Intermediary Cost and Coexistence Puzzle

by

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Abstract

The coexistence puzzle is explained via an interaction between intermediary cost and uncertainty with regards to consumption trade. If a trade opportunity as a buyer is more likely to arise, ex-ante net return on bond at the margin would be negative up to a certain amount of transaction and, therefore, agents are willing to hold money in the presence of an interest-bearing bond.

Keywords: intermediary cost, interest-bearing asset, coexistence puzzle

JEL classification: E40, E42

1. Introduction

The coexistence puzzle has been one of the long-standing issues for monetary economists at least since Hicks (1935). Gherity (1993) and Burdekin and Weidenmier (2008) report that several types of bonds issued during the U.S. Civil War were circulated at par (face value without interest) as media of exchange until shortly before maturity, but failed to drive money out of circulation.

This paper provides an explanation for the coexistence puzzle based on the interaction between intermediary cost and uncertainty with regards to consumption trade as the key determinant of portfolio choice between money and interest-bearing assets. Specifically, we embed the “shoeleather” or intermediary cost incurred in the purchase of government bonds

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into a general equilibrium model wherein money is essential. A one-period risk-free coupon bond can be freely liquidated at par for consumption-good purchase and intermediary cost is strictly less than exogenous coupon payment at maturity.\(^1\) Therefore, ex-post return on a coupon bond net of intermediary cost is strictly positive.

However, ex-ante return on the bond net of intermediary cost is not always positive. For example, net return on the bond can be negative at the margin if it is used for consumption purchase and its return from early redemption is sufficiently small, which was the case of Civil-War bonds that were circulated at face value without interest. Then people are willing to hold money for consumption purchase even in the presence of an interest-bearing bond. In general, ex-ante return on the bond net of intermediary cost depends on the likelihood that the bond is liquidated for consumption purchase, which is determined by trade opportunity as a buyer and expected amount of transaction. The novelty here is that the commonly-observed intermediary cost combined with uncertainty with regards to consumption transaction turns out to be a fundamental ingredient of the coexistence result.\(^2\)

2. **Model**

The environment is that of Lagos and Wright (2005) with competitive markets as in Berentsen et al. (2005). Time is discrete and continues forever. In each period, a unit mass of infinitely-lived agents trade in three Walrasian markets for consumption good, called market 1, 2, and 3, which open sequentially. Trading histories are private and agents cannot commit to their future actions, which make a medium of exchange essential. Each agent maximizes discounted expected utility with the discount factor \(\beta \in (0, 1)\) between one period and the next.

\(^1\)Based on the observation of a non-trivial fraction of U.S. households with no interest-bearing assets, Mulligan and Sala-i-Martin (1996) and Lucas (2000) argue that such an intermediary cost would be sizable.

\(^2\)Among recent related works using search-theoretic models of money are Aiyagari et al. (1996), Shi (2005), Boel and Camera (2006), Zhu and Wallace (2007), Berentsen and Waller (2008), Marchesiani and Senesi (2009), and Lagos (2010).
There is one perishable and divisible good which can be potentially produced and consumed by all agents. There are two other objects, called money and bond. Money is durable, divisible, and its stock is exogenously given by $M > 0$. A one-period risk-free bond takes the form of a book-entry coupon bond issued by the government that has a technology for record keeping on intra-day bond transactions, but not on agents’ trading histories. The purchase of a bond at a price of one unit of money requires an agent physical effort which incurs disutility $\gamma > 0$ per unit of bond. In the real world, this cost can be interpreted as the time spent in purchasing a bond as well as the transaction fee charged by security dealers. Hence, hereinafter we call it an intermediary cost, although we do not introduce financial intermediary explicitly. Upon request by agents for a book-entry bond liquidation before maturity, the government can cash in a bond at par (i.e., one unit of money) with no coupon payment. This is consistent with the historical facts documented in Gherity (1993) and Burdekin and Weidenmier (2008).\footnote{If a bond takes the form of a discount bond rather than a coupon bond, liquidating a bond can be interpreted as giving up remaining accrued interest.}

The sequence of events within a period is as follows. When entering market 1 with a given amount of money, each agent receives an individual trading shock such that she will become, with equal probability, either a buyer or a seller of consumption good. Agents get utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of good where $u''(q) < 0 < u'(q)$, $u'(\infty) = 0$, $u(0) = 0$, and $u'(0) = \infty$. Production of $q \in \mathbb{R}_+$ units of good incurs disutility $q$, which is also the case in market 2 and 3 described below.

