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CHARACTERIZATIONS OF SOME STRATEGYPROOF MECHANISMS IN THE QUEUEING PROBLEM

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ABSTRACT. We investigate mechanisms satisfying queue efficiency, equal treatment of equals and strategyproofness in the context of queueing models. We give two results here. First, we present a simpler proof of Kayi and Ramaekers' [9] characterization of the symmetrically balanced VCG mechanism. Second, we use independence axioms, introduced by Chun [2] and Maniquet [10], to characterize the pivotal and the reward-based pivotal mechanisms (Mitra and Mutuswami [12]).

JEL Classifications: C72, D63, D82.

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1. INTRODUCTION

This paper is on queueing models which have been analyzed in Economics in a recent set of papers (Chun [1], [2], Kayi and Ramaekers [9], Maniquet [10], Mitra [11], Mitra and Mutuswami [12] among others). We provide two results here. First, we give a simpler proof of the characterization of the *symmetrically balanced VCG mechanism* obtained by Kayi and Ramaekers [9]. Second, we provide alternate characterizations of two mechanisms from the class of k-pivotal mechanisms identified by Mitra and Mutuswami [12], the *pivotal mechanism* and the *reward-based pivotal mechanism*.

The key axioms in our analysis are *queue efficiency* and *strategyproofness*. The former requires that the selected queue minimize the aggregate waiting cost. The latter requires that truthful reporting be a weakly dominant strategy for all agents. In our context, a classic result of Holmström [8] implies that a mechanism satisfies these two properties if and only if it is a VCG mechanism.[1] Our focus, therefore, is on particular mechanisms from the class of VCG mechanisms.

A brief word about the motivation follows. As noted above, Kayi and Ramaekers [9] show that the symmetrically balanced VCG mechanism is the unique VCG mechanism satisfying ETE and BB. Their proof is complicated

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[1]The family of VCG mechanisms is due to Vickrey [15], Clarke [4] and Groves [6].

because they work with *social choice correspondences*. To be precise, at every profile of types, the mechanism chooses a subset from the set of all possible queues. This formulation means that they cannot use Holmström's result which only applies to *social choice functions*. In contrast, we assume the existence of a tie-breaking rule which selects one queue whenever there is more than one efficient queue and this allows us to use Holmström's result to provide an alternate proof.

One might ask naturally how much generality is lost in our approach. In this regard, we note here that the correspondence which selects the set of efficient queues at each profile is "essentially" single-valued. As will become clear, the efficient queue is unique at all profiles where no two agents have identical waiting costs. We can show that the set of all such profiles is an open and dense set in $\mathbb{R}^n_+$ and thus the efficiency correspondence is generically single-valued. Hence, in our opinion, imposing a tie-breaking rule on profiles where the efficiency correspondence is not single-valued does not amount to a significant loss of generality. We emphasize that our result applies for any choice of tie-breaking rule.[2]

The k-pivotal mechanisms were introduced by Mitra and Mutuswami [12] and generalize the pivotal mechanism which appears in many contexts. Their significance lies in the fact that they are immune to a particular form of group deviation.[3] Mitra and Mutuswami [12] characterize this family using pairwise strategyproofness, queue efficiency, equal treatment of equals and weak linearity. Since, as noted by Mitra and Mutuswami [12] themselves, weak linearity does not have a strategic or normative interpretation, we provide alternate characterizations of two mechanisms from this class using appropriate independence axioms introduced by Chun [2] and Maniquet [10].[4]

This paper is organized as follows. In Section 2, we introduce the model and characterize the subset of VCG mechanisms which also satisfy *equal treatment of equals*. We use this result to prove our two main results in Section 3 and we conclude in Section 4.

2. THE MODEL

Let $N = \{1, \ldots, n\}$, $n \geq 2$, be the set of agents. Each agent has one job to process by a machine but the machine can process only one job at a time. All jobs take the same time to process which is normalized to one.

[2]To be precise, agents with the same waiting cost end up with the same utility, no matter which tie-breaking rule is used to select the efficient queue.

[3]Mitra and Mutuswami [12] show that the k-pivotal mechanisms are *weak group strategyproof* which implies that no coalition deviate in a manner benefiting all deviating members strictly. Pairwise strategyproofness is weak group strategyproofness restricted to coalitions of size at most two.

