096, 0.023)	0.26 (0.045, 0.000)
1	All	0.03 (0.014, 0.047)	0.14 (0.024, 0.000)
	I = (highest-value)	0.06 (0.036, 0.101)	0.28 (0.058, 0.000)
	I = (second highest-value)	-0.02 (0.012, 0.112)	0.03 (0.019, 0.165)
	I = (lowest-value)	0.03 (0.012, 0.005)	0.13 (0.036, 0.001)
2	All	0.00 (0.002, 0.453)	0.01 (0.004, 0.063)
	O = (highest-value)	-0.01 (0.003, 0.000)	-0.03 (0.005, 0.000)
	O = (second highest-value)	0.02 (0.006, 0.008)	0.03 (0.010, 0.001)
	O = (lowest-value)	0.00 (0.001, 0.024)	0.02 (0.006, 0.005)

Notes. I stands for an insider and O represents an outsider. The first number in parentheses is a standard error of sample mean and the second number is p-value from t-test for the null hypothesis that the mean is equal to zero.

Table 7. Regression analysis of observed and theoretical revenues

Variables	Observed revenues: English	Observed revenues: SBSP	Theoretical revenues: English	Theoretical revenues: SBSP
$k = 1$	10.466***	10.491***	12.424***	13.554***
	(1.983)	(2.477)	(.996)	(1.374)
$k = 2$	18.716***	9.838***	21.897***	21.635***
	(1.738)	(2.218)	(.873)	(1.23)
$s_{(1)}$	0.889***	0.766***	1.027***	0.638***
	(.039)	(.055)	(.02)	(.03)
$s_{(2)}$	2.087***	1.837***	2.330***	2.515***
	(.038)	(.051)	(.019)	(.029)
$s_{(3)}$	0.796***	0.761***	0.661***	0.629***
	(.04)	(.054)	(.02)	(.03)
constant	-7.739***	18.436***	-16.151***	-14.889***
	(2.98)	(3.883)	(1.497)	(2.154)
# of obs.	1125	975	1125	975
R^2	0.907	0.837	0.978	0.960
p-value H_0: ($k=0$) = ($k=1$)	0.000	0.000	0.000	0.000
p-value H_0: ($k=1$) = ($k=2$)	0.000	0.749	0.000	0.000

Note. Standard errors are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. $s_{(1)}$ = min[**s**], $s_{(3)}$=max[**s**], $s_{(2)}$=med[**s**]

Table 8. Regressions of bids on signals in the SBSP auctions

Variables	***(k = 0)***	***(k = 1)***		***(k = 2)***	
s_i	2.754***	2.309	2.309***	2.756	2.756***
	(0.16)	(0.130)***	(0.13)	(0.219)***	(0.22)
$1[s_i > 82.109]$			-90.933		
			(79.09)		
$1[s_i > 82.109] \times s_i$			1.125		
			(0.89)		
$1[s_i > 500/6]$					-352.69
					(226.80)
$1[s_i > 500/6] \times s_i$					4.006
					(2.50)
Constant	48.797***	75.876	75.876***	39.836	39.836***
	(8.57)	(7.273)***	(7.29)	(12.048)***	(12.07)
R^2	0.71	0.6	0.7	0.58	0.65
# of obs.	630	447[▪]	540	412[§]	495
Ho:(**b**,*c)=(* ***b,o)***	(s=3.5)	(s=3.43578)	n/a	(s=17/5)	(s=17/5, 1[.]=-300, s+1[.]s = 7)
F test	16.21	54.56		5.5	3.64
p value	0.00	0.00		0.00	0.02

Notes: Robust standard errors clustered by individual subjects are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. ▪ Restricts sample to $s_i \leq 82.109$. § Restricts sample to $s_i \leq 500/6$.

Table 9. Censored regressions of bids in the English auctions

	($k = 0$)		($k = 1$)			($k = 2$)	
	first drop-out	second drop-out	first drop-out	second drop-out		first drop-out	second drop-out
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
s_i	2.878***	1.893***	3.258***	2.394***	2.456***	3.48***	1.268***
	(0.14)	(0.10)	(0.16)	(0.15)	(0.16)	(0.14)	(0.10)
p/4		1.591***		1.369***	1.409***		2.812***
		(0.12)		(0.21)	(0.21)		(0.19)
1[1st drop-out = insider]× s_i				-1.010***	-1.076***		
				(0.21)	(0.22)		
1[1st drop-out = insider] ×p/4				1.240***	1.093***		
				(0.33)	(0.35)		
1[1st drop-out = insider]					10.775		
					(8.45)		
Constant	77.893***	46.28***	65.669***	22.705***	17.285***	32.089***	21.836***
	(9.61)	(5.60)	(10.13)	(3.82)	(6.28)	(8.04)	(6.79)
sigma	61.947	30.677	58.226	26.358	26.378	56.556	23.396
	(8.365)***	(2.896)***	(6.230)***	(1.794)***	(1.810)***	(8.449)***	(1.785)***
pseudo-R^2	0.13	0.20	0.16	0.23	0.23	0.16	0.24
# of obs.	630	420	570	388	388	630	343
Ho:(**b**,*c)=(* ***b,o)***	(s=4)	(s=3,p/4=1)	(s=4)	(s=3,p/4=1, 1[.]s+s=2, 1[.]p/4+p/4=2)	(s=3,p/4=1, 1[.]s+s=2, 1[.]p/4+p/4=2, 1[.]=0)	(s=4)	(s=2,p/4=2)
F test	40.17	45.63	21.11	36.65	30.63	9.43	48.41
p value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Robust standard errors clustered by individual subjects are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively.