With money balance after the trade in market 1, agents move on to market 2 and choose a portfolio of money and bond. Upon demand by agents, the government sells a coupon bond at a price of one unit of money. Then, an agent becomes either a buyer with probability $\rho_b$ or a seller with probability $\rho_s = 1 - \rho_b$. An agent receives utility $\varepsilon_i u(q)$ from consuming $q$ units of good where $\varepsilon_i \in \{\varepsilon_h, \varepsilon_l\}$ with $\varepsilon_h > \varepsilon_l$ represents an aggregate preference shock that is realized together with individual trading shocks (to be a buyer or a seller). In particular,
$\epsilon_i = \epsilon_h$ with probability $\delta_h$ and $\epsilon_i = \epsilon_l$ with probability $\delta_l = 1 - \delta_h$. As a buyer, an agent can freely liquidate a bond at par for consumption purchase, in which case she essentially gives up an interest-bearing coupon. The proceeds of bond sales are used to produce goods by the government which has access to linear technology. Specifically, the government can produce $\theta > \gamma$ units of good per unit of money at no cost.

At the beginning of market 3, the government redeems a bond with a unit of money and $\theta$ units of good (exogenous coupon rate) according to its record which is wiped out immediately after redemption. Other than selling and redeeming bonds, there is no activity of the government so that its budget is always in balance. In market 3, all agents can consume, produce, and get utility $U(q)$ from consuming $q \in \mathbb{R}_+$ units of good where $U(\cdot)$ satisfies all the nice properties mentioned above. Agents also choose the balance of money to be carried into the following period.

3. Equilibrium

Let $p_{j,t}$ and $q_{j,t}$ denote price and quantity of good traded in market $j \in \{1, 2, 3\}$ at period $t$, respectively. We will study equilibria in which real balance of money at the end of period is constant with a fixed stock of money $M$. For this reason, we will drop the time subscript $t$ and index the next-period variable by $+1$, if there is no risk of confusion.

Let $V_3(m_3, g_3)$ denote the expected value for an agent who enters market 3 with $(m_3, g_3)$ where $m_3$ and $g_3$ denote money and government-bond balances brought into market 3, respectively. Let $\phi$ denote the price of money in terms of good in market 3, $p_3 = 1/\phi$. Then, the problem for an agent entering market 3 with $(m_3, g_3)$ is

$$V_3(m_3, g_3) = \max_{(q_3^b, q_3^s, m_{1,+1})} \left[ U(q_3^b) - q_3^s + \beta V_1(m_{1,+1}) \right]$$

(1)

As discussed in Berentsen et al. (2005), the different preference in market 3 is simply a technical device to ensure a degenerate distribution at the beginning of each period.
subject to \( q_3^b + \phi m_{1,+1} = q_3^s + \phi (m_3 + g_3) + \theta g_3 \). Here \( q_3^b \) and \( q_3^s \) denote consumption and production in market 3, respectively, and \( V_1(m_{1,+1}) \) is the expected value of entering market 1 in the next period with \( m_{1,+1} \). Substituting \( q_3^s \) from the constraint, we have

\[
V_3(m_3, g_3) = \phi (m_3 + g_3) + \theta g_3 + \max_{(q_3^s, m_{1,+1})} \left[ U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_1(m_{1,+1}) \right].
\]

The first-order conditions and envelope conditions are respectively as follows:

\[
U'(q_3^b) = 1 	ag{2}
\]
\[
\phi \geq \beta V_1'(m_{1,+1}) =: \text{if } m_{1,+1} > 0 	ag{3}
\]
\[
V'_{3,1}(m_3, g_3) = \phi; \quad V'_{3,2}(m_3, g_3) = \phi + \theta. 	ag{4}
\]

As in Lagos and Wright (2005), all agents consume \( q_3^b = q_3^s = \arg\max [U(q_3^b) - q_3^b] \) regardless of \((m_3, g_3)\) and exit market 3 with an identical balance of money.