[4]Weak linearity requires that transfers vary in a linear fashion whenever an agent changes her announcement in a manner which does not change the efficient queue.

A *queue* is an onto function $\sigma : N \to \{1, \ldots, n\}$ denoting the order in which jobs are processed. We denote $\sigma(i)$ as σ_i. The set of all possible queues is $\Sigma(N)$. The set of all *predecessors of agent i* is $P_i(\sigma) = \{j \in N \mid \sigma_j < \sigma_i\}$ and the set of all *followers of agent i* is $F_i(\sigma) = \{j \in N \mid \sigma_j > \sigma_i\}$. When the context is clear, we abuse notation and use P_i and F_i.

Each agent i is identified with her waiting cost per unit of time $\theta_i \in \mathbb{R}_+$ which is known only to the agent. A *profile of waiting costs*, $\theta = (\theta_i)_{i \in N}$, is a collection of the waiting costs of all agents. For all $i \in N$, let $\theta_{N \setminus \{i\}}$ denote the collection of waiting costs of all agents other than i. An agent's utility depends on her waiting cost θ_i and the transfer t_i she receives. We assume that preferences are quasi-linear and given by $u_i(\sigma_i, t_i; \theta_i) = t_i - (\sigma_i - 1)\theta_i$.

A *mechanism* $\mu = (\sigma, t)$ associates to each profile θ, a tuple $\mu(\theta) \equiv (\sigma(\theta), t(\theta))$ where $\sigma(\theta)$ is the selected queue and $t(\theta) = (t_1(\theta), \ldots, t_n(\theta))$ is the vector of transfers. Let $\mu_i(\theta) = (\sigma_i(\theta), t_i(\theta))$ denote i's allocation for the profile θ and let $u_i(\theta; \theta'_i) = -(\sigma_i(\theta) - 1)\theta'_i + t_i(\theta)$ denote i's utility when the announced profile is θ and her own waiting cost is θ'_i.

2.1. **Axioms.** We say that a queue σ is *efficient for the profile* θ if $\sigma \in \operatorname{argmin}_{\sigma' \in \Sigma(N)} \sum_{i \in N} (\sigma'_i - 1)\theta_i$. In words, an efficient queue minimizes the aggregate waiting costs of the agents. It is easy to show that σ is efficient for the profile θ if and only if $\sigma_i < \sigma_j$ whenever $\theta_i > \theta_j$. Let $E(\theta)$ denote the set of efficient queues for the profile θ. Note that $E(\theta)$ is always non-empty and is a singleton if no two agents have the same waiting cost.

Remark 2.1. Since $E(\theta)$ can be a correspondence, our definition of a mechanism implicitly assumes the existence of a *tie-breaking rule* which selects an efficient queue whenever there is more than one such queue. We assume that there is an order of the agents, $\succ$, which is used to break ties. The same order is used to break ties when a queue involving subsets of agents has to be selected.

Definition 2.1. A mechanism μ is *queue efficient* (EFF) if for all θ, $\sigma(\theta) \in E(\theta)$.

The next axiom is *strategyproofness* which requires that an agent should not benefit strictly by misrepresenting her waiting cost no matter what she believes other agents to be doing. Call θ, θ' *S-variants* if $\theta_i = \theta'_i$ for all $i \in N \setminus S$.

Definition 2.2. A mechanism μ is *strategyproof* (SP) if for all $i \in N$, all i-variants θ, θ', $u_i(\theta; \theta_i) \geq u_i(\theta'; \theta_i)$.

Remark 2.2. Holmström [8] shows that when the domain of preferences is convex, the *Vickrey-Clarke-Groves (VCG) mechanisms* are the only ones satisfying EFF and SP. Since preferences are quasi-linear, the preferences are completely specified by the domain of the profile of waiting costs. Since this is $\mathbb{R}^n_+$, it follows that a mechanism satisfies EFF and SP if and only if it is a VCG mechanism.

We now formally define these mechanisms.