Table 10. Nonlinear least squares of naïve bidding

	English auction: first and second drop-out prices			Second-price auction		
	k=0	*k=1*	*k=2*	*k=0*	*k=1*	*k=2*
α	0.672	0.512	0.280	0.488	0.729	0.402
	(.069)***	(.070)***	(.067)***	(.086)***	(.079)***	(.122)***
R^2	0.92	0.93	0.91	0.94	0.95	0.92
N	421	376	404	630	540	495
$H_0: \alpha_k = \alpha_{k'}$	$\alpha_0 = \alpha_2$	$\alpha_0 = \alpha_1$	$\alpha_1 = \alpha_2$	$\alpha_0 = \alpha_2$	$\alpha_0 = \alpha_1$	$\alpha_1 = \alpha_2$
F test:	16.64	2.64	5.73	0.34	4.32	5.15
p value	0.00	0.11	0.02	0.56	0.04	0.03

Note. Robust standard errors clustered by individual subjects are reported in parentheses. *,**, and *** represent 10%, 5%, and 1% significance level, respectively.

Online Appendix

Online Appendix I: Proof of Proposition 1 and Analysis for Example 1

Proof of Proposition 1. Let $B : [0, \bar{s}] \to \mathbb{R}_+$ denote the symmetric bidding strategy of outsiders. The restriction to undominated equilibrium implies that $B(\bar{s}) \leq v(\mathbf{s})$ while all insiders bid their values. Suppose for contradiction that $B(\bar{s}) < v(\mathbf{s})$. If an outsider i with signals $s_i = \bar{s}$ deviates to some bid greater than $B(\bar{s})$, it only increases his chance of winning against insiders, in which case his payoff is given by $v_i(\bar{s}, s_{-i}) - \max_{j \in I} v_j(\bar{s}, s_{-i}) = \bar{s} - \max_{j \in I} s_j > 0$ (unless $\max_{j \in I} s_j = \bar{s}$). Thus, this deviation is profitable. $\square$

Equilibrium of the Second-Price Auction with $I = \{2, 3\}$. We aim to find bidder 1's bid that is the best response to the value bidding of the two insiders, bidders 2 and 3. By symmetry, it suffices to focus on the case in which $s_2 \geq s_3$ so $v_2(s) \geq v_3(s)$, meaning that bidder 2 bids higher than bidder 3 does. Then, by bidding b, bidder 1 wins the object to obtain a payoff equal to $v_1(s) - v_2(s) = s_1 - s_2$ when $b \geq v_2(s)$ and $s_2 \geq s_3$, which can be rewritten as $s_3 \leq \min\{b - 2s_2 - s_1, s_2\}$. Given this and the uniform distribution of signals, bidder 1's payoff from biding b is given as

$$\pi(b; s_1) = \int_0^{\min\{1, \frac{b-s_1}{2}\}} (s_1 - s_2) \min\{b - as_2 - s_1, s_2\} ds_2.$$

This expression is maximized by setting $b = B_1(s_1)$ with $B_1(s_1)$ defined in (3).

Equilibrium of the Second-Price Auction with $I = \{3\}$. Let $B : [0, 1] \to \mathbb{R}_+$ denote a symmetric, non-decreasing bidding strategy for the two outsiders. We consider the maximization problem faced by bidder 1 with any fixed signal $s_1 \in [0, 1]$, when bidder 2 follows the bidding strategy B and bidder 3 bids his value. By bidding b, he wins if $s_2 \leq B^{-1}(b)$ and $v_3(s) \leq b$. Then, his payment is equal to $v_3(s)$ if $v_3(s) \geq B(s_2)$, and equal to $B(s_2)$ otherwise. In the former case (the darker gray area A_3 in Figure A), his (ex-post) payoff is $v_1(s) - v_3(s) = s_1 - s_3$ while in the latter case (the lighter gray area A_2 in the graph below), his payoff is $v_1(s) - B(s_2) = 2s_1 + s_2 + s_3 - B(s_2)$.

Given this and the uniform distribution of signals, the expected payoff of bidder 1 with

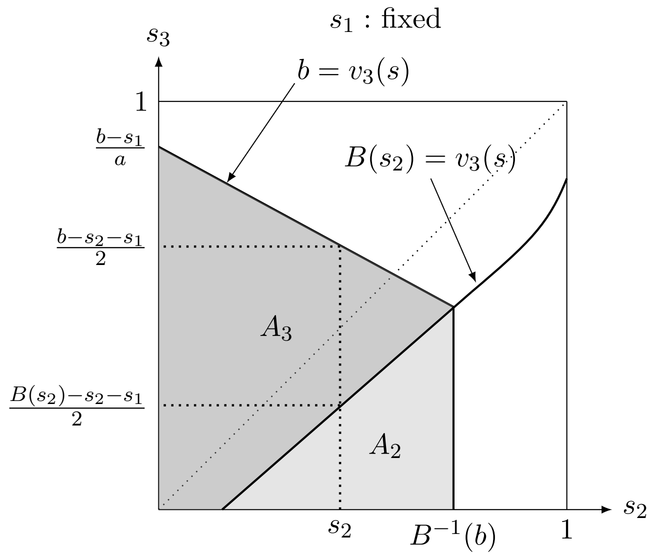


Figure A: An outsider's payoff from bidding b in the second-price auction

s_1 can be written as

$$\pi(b; s_1) = \int_0^{B^{-1}(b)} \left[\int_{\max\{\frac{B(s_2)-s_2-s_1}{2},0\}}^{\min\{\frac{b-s_2-s_1}{2},1\}} (s_1 - s_3) ds_3 \right] ds_2$$
$$+ \int_0^{B^{-1}(b)} \left[\int_0^{\max\{\frac{B(s_2)-s_2-s_1}{2},0\}} (2s_1 + s_2 + s_3 - B(s_2)) ds_3 \right] ds_2.$$