Definition 2.3. A mechanism $\mu = (\sigma, t)$ is a VCG mechanism if it satisfies EFF and the transfers at all profiles θ given by

$$t_i(\theta) = -\sum_{j \in F_i(\sigma(\theta))} \theta_j + f_i(\theta_{N\setminus\{i\}}). \tag{2.1}$$

It is to be noted that the usual way of writing the VCG transfers is $t_i(\theta) = -\sum_{j \neq i}(\sigma_j(\theta) - 1)\theta_j + h_i(\theta_{N\setminus\{i\}})$. This is equivalent to our formulation. For details, see our companion paper (Chun, Mitra and Mutuswami [3]).

A desirable property is *budget balance* requiring there be no net transfers into or out of the economy.

Definition 2.4. A mechanism μ is *budget balanced* (BB) if for all θ, $\sum_{i=1}^{n} t_i(\theta) = 0$.

Finally, *equal treatment of equals* is an equity property requiring that two agents with the same waiting cost end up with the same (net) utilities.

Definition 2.5. A mechanism μ satisfies *equal treatment of equals* (ETE) if for all θ, and all $i, j \in N$, $\theta_i = \theta_j$ implies that $u_i(\theta; \theta_i) = u_j(\theta; \theta_j)$.

We also use some additional axioms in our characterizations of the pivotal and reward-based pivotal mechanisms and we discuss them later.

2.2. **Anonymous VCG mechanisms.** In this subsection, we characterize the subset of VCG mechanisms satisfying EFF, SP and ETE. We use this proposition to prove our two main results.

Definition 2.6. A VCG mechanism $\mu = (\sigma, t)$ is *anonymous* if for all θ and all $i \in N$,

(1) $t_i(\theta) = -\sum_{j \in F_i(\sigma(\theta))} \theta_j + f_i(\theta_{N\setminus\{i\}})$, and
(2) f_i is symmetric, i.e., $f_i(x) = f_i(y)$ whenever x and y are permutations of one another.
(3) for all $i, j \in N$, and all θ such that $\theta_i = \theta_j$, $f_i(\theta_{-i}) = f_j(\theta_{-j})$.

Remark 2.3. Given (2) and (3), we can write $f_i = f$ for all $i \in N$.

Our characterization result follows. It must be emphasized that even though we require SP and ETE, the choice of tie-breaking rule does not matter since agents having the same waiting costs end up with the same utility.

Proposition 2.1. *A mechanism satisfies EFF, ETE and SP if and only if it is an anonymous VCG mechanism.*

Proof. : Since the sufficiency is obvious, we only prove the necessity part here. It follows from Remark 2.2 that if (σ, t) satisfies EFF and SP, then it is a VCG mechanism. Hence, the transfers are given by

$$\forall \theta, \forall i \in N: \quad t_i(\theta) = -\sum_{j \in F_i(\sigma(\theta))} \theta_j + f_i(\theta_{N\setminus\{i\}}).$$

Let $\theta_{N\setminus\{i\}}$ and $\theta'_{N\setminus\{i\}}$ be two profiles (in $\mathbb{R}_+^{n-1}$) such that there exist two agents k, j such that (i) $\theta'_j = \theta_k, \theta'_k = \theta_j$, and (ii) $\theta_l = \theta'_l$ for all $l \neq j, k$. In other words, the two profiles differ only in that the announcements of two agents are interchanged. We will show that for all $i \in N$, $f_i(\theta_{N\setminus\{i\}}) = f_i(\theta'_{N\setminus\{i\}})$. Suppose not, and let $x = f_i(\theta_{N\setminus\{i\}}) \neq f_i(\theta'_{N\setminus\{i\}}) = y$.

Consider $(\theta_j, \theta_{N\setminus\{i\}}) \in \mathbb{R}_+^n$ where i and j announce θ_j and the other agents announce $\theta_{N\setminus\{i,j\}}$. Using ETE between i and j, it follows that

$$f_i(\theta_{N\setminus\{i\}}) = f_j(\theta_j, \theta_{N\setminus\{i,j\}}) = x. \tag{2.2}$$

Next, consider $(\theta_j, \theta_k, \theta_{N\setminus\{i,j\}})$ where i announces θ_j, j and k announce θ_k and the others announce $\theta_{N\setminus\{i,j,k\}}$. Using ETE between j and k, it follows that

$$f_j(\theta_j, \theta_{N\setminus\{i,j\}}) = f_k(\theta_j, \theta_k, \theta_{N\setminus\{i,j,k\}}). \tag{2.3}$$

It follows from the above two equations that

$$f_k(\theta_j, \theta_k, \theta_{N\setminus\{i,j,k\}}) = x. \tag{2.4}$$

Now, consider $(\theta_k, \theta_{N\setminus\{i\}})$ where i and k announce θ_k and the rest announce $\theta_{N\setminus\{i,k\}}$. Using ETE between i and k, we obtain

$$f_i(\theta_{N\setminus\{i\}}) = f_k(\theta_k, \theta_{N\setminus\{i,k\}}) = x. \tag{2.5}$$

It follows from the previous two equations that

$$f_k(\theta_j, \theta_k, \theta_{N\setminus\{i,j,k\}}) = f_k(\theta_k, \theta_j, \theta_{N\setminus\{i,j,k\}}) = x. \tag{2.6}$$

Now starting from $\theta'_{N\setminus\{i\}}$, we can do the same analysis to obtain

$$f_k(\theta_j, \theta_k, \theta_{N\setminus\{i,j,k\}}) = f_k(\theta_k, \theta_j, \theta_{N\setminus\{i,j,k\}}) = y. \tag{2.7}$$

This gives us the contradiction. Now let $\theta_{N\setminus\{i\}}$ and $\theta'_{N\setminus\{i\}}$ be permutations of one another. It is easy to see that we can get from one profile to any permutation of the profile by interchanging sequentially the announcements of two agents in at most $n-2$ steps.[5] From the above analysis, f_i cannot change at any step and hence, we must have $f_i(\theta_{N\setminus\{i\}}) = f_i(\theta'_{N\setminus\{i\}})$.

Note that ETE implies that for all $i, j \in N, i \neq j$, $f_i(\theta_j, \theta_{N\setminus\{i,j\}}) = f_j(\theta_i, \theta_{N\setminus\{i,j\}})$ if $\theta_i = \theta_j$. This along with symmetry of f_i implies that all the f_i functions are the same and so we can put $f_i = f$ for all i. □

Remark 2.4. In the context of a related but different model of allocating heterogeneous goods, Pápai [13] showed that the family of *anonymous VCG mechanisms* can be characterized using the property of envy-free (see Observation 3, p 376 of her article). We provide a complete proof here to show

[5] Let $\theta, \theta' \in \mathbb{R}^n$ be permutations of one another. First, find θ_k such that $\theta_k = \theta'_1$. Construct the profile $\hat{\theta}^1$ such that $\hat{\theta}^1_1 = \theta_k, \hat{\theta}^1_k = \theta_1$ and $\hat{\theta}^1_j = \theta_j$ for $j \neq 1, k$. Then proceed to θ_2 and so on. At the end of step j, the first j components of $\hat{\theta}^j$ must coincide with θ' and hence, in at most $n-1$ steps we would have moved from θ to θ'.

that these mechanisms can be characterized using the weaker property of equal treatment of equals.

Remark 2.5. Hashimoto and Saitoh [7] show that anonymity in welfare and SP implies EFF. The following example shows that anonymity in welfare cannot be weakened to ETE. Let $N = \{1, 2\}$ and suppose that $\sigma_i(\theta) = i$ for all profiles θ. The transfers are given by

$$t_i(\theta) = \begin{cases} -\theta_2/2 & \text{if } i = 1, \\ \theta_1/2 & \text{if } i = 2. \end{cases}$$

It is straightforward to verify that this mechanism violates EFF but satisfies both SP and ETE.

3. Characterization Results

We now analyze some interesting mechanisms from the class of anonymous VCG mechanisms characterized in Theorem 2.1.

3.1. **Symmetrically balanced VCG mechanism.** The *symmetrically balanced VCG mechanism* is the VCG mechanism for which the transfers are given by
(3.1)

$$\forall i \in N: \quad t_i^{\lambda^*}(\theta) = \sum_{l \in P_i(\sigma(\theta))} \left(\frac{\sigma_l(\theta) - 1}{n - 2} \right) \theta_l - \sum_{l \in F_i(\sigma(\theta))} \left(\frac{n - \sigma_l(\theta)}{n - 2} \right) \theta_l.$$

This mechanism is obtained by choosing

$$\forall i \in N : f(\theta_{N \setminus \{i\}}) = \sum_{j \in N \setminus \{i\}} \left(\frac{\sigma_j(\theta_{N \setminus \{i\}}) - 1}{n - 2} \right) \theta_j. \tag{3.2}$$

Kayi and Ramaekers [9] established the following result. As discussed in the introduction, we provide an alternate proof here.