The first (resp., second) integration corresponds to bidder 1's payoff in the area A_3 (resp,. A_2). Then, the requirement to maximize this payoff by setting $b = B(s_1)$ yields a differential equation that can be used to solve for the function B. While we omit the detailed expression for this differential equation, it yields a linear solution below a threshold signal $\overline{s} \simeq 0.82$:

$$B(s_1) = \frac{1}{10}\left(\sqrt{129} + 23\right) s_1 \text{ for } s_1 \in [0, \overline{s}],$$

where the threshold $\overline{s}$ solves equation $\frac{B(s_1)-s_1}{2} = 1$. For $s_1 \geq \overline{s}$, the first order condition yields the following differential equation:

$$\frac{1}{B'(s_1)} \int_0^{\frac{B(s_1)-2s_1}{2}} (3s_1 + s - B(s_1)) ds + \int_{B(s_1)-2-s_1}^{s_1} \frac{1}{2}\left(s_1 - \frac{B(s_1) - s_1 - s}{2}\right) ds = 0$$

Unfortunately, an analytical solution for this differential equation is unavailable. Instead, we rely on a numerical method to draw the graph of equilibrium bidding strategy for the case of $k = 1$ in Figure 1.

Online Appendix II

Sample Instructions: English auction with one insider

This is an experiment in the economics of decision-making. Research foundations have provided funds for conducting this research. Your earnings will depend partly on your decisions and partly on the decisions of the other participants in the experiment. If you follow the instructions and make careful decisions, you may earn a considerable amount of money.

At this point, check the name of the computer you are using as it appears on the top of the monitor. At the end of the experiment, you should use your computer name to claim your payments. At this time, you will receive £5 as a participation fee simply for showing up on time. In addition, you will receive £10 as an initial balance you will use in this experiment. Any positive or negative earnings incurred during the experiment will be added into this balance. Details of how you will make decisions will be provided below. During the experiment we will speak in terms of experimental tokens instead of pounds. Your payments will be calculated in terms of tokens and then exchanged at the end of the experiment into pounds at the following rate:

40 Tokens = 1 Pound

In this experiment, you will participate in 17 independent and identical (of the same form) auction rounds. In each round you will act as a bidder in an auction and compete for a single hypothetical object with other two participants in your group. Note that the first two rounds are practice rounds in which your earnings will not be counted for actual payoffs. The remaining 15 auction rounds are real and any positive or negative earnings will be counted for actual payoffs. If your balance during the experiment goes below zero, you will become inactive and be excluded for any remaining auction rounds and will receive only £5 participation fee at the end of the experiment.

An auction round

Next, we will describe in detail the process that will be repeated in all 17 rounds. Each round starts by having the computer randomly form three-participant groups. In each group, one participant is played by the computer (called a computer participant), while the other two participants are played by persons. The groups formed in each round depend solely upon chance and are independent of the groups formed in any of the other rounds. That is, in any group each active person is equally likely to be chosen for that group. In a case where any other person was excluded due to its negative balance, there is a chance that you may become inactive in a particular round when you are matched with that person who was excluded.

In the beginning of each round, each participant will be assigned a *signal* that will be randomly drawn from the set of integer numbers of tokens between 0 and 100 (numbers not including decimals). That is, any number from the set {0, 1, 2, …, 100} will be equally likely to be drawn. A signal you will be assigned in each round is independent of signals other participants will be assigned and is independent of a signal assigned to you in any of the other rounds. This will be done by the computer.

The result of your draw of a signal will be your private information and will not be shared with another person in your group during each round. On the other hand, the computer participant will know not only its own signal but also the signals of other two human participants.

The value of the object for each participant is determined by signals received by that participant and the other participants in the same group. Specifically, each participant's value is the sum of his or her own signal multiplied by two and signals received by the other two participants. The determination of your value can be summarized by the following formula:

$$\text{Your value} = 2 \times (\text{your signal}) + (\text{Other 1's signal}) + (\text{Other 2's signal})$$

The information about your signal and value will be displayed at the top of the screen (see Attachment 1). Note that Other 2 in the screen is the computer participant. Because the signals the other participants received are not shared with you, you will not know the exact number of your value.

To illustrate this more, consider an example in which your signal is 84 and the signals of the other two participants in your group, Other 1 and Other 2, are 43 and 26, respectively. The value for each participant will then be calculated to be

$$\text{Your value} = 2 \times 84 + 43 + 26 = 237$$

$$\text{Other 1's value} = 2 \times 43 + 84 + 26 = 196$$

$$\text{Other 2's value} = 2 \times 26 + 84 + 43 = 179$$

Because each human participant does not know the signals the other two participants received, each person will be only informed of his or her own value as

$$\text{Your value} = 2 \times 84 + ??? + ???$$

After every participant is assigned a signal, the bidding process will get started with ascending price clocks (number boxes) shown in the middle of the computer screen (see Attachment 1). The left-hand clock represents your bidding and the middle clock represents the bidding of another human participant, while the right-hand clock represents the bidding of the computer participant.

In the beginning of the bidding process, the three clocks will simultaneously start at -4 and synchronously move upwards by 1 unit per half second until one participant drops out. If one participant stops his clock, the remaining two participants will observe, at the next bid increment, that participant's clock having been stopped and turning red (see Attachment 2). There will then be 3 seconds of time pause. From then on the two remaining clocks will synchronously increase by 1 unit per second. If one more participant drops out, the auction will then be over. The last remaining participant will become a winner of the object and will pay *the price at which the second participant dropped out*. If all remaining participants dropped out at the same price level or if the price level reached 500 (the maximum bid allowed), the winner will then be selected at random from the set of active participants and pay the price at which this event occurred.

As soon as the price on your clock reaches the level you want to drop out, move the mouse over your clock (number box) and click on it. This will make you drop out of the bidding, that is, your clock stop. Once you have dropped out, you will not be allowed to re-enter the auction in this round. Note that you cannot stop your clock before the clock reaches 0 (the minimum bid allowed).

The computer participant will use a simple rule of drop-out decision: it will drop out at a price equal to its own value. The computer participant will always abide by this rule.

When the first round ends, the computer will inform you of the results of this round, which include bids you and other participants dropped out at, signals you and other participants received, values of the object, payments and earnings in the round (see Attachment 3)[1]. This completes the first auction round. To move on to the second round, press the OK button at the bottom right hand side of the screen (see Attachment 3).

After letting you observe the results of the first round, the second round will start by having the computer randomly form new groups of three participants and select signals for participants. You will be again asked to take part in the bidding process. After every participant has made a decision, you will observe the results of the second round. This process will be repeated until all the 17 independent and identical auction rounds are completed. At the end of the last round, you will be informed that the experiment has ended. Note again that the first two rounds are practice rounds in which your earnings will not be counted for actual payoffs.

Earnings

Your earnings in each round can be summarized by the following formula:

Earnings = (winning revenue) – (winning cost)

The winning revenue is the value assigned to you if you won the object and zero otherwise. The winning cost is the price paid by you. If you did not win the object, the winning cost is equal to zero. The difference between the winning revenue and winning cost will determine your earnings in each round.

Consider an example to understand the determination of earnings more easily. Suppose that signals assigned to you, another human participant, and the computer participant are 84, 43, and 26, respectively as below. Further, suppose that Other 1 dropped at 142 and Other 2 (the computer participant) dropped at 179 (equal to its own value), while you remained active.

[1] In the history box of bidding screen (see the bottom box in Attachment 2), in case you were the winner in a previous round, you bidding in that round is denoted by -999. This is simply a feature in the programming.

Participant	Signal	Value	Drop-out price	Earnings
You	84	$2 \times 84 + 43 + 26 = 237$	--	237 – 179 = 58
Other 1	43	$2 \times 43 + 84 + 26 = 196$	142	0
Other 2	26	$2 \times 26 + 84 + 43 = 179$	179	0

Because you were the last remaining bidder, you won the object and paid the price at which the second participant dropped out, 179. Your winning revenue is your own value, 237. Your winning cost is the price paid by you, 179. Therefore, your earnings will be given by

Your earnings = 237 – 179 = 58.

The other two participants' earnings will be equal to zero because they did not win the auction.

Consider another example in which your signal is 54, while signals of another human participant and the computer participant are 60 and 15, respectively. Suppose that Other 2 (the computer participant) dropped at 144 (its own value) and Other 1 dropped at 204, while you remained active.

Participant	Signal	Value	Drop-out price	Earnings
You	54	$2 \times 54 + 60 + 15 = 183$	--	183 – 204 = – 21
Other 1	60	$2 \times 60 + 54 + 15 = 189$	204	0
Other 2	15	$2 \times 15 + 54 + 60 = 144$	144	0

In this case, your value is 183, while the price you pay is 204. Thus, your earnings will be then 183 – 204 = – 21.

Your payoffs in the experiment will be the sum of your earnings over the 15 rounds after the first two practice rounds, whose tokens will be converted into pounds at the end of the experiment, *plus* the initial balance £10. In addition, you will receive £5 participation fee. You will receive your payment as you leave the experiment.

Rules

Please do not talk with anyone during the experiment. We ask everyone to remain silent until the end of the experiment. Your participation in the experiment and any information about your earnings will be kept strictly confidential. If there are no further questions, you are ready to start. An instructor will activate your program.