Theorem 3.1. Let $n \geq 3$. A mechanism satisfies EFF, ETE, SP and BB if and only if it is the symmetrically balanced VCG mechanism.

Proof. Since the sufficiency is clear, we only show the necessity here. Let $\theta' = (\theta'_i)_{i \in N}$ be such that $\theta'_1 > \cdots > \theta'_n$. We shall show that

$$f(\theta'_{-n}) = \sum_{i=1}^{n-1} \left(\frac{i - 1}{n - 2} \right) \theta'_i. \tag{3.3}$$

By EFF, $\sigma_i(\theta') = i$ for all $i \in N$. By Proposition 2.1 and budget balance,

$$\sum_{i \in N} f(\theta'_{N \setminus \{i\}}) = \sum_{i \in N} (i - 1)\theta'_i. \tag{3.4}$$

Let $\Theta = \{\theta'_1, \cdots, \theta'_n\}$. For $k = 1, \ldots, n$, let Θ^k be the set of profiles θ such that

(1) $\theta_i \in \Theta$ for all $i \in N$,
(2) $i < j < k \Longrightarrow \theta_i > \theta_j > \theta'_n$,

(3) $\theta_i = \theta'_n$ if $i \geq k$.

We will prove the following hypothesis by induction. For all $k = 1, \dots, n$, and all $\theta \in \Theta^k$,

$$f(\theta_{-n}) = \sum_{r=1}^{k-1} \left(\frac{r-1}{n-2}\right) \theta_r + \frac{(n+k-3)(n-k)}{2(n-2)} \theta'_n. \tag{3.5}$$

Step 1: When $k = 1$, the set Θ^1 is a singleton and $\theta = (\theta'_n, \dots, \theta'_n)$. By BB,

$$\sum_{i \in N} f(\theta_{-i}) = \frac{n(n-1)}{2} \theta'_n.$$

Since f is symmetric, we can write the above as

$$nf(\theta_{-n}) = \frac{n(n-1)}{2} \theta'_n \text{ or } f(\theta_{-n}) = \frac{n-1}{2} \theta'_n.$$

It is easily verified that the above expression for $f(\theta_{-n})$ is identical to the one given by (3.5).

Induction Step: Suppose that the hypothesis is true for all $k \leq K-1$. Let $\theta = (\theta_1, \cdots, \theta_{K-1}, \theta_n, \cdots, \theta_n) \in \Theta^K$. Observe that any efficient queue σ for this profile must be such that $\sigma_i = i, i = 1, \dots, K-1$. BB implies that

$$\sum_{i \in N} f(\theta_{-i}) = \sum_{i=1}^{K-1} (i-1)\theta_i + \frac{(n-K+1)(n+K-2)}{2} \theta'_n. \tag{3.6}$$

Let $i \in \{1, \dots, K-1\}$. Define θ^i by

$$\theta^i_j = \begin{cases} \theta_j & \text{if } j < i, \\ \theta_{j+1} & \text{if } n > j \geq i, \\ \theta'_n & \text{otherwise.} \end{cases}$$

Thus, the first $i-1$ elements of θ^i are the same as θ, for the next $n-i$ elements, the jth element of θ^i is the $j+1$ element of θ and the last element of θ^i is θ'_n. Observe that $\theta^i_j = \theta'_n$ for all $j \geq K-1$ and hence $\theta^i \in \Theta^{K-1}$.