Attachment 1

Period

6 of 17

Your signal: 28

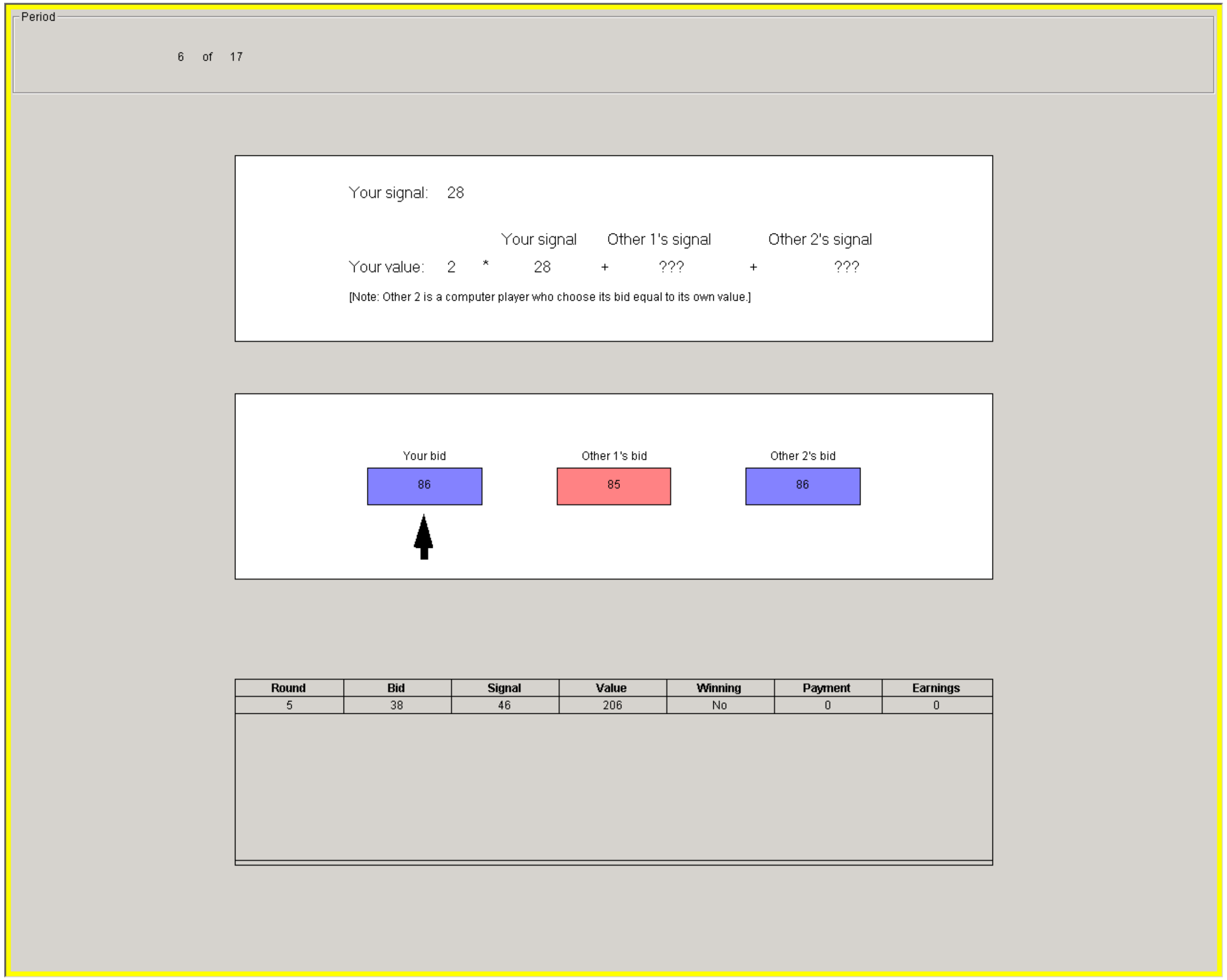
			Your signal		Other 1's signal		Other 2's signal
Your value:	2	*	28	+	???	+	???

[Note: Other 2 is a computer player who choose its bid equal to its own value.]

Your bid	Other 1's bid	Other 2's bid
20	20	20

Round	Bid	Signal	Value	Winning	Payment	Earnings
5	38	46	206	No	0	0

Attachment 2



Round	Bid	Signal	Value	Winning	Payment	Earnings
5	38	46	206	No	0	0

<u>Attachment 3</u>

Period

1 of 17

Bid choices and outcomes of your group in this round are summarized below.

Please press OK to continue.

Players	Bid	Signal	Value	Winning	Payment	Earnings
You	--	9	23	Yes	24	-1
Other 1	24	5	19	No	0	0
Other 2	14	0	14	No	0	0

OK

Online Appendix III

Regression analysis of observed revenues: alternative specifications

Variables	**English**	**SBSP**
$k = 1$	10.847***	10.861***
	(1.986)	(2.468)
$k = 2$	18.853***	10.435***
	(1.74)	(2.214)
$s_{(1)}$	0.975***	0.288
	(.227)	(.304)
$s_{(2)}$	2.037***	1.710***
	(.153)	(.197)
$s_{(3)}$	0.466**	1.075***
	(.226)	(.301)
$s_{(1)}^2$	0.006***	0.009***
	(.002)	(.003)
$s_{(2)}^2$	0.001	0.003
	(.002)	(.002)
$s_{(3)}^2$	0.002	-0.004
	(.002)	(.002)
$s_{(1)} \times s_{(2)}$	-0.007*	-0.014***
	(.003)	(.004)
$s_{(1)} \times s_{(3)}$	0	0.009**
	(.003)	(.004)
constant	2.36	15.850*
	(6.665)	(9.04)
# of obs.	1125	975
R^2	0.908	0.839
p-value H_0: $(k=0) = (k=1)$	0.000	0.000
p-value H_0: $(k=1) = (k=2)$	0.000	0.834

Note. Standard errors are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. $s_{(1)}$ = min[**s**], $s_{(3)}$=max[**s**], $s_{(2)}$=med[**s**]

Regression analysis of observed revenues: alternative specifications

Variables	**English**	**SBSP**
$k = 1$	15.665**	17.633*
	(7.936)	(9.763)
$k = 2$	29.069*	24.29
	(15.381)	(18.879)
$s_{(1)}$	0.784***	0.626***
	(.048)	(.066)
$s_{(2)}$	2.053***	1.716***
	(.043)	(.059)
$s_{(3)}$	0.707***	0.622***
	(.051)	(.066)
$1[s_{(1)} = s_{(Insider)}]$	-10.971	-16.276*
	(7.892)	(9.833)
$1[s_{(2)} = s_{(Insider)}]$	-5.224	-19.698*
	(8.303)	(10.228)
$1[s_{(3)} = s_{(Insider)}]$	-23.238**	-31.405***
	(9.224)	(11.549)
$1[s_{(1)} = s_{(Insider)}]\times s_{(1)}$	0.245***	0.261***
	(.067)	(.092)
$1[s_{(2)} = s_{(Insider)}]\times s_{(2)}$	0.082	0.332***
	(.058)	(.075)
$1[s_{(3)} = s_{(Insider)}]\times s_{(3)}$	0.181***	0.310***
	(.066)	(.09)
constant	3.565	38.074***
	(4.217)	(5.168)
# of obs.	1125	975
R^2	0.91	0.844
p-value H_0: $(k=0) = (k=1)$	0.049	0.071
p-value H_0: $(k=1) = (k=2)$	0.084	0.484

Note. Standard errors are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. $s_{(1)} = \min[\mathbf{s}]$, $s_{(3)}=\max[\mathbf{s}]$, $s_{(2)}=\text{med}[\mathbf{s}]$

Online Appendix IV: Maximum Likelihood Approach

The maximum likelihood approach addresses the censoring that exists in the observed data. It distinguishes between the observed drop-out bid $d_{s,ir}$ and the reservation bid $p_{s,ir}$. For ease of exposition denote, denote $g_1(i,r)$ for all $i \in N$ as the individual who first drops out from i's group in round r and the corresponding group as $g(i,r)$. Let $G_1(r)$ be the collection of all first bidders at round r. Therefore, for all $i \in G_1(r)$ we know $p_{1,ir} = d_{1,ir}$ but for the other two active bidders $j \in g(i,r) \setminus g_1(i,r)$ we only know that $p_{1,jr} > p_{1,g_1(j,r)r}$, implying that we observe a right-censored variable of their true drop-out price. For the second stage, we know that for bidder $g_2(i,r)$, who drops out second, his/her reservation bid is $p_{2,g_2(i,r)r} = d_{2,g_2(i,r)r}$; however, for the remaining bidder j we only know that $d_{2,jr} > d_{2,g_2(i,r)r}$, which again implies a right-censored variable.

Consider $\epsilon^k_{s,ir} = p^k_{s,ir} - \Gamma^k_{s,ir}$ where $\Gamma^k_{s,ir}$ follows from the right hand side of the regression equation. We know that $d_{s,g_1(i,r)r} = p_{s,ir}$ if and only if $i \in g_s(i,r)$. If we define $e^k_{s,ir} = d^k_{s,g_1(i,r)r} - \Gamma^k_{s,ir}$ $d_{s,g_1(i,r)r} = p_{s,ir}$ if and only if $e^k_{s,ir} = \epsilon^k_{s,ir}$.

Denoting $\theta^k_s = \left(\alpha_{\mathbf{s}}{}^k, \beta_{\mathbf{s}}{}^k, \delta_{\mathbf{s}}{}^k, \{\sigma_{s,i} : i : 1 \to N_k\}\right)$ and $D_s \equiv \left(d_{s,g_s(i,r)}\right)_{\forall i \in N_k, r \in R}$ the information on drop-out prices from the experiment, the density function associated with the first bidding function $p_{1,ir}$ is given by

$$f_{p_{1,ir}}(b \mid \cdot) = f(p_{1,ir} = d_{1,g_1 i,rr} \mid \cdot)^{\mathbf{1}_{[i \in g_1(i,r)]}} (1 - F(p_{1,ir} \leq d_{1,g_1 i,rr} \mid \cdot))^{\mathbf{1}_{[i \notin g_1(i,r)]}}.$$

Therefore, the maximum likelihood function is

$$L^k_1\left(\theta^k_1; D_1\right) = \prod_{r \in R} \prod_{i \in N_k} \left[\frac{1}{\sigma_i} \phi\left(\frac{e^k_{1,ir}}{\sigma_i}\right)\right]^{\mathbf{1}_{[i \in g_1(i,r)]}} \left[1 - \Phi\left(\frac{e^k_{1,ir}}{\sigma_i}\right)\right]^{\mathbf{1}_{[i \notin g_1(i,r)]}} \tag{14}$$

On the other hand, the maximum likelihood associated with the second bidders is

$$L^k_2\left(\theta^k_2; D_2\right) = \prod_{r \in R} \prod_{i \in N_k \setminus G_1(r)} \left[\frac{1}{\sigma_i} \phi\left(\frac{e^k_{2,ir}}{\sigma_i}\right)\right]^{\mathbf{1}_{[i \in g_2(i,r)]}} \left[1 - \Phi\left(\frac{e^k_{2,ir}}{\sigma_i}\right)\right]^{\mathbf{1}_{[i \notin g_2(i,r)]}} \tag{15}$$

This specification corresponds to a Partial maximum likelihood estimator. From Wooldridge (2003) we know that, once the variance matrix is corrected for within-subject dependence, the pooled partial maximum likelihood estimation analysis is consistent and asymptotically normal.