For $i = 1, \dots, K-1$, θ^i_{-n} is a permutation of θ_{-i}. Hence the symmetry of f implies that $f(\theta_{-i}) = f(\theta^i_{-n})$. Symmetry also implies that for any $i, j \in \{K, \dots, n\}$, $f(\theta_{-i}) = f(\theta_{-j})$. Hence, for $i = K, \dots, n$, we can write $f(\theta_{-i}) = f(\theta_{-n})$. Therefore, we can write (3.6) as

$$\sum_{i=1}^{K-1} f(\theta^i_{-n}) + (n-K+1) f(\theta_{-n}) = \sum_{i=1}^{K-1} (i-1)\theta_i + \frac{(n-K+1)(n+K-2)}{2} \theta'_n. \tag{3.7}$$

Using the induction hypothesis on the profiles θ^i, we have

$$f(\theta^i_{-n}) = \sum_{r=1}^{K-2} \left(\frac{r-1}{n-2}\right) \theta^{i-1}_r + \frac{(n+K-4)(n-K+1)}{2(n-2)} \theta'_n. \tag{3.8}$$

Hence,

$$\begin{aligned}(n-K+1)f(\theta_{-n}) &= \sum_{i=1}^{K-1}(i-1)\theta_i - \sum_{i=1}^{K-1}\sum_{r=1}^{K-2}\left(\frac{r-1}{n-2}\right)\theta_r^i \\ &+ \left[\frac{(n-K+1)(n+K-2)}{2} - \frac{(K-1)(n+K-4)(n-K+1)}{2(n-2)}\right]\theta'_n.\end{aligned}$$

This simplifies to

$$\begin{aligned}(n-K+1)f(\theta_{-n}) &= \sum_{i=1}^{K-1}(i-1)\theta_i - \sum_{i=1}^{K-1}\sum_{r=1}^{K-2}\left(\frac{r-1}{n-2}\right)\theta_r^i \\ &+ \frac{(n-K+1)(n-K)(n+K-3)}{2(n-2)}\theta'_n.\end{aligned}$$

Recall that $\theta_j^i = \theta_j, j = 1, \dots, i-1$ and $\theta_j^i = \theta_{j+1}, j = i, \dots, K-2$. Note, however, that the order in the efficient queue is the same for $i = 1, \dots, K-2$ in the profiles θ and θ^i. Using these observations,

$$\sum_{r=1}^{K-2}\left(\frac{r-1}{n-2}\right)\theta_r^i = \sum_{r=1}^{i-1}\left(\frac{r-1}{n-2}\right)\theta_r + \sum_{r=i}^{K-2}\left(\frac{r-1}{n-2}\right)\theta_{r+1}.$$

Note that θ_r appears in exactly $K-2$ of the profiles of the type θ^i. In particular, θ_r appears in all profiles except for θ^r. For $i = 1, \dots, r-1$, θ_r is the waiting cost of agent $r-1$ (in the profile θ^i); in the other profiles, it is the waiting cost of agent r. It thus follows that the coefficient of θ_r in the expression

$$\sum_{i=1}^{K-1}(i-1)\theta_i - \sum_{i=1}^{K-1}\sum_{j=1}^{K-2}\left(\frac{j-1}{n-2}\right)\theta_j^i$$

is

$$r-1-\frac{(r-2)(r-1)}{n-2} - \frac{(K-r-1)(r-1)}{n-2} = \frac{(r-1)(n-K+1)}{n-2}.$$

We thus have

$$(n-K+1)f(\theta_{-n}) = \sum_{r=1}^{K-1}\frac{(r-1)(n-K+1)}{n-2}\theta_r + \frac{(n-K+1)(n-K)(n+K-3)}{2(n-2)}\theta'_n.$$

Dividing across by $(n-K+1)$ establishes the induction step.

We can now establish (3.3) by considering the case $k = n$. In this case, the second term drops out of (3.5) and the expression is exactly what we want to establish. Since $f(\theta'_{-n})$ does not depend on θ'_n, it follows that (3.3) also applies to any profile where the waiting costs of agents in $N \setminus \{n\}$ have the same ordinal ranking. Observe that (3.2) reduces to (3.3).

Now let $i \neq n$. Rename the agents so that agents i and n interchange names with the others retaining their original names. We can do the same argument, and at the end interchange names again, to conclude that $f(\theta'_{-i})$ is given by (3.2). To complete the proof, we need to consider the case when $\theta'_1 \geq \cdots \geq \theta'_n$. In this case, the same proof goes through except that we have to use a tie-breaking rule to select an efficient queue. It can be verified that no matter what tie-breaking rule is used, $f(\theta'_{-i})$ will still be given by (3.2). □