Online Appendix V

Table. Frequencies of negative surplus and average surplus

# of insiders	Ranking of values (*I* or *O*)	Second-price auction (1) Freq. negative	Second-price auction (2) Average	English auction (3) Freq. negative	English auction (4) Average	H_0: (1) = (3)	H_0: (2) = (4)
0	All	0.25	36.07	0.17	44.15	0.03	0.10
		(210)		(210)			
1	All	0.32	12.96	0.22	23.32	0.03	0.00
		(184)		(197)			
	I = (highest-value)	0.96	-21.78	0.95	-13.18	0.88	0.10
		(27)		(22)			
	I = (second highest-value)	0.11	19.29	0.01	28.07	0.02	0.05
		(70)		(72)			
	I = (lowest-value)	0.28	18.66	0.20	27.80	0.25	0.07
		(87)		(103)			
2	All	0.25	18.12	0.21	18.46	0.29	0.90
		(158)		(227)			
	O = (highest-value)	0.00	30.46	0.00	26.46	-	0.09
		(118)		(180)			
	O = (second highest-value)	1.00	-10.87	1.00	-10.00	-	0.75
		(30)		(44)			
	O = (lowest-value)	1.00	-40.50	1.00	-43.67	-	0.81
		(10)		(3)			

Note. The Bidder's surplus is defines as valuation minus price paid. The two columns on the right side report the *p*-value of the *t*-test for the null hypothesis that outcomes between the second-price auction and the English auction are equivalent. *I* denotes an insider and *O* an outsider. The number of observations is in parentheses. It excludes cases where Insiders win the auction. - No sufficient variation to compute significance tests

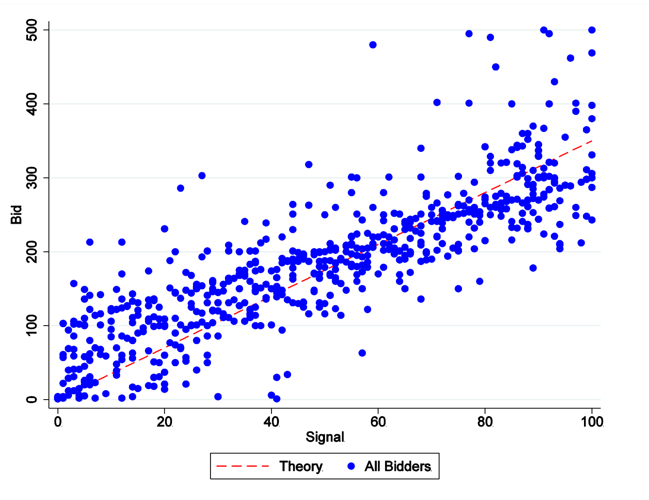
Table. Theoretical predictions on frequencies of negative surplus and average surplus

		Second-price auction		English auction			
		(1)	(2)	(3)	(4)		
# of insiders	Ranking of values (*I* or *O*)	Freq. negative	Average	Freq. negative	Average	H_0: (1) = (3)	H_0: (2) = (4)
0	All	0.07	62.06	0.00	55.53	0.00	0.16
		(161)		(173)			
1	All	0.06	32.16	0.00	34.22	0.00	0.45
		(142)		(154)			
	I = (highest-value)	0.75	-2.00	-	-	-	-
		(4)		(0)			
	I = (second highest-value)	0.00	29.32	0.00	28.79	-	0.87
		(65)		(72)			
	I = (lowest-value)	0.08	38.99	0.00	38.99	-	0.56
		(73)		(82)			
2	All	0.07	29.42	0.00	26.60	0.00	0.24
		(120)		(179)			
	O = (highest-value)	0.00	31.82	0.00	26.60	-	0.03
		(112)		(179)			
	O = (second highest-value)	1.00	-	-	-	-	-
		(8)		(0)			
	O = (lowest-value)	-	-	-	-	-	-
		(0)		(0)			

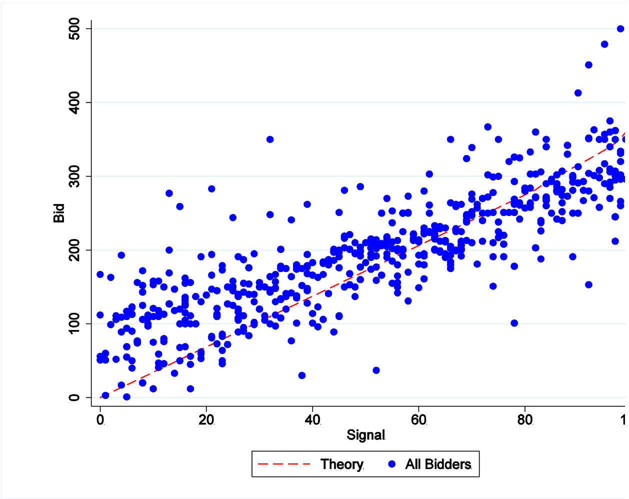
Note. The Bidder's surplus is defines as valuation minus price paid. The two columns on the right side report the *p*-value of the *t*-test for the null hypothesis that outcomes between the second-price auction and the English auction are equivalent. *I* denotes an insider and *O* an outsider. The number of observations is in parentheses. It excludes cases where Insiders win the auction. - No sufficient variation to compute significance tests