3.2. **Pivotal and reward-based pivotal mechanisms.** In the queueing context, by generalizing the idea of the pivotal mechanism, Mitra and Mutuswami [12] introduced the k-pivotal mechanisms and showed that they are *weak group strategyproof*.[6] In addition, it can be shown that the k-pivotal mechanisms also satisfy the normative criterion of *no-envy*.[7]

Definition 3.1. A mechanism $(\sigma, \bar{t}^k)$ is a *k-pivotal mechanism* if there exists $k \in \{1, \ldots, n\}$ such that for each profile θ, $\sigma(\theta) \in E(\theta)$ and the transfers are given by

$$\bar{t}_i^k(\theta) = \begin{cases} -\sum_{j:\sigma_i(\theta)<\sigma_j(\theta)\leq k} \theta_j & \text{if } \sigma_i(\theta) < k, \\ 0 & \text{if } \sigma_i(\theta) = k, \\ \sum_{j:k\leq\sigma_j(\theta)<\sigma_i(\theta)} \theta_j & \text{if } \sigma_i(\theta) > k. \end{cases}$$

Mitra and Mutuswami [12] characterize the k-pivotal mechanisms through the axioms of pairwise strategyproofness, EFF, ETE and weak linearity. As discussed in the introduction, weak linearity is a technical assumption, lacking interpretation in strategic or normative terms. Here, we provide axiomatic characterization of two mechanisms in the class of k-pivotal mechanisms without using weak linearity.

Definition 3.2. The *pivotal* mechanism $(\sigma, t^{\mathbf{p}})$ is such that for all profiles θ, $\sigma(\theta) \in E(\theta)$ and

$$t_i^{\mathbf{p}}(\theta) = -\sum_{j\in F_i(\sigma(\theta))} \theta_j \text{ for all } i \in N. \tag{3.9}$$

Definition 3.3. The *reward-based pivotal* mechanism $(\sigma, t^{\mathbf{r}})$ is such that for all profiles θ, $\sigma(\theta) \in E(\theta)$ and

$$t_i^{\mathbf{r}}(\theta) = \sum_{j\in P_i(\sigma(\theta))} \theta_j \text{ for all } i \in N. \tag{3.10}$$

Remark 3.1. The *pivotal mechanism* is the n-pivotal mechanism while the *reward-based pivotal mechanism* is the 1-pivotal mechanism.

The independence axioms that we use to characterize the pivotal and the reward-based pivotal mechanisms are based on the idea that if an agent's waiting cost changes but the efficient queue remains unchanged, then some agents should remain unaffected. Note that even though the efficient queue is unchanged, the aggregate waiting cost of society will change. Hence, it is not clear which agents should remain unaffected. Chun [2] and Maniquet [10] take differing approaches.

[6] Formally, weak group strategyproofness implies there are no S-variants θ, θ' such that $u_i(\theta'; \theta_i) > u_i(\theta; \theta_i)$ for all $i \in S$. We have pairwise strategyproofness if we also require $|S| \leq 2$.

[7] No-envy, introduced by Foley [5], requires the allocation to be such that no agent strictly prefers having another agent's allocation.

Maniquet [10] uses *independence of preceding costs* which requires that an increase in an agent's waiting cost which leaves the efficient queue is unchanged should affect only the agent and her predecessors. This reflects the idea that when an agent's waiting cost increases, while the aggregate waiting cost increases, this cannot be attributed to the agents' successors. On the other hand, Chun [2] uses *independence of following costs* which requires that a decrease in an agent's waiting cost (so that the efficient queue is unchanged) not affect the predecessors. This is based on a similar idea: a decrease in an agent's waiting cost decreases the aggregate waiting cost. Since the agent's predecessors are not involved in generating this benefit, they should not be affected. We define the axioms formally now.

Definition 3.4. A mechanism μ satisfies *independence of preceeding costs* (IPC) if for all i-variants θ and θ' such that $\theta'_i > \theta_i$ and $\sigma(\theta) = \sigma(\theta')$, $u_l(\theta;\theta_l) = u_l(\theta';\theta_l)$ for all $l \in F_i(\sigma(\theta))$.

Definition 3.5. A mechanism μ satisfies *independence of following costs* (IFC) if for all i-variants θ and θ' such that $\theta'_i < \theta_i$ and $\sigma(\theta) = \sigma(\theta')$, $u_l(\theta;\theta_l) = u_l(\theta';\theta_l)$ flor all $l \in P_i(\sigma(\theta))$.

Remark 3.2. It is obvious that the *pivotal* mechanism satisfies IPC, and the *reward-based pivotal* mechanism satisfies IFC.

Our last axiom is a mild regularity condition on the transfers saying that if all agents have zero waiting costs, then their utility from the mechanism should also be zero.

Definition 3.6. A mechanism μ satisfies the *zero transfer condition* (ZTC) if $u_i(\theta;\theta_i) = 0$ for all $i \in N$ whenever $\theta = (0, \ldots, 0)$.

The following theorem characterizes the pivotal and the reward-based pivotal mechanisms.

Theorem 3.2.

(1) The *pivotal* mechanism $(\sigma, t^{\mathbf{p}})$ is the only mechanism that satisfies EFF, ETE, SP, IPC, and ZTC.
(2) The *reward-based pivotal* mechanism $(\sigma, t^{\mathbf{r}})$ is the only mechanism that satisfies EFF, ETE, SP, IFC, and ZTC.

Proof. We first prove (1). It is obvious that the pivotal mechanism satisfies EFF, ETE, SP, IPC, and ZTC. Conversely, let (σ, t) be a mechanism satisfying the five axioms. Without loss of generality, suppose that θ is such that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$ and that for each $i \in N$, $\sigma_i = i$. By Proposition 2.1, the transfer can be expressed as:

$$\forall i \in N: \quad t_i(\theta) = -\sum_{j \in F_i(\sigma(\theta))} \theta_j + f(\theta_{N\setminus\{i\}}).$$

In particular, $t_n(\theta) = f(\theta_{N\setminus\{n\}})$. By IPC, for all $N \setminus \{n\}$-variants θ, θ' such that $\theta'_i \geq \theta'_n = \theta_n$ for all $i \neq n$, $t_n(\theta) = t_n(\theta')$. This implies that there exists

$c \in \mathbb{R}$ such that for all $\theta_{N\setminus\{n\}}$, $f(\theta_{N\setminus\{n\}}) = c$. By ZTC, $c = 0$. Altogether, we conclude that for all $i \in N$, $t_i(\theta) = -\sum_{j\in F_i(\sigma(\theta))}\theta_j = t_i^{\mathbf{p}}$, as desired.

Now we prove (2). It is obvious that the reward-based pivotal mechanism satisfies EFF, ETE, SP, IFC, and ZTC. Conversely, let (σ, t) be a mechanism satisfying the five axioms. Without loss of generality, suppose that θ is such that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$ and that for each $i \in N$, $\sigma_i = i$. By Theorem 2.1, the transfer can be expressed as:

$$\forall i \in N: \quad t_i(\theta) = -\sum_{j\in F_i(\sigma(\theta))}\theta_j + f(\theta_{N\setminus\{i\}}).$$

In particular, $t_1(\theta) = -\sum_{j\in F_1(\sigma(\theta))}\theta_j + f(\theta_{N\setminus\{1\}})$. By IFC, for all θ, θ' such that $\theta_1 = \theta'_1$, $t_1(\theta) = t_1(\theta')$ which implies that there exists $c \in \mathbb{R}$ such that for all $\theta_{N\setminus\{1\}}$, $f(\theta_{N\setminus\{1\}}) = \sum_{j\in F_1(\sigma(\theta))}\theta_j + c$. By ZTC, $c = 0$. Altogether, we have that for all $i \in N$,

$$\begin{aligned} t_i(\theta) &= -\sum_{j\in F_i(\sigma(\theta))}\theta_j + f(\theta_{N\setminus\{i\}}) \\ &= -\sum_{j\in F_i(\sigma(\theta))}\theta_j + \sum_{j\in N\setminus\{i\}}\theta_j \\ &= \sum_{j\in P_i(\sigma(\theta))}\theta_j \\ &= t_i^{\mathbf{r}}, \end{aligned}$$

which is the desired conclusion. □

4. Conclusions

We have provided an alternate proof of Kayi and Ramaekers' [9] characterization of symmetrically balanced VCG mechanism. We have also provided characterizations of the pivotal and the reward-based pivotal mechanisms using independence axioms. A remaining task is to provide a characterization of the entire class of k-pivotal mechanisms without using weak linearity.

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