Figure 3. Scatter plots of bids and signals: Second-price auctions

A. (# of insiders) = 0



B. (# of insiders) = 1



C. (# of insiders) = 2

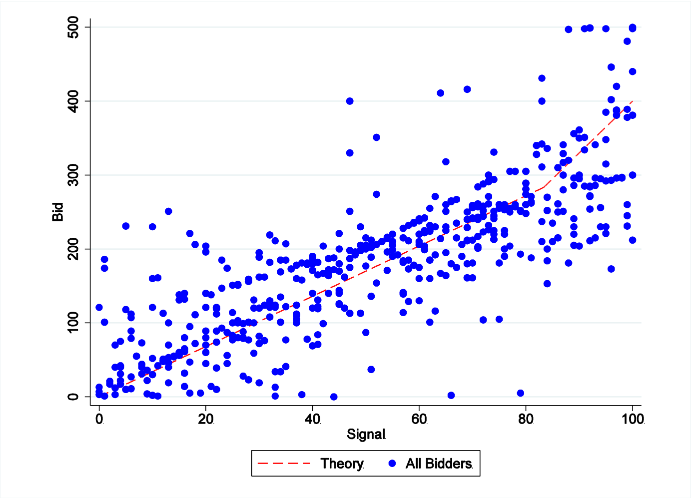
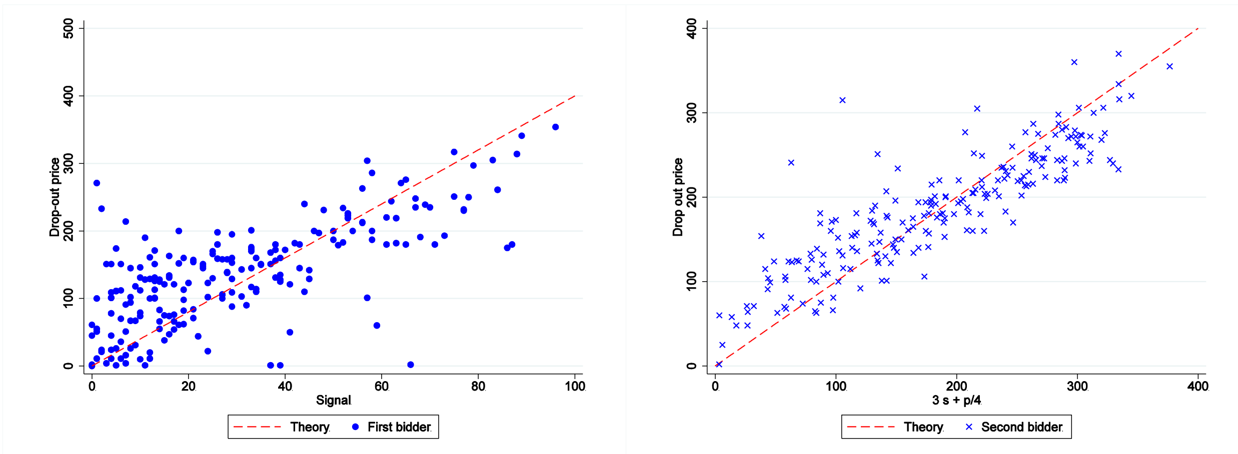
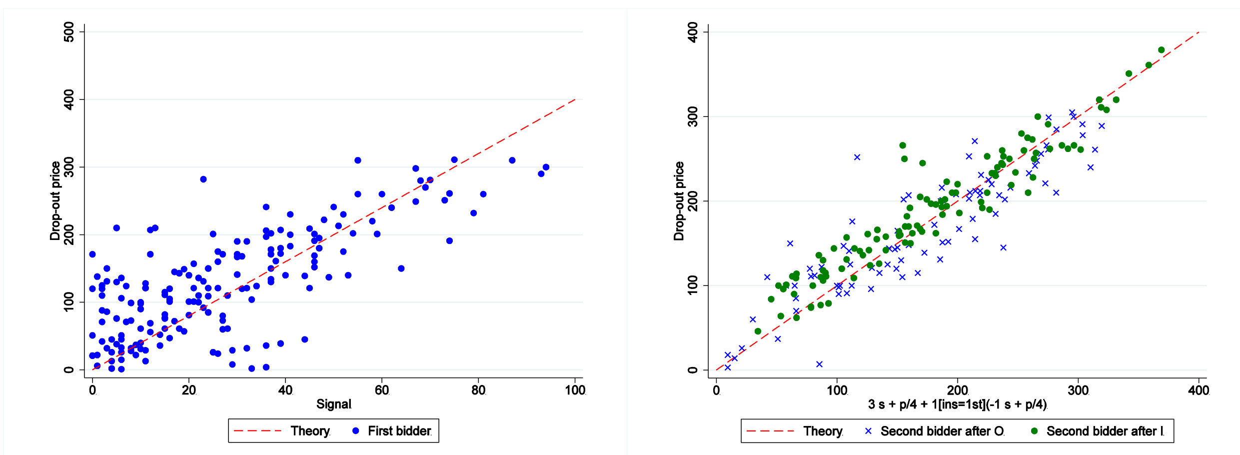


Figure 4. Scatter plots of bids and signals: English auctions

A. (# of insiders) = 0



B. (# of insiders) = 1



C. (# of insiders) = 2