We next turn to market 2. Let \( V_2(m_2) \) be the expected value for an agent entering market 2 with \( m_2 \). We let \( \Gamma(m_2) \) be a set of feasible portfolios for an agent with \( m_2 \), defined by

\[
\Gamma(m_2) = \{ a = (\tilde{m}_2, g_2) \in \mathbb{R}_+^2 : \tilde{m}_2 + g_2 \leq m_2 \}.
\]

Then the portfolio-choice problem is

\[
V_2(m_2) = \max_{a \in \Gamma(m_2)} J(a) \tag{5}
\]

where \( J(a) \), the expected payoff from choosing \( a = (\tilde{m}_2, g_2) \) prior to the realization of individual trading shocks and aggregate preference shocks in market 2, satisfies

\[
J(a) = \rho_b \left\{ \sum_{i \in \{k,l\}} \delta_i \max_{q_{2i}^b} \left[ \varepsilon_i u(q_{2i}^b) + V_3 \left( (\tilde{m}_2 - p_2 q_{2i}^b) \mathbb{I}^c, g_2 - (p_2 q_{2i}^b - \tilde{m}_2) \mathbb{I}_{\{\tilde{m}_2 < p_2 q_{2i}^b \leq m_2\}} \right) \right] \right\}
+ \rho_s \left\{ \max_{q_{2i}^s} \left[ -q_{2i}^s + V_3 ((\tilde{m}_2 + p_2 q_{2i}^s), g_2) \right] \right\} - \gamma g_2. \tag{6}
\]

Here \( \mathbb{I}(\chi) = 1 \) if and only if \( \chi \) is true and \( \mathbb{I}^c = 1 - \mathbb{I} \). Noting that, upon request by agents,
the government can redeem a book-entry coupon bond at the face value before maturity, \( p_2 q_{2i}^b \in (\tilde{m}_2, m_2] \) implies that \( (p_2 q_{2i}^b - \tilde{m}_2) \) amount of bonds is cashed in at par without coupon payment.

Conditional on the realization of \( \varepsilon_i \) and individual trading shocks, an agent chooses \( q_{2i}^b \) and \( q_{2i}^s \), taking \( p_2 \) as given. More specifically, as a seller, an agent solves the second term of (6), which yields the first-order condition

\[
p_2 = (1/\phi) = p_3. \tag{7}
\]

That is, whatever demand arises, sellers are willing to supply goods at a price \( p_2 = p_3 \). Similarly, as a buyer, an agent solves the first term of (6) and chooses \( q_{2i}^b \) which satisfies

\[
\frac{\varepsilon_i u'(q_{2i}^b)}{p_2} \geq \phi \mathbb{I}^c + (\phi + \theta) \mathbb{I}_{\{\tilde{m}_2 < p_2 q_{2i}^b \leq m_2\}} \tag{8}
\]

where the equality holds when the constraint \( (p_2 q_{2i}^b \leq \tilde{m}_2 \text{ for } \mathbb{I} = 0 \text{ and } p_2 q_{2i}^b \leq m_2 \text{ for } \mathbb{I} = 1) \) is inactive. Let \( q_{2i}^b \) be the solution to (8) with equality and \( \mathbb{I}_{\{\tilde{m}_2 < p_2 q_{2i}^b \leq m_2\}} = 1 \). Then, an agent with \( m_2 \) chooses \( \tilde{m}_2 \) which satisfies

\[
(\gamma - \rho_s \theta) \geq \rho_b \sum_{i \in \{h,l\}} \delta_i \left\{ (\phi + \theta) - \varepsilon_i u'(q_{2i}^b) \tilde{m}_2 \right\} \mathbb{I}_{\{p_2 q_{2i}^b \leq \tilde{m}_2 \leq m_2\}} \tag{9}
\]

where the equality holds when the constraint \( (\tilde{m}_2 \leq m_2) \) is inactive. This implies that a higher intermediary cost \( \gamma \) makes the bond less attractive and hence \( \tilde{m}_2 \) will increase: i.e.,

\[ g_2 = m_2 - \tilde{m}_2 \text{ will decrease}. \]

5 The left-hand side of (9) represents the expected net marginal return from holding money in terms of the intermediary cost saving, net of the forgone coupon payment in case she becomes a seller. The right-hand side of it represents the expected net marginal (opportunity) cost of holding money in terms of the forgone expected marginal return from holding bond, net of the marginal liquidity return from holding money. This is in line with the asset-pricing equations in Lagos (2011) in which money and equity shares can be used as means of payment. In our model, equity share is replaced by a one-period book-entry coupon bond that the government issues upon demand by agents at its face value with an exogenous coupon rate.
Finally, $V_1(m_1)$, the expected value for an agent entering market 1 with $m_1$, satisfies the following Bellman equation:

$$V_1(m_1) = \frac{1}{2} \left\{ \max_{q_i^b} \left[ u(q_i^b) + V_2(m_1 - p_1q_i^b) \right] \right\} + \frac{1}{2} \left\{ \max_{q_i^s} \left[ V_2(m_1 + p_1q_i^s) - q_i^s \right] \right\}. \quad (10)$$

As in market 2, a seller chooses $q_i^s$ which solves the second term on the right-hand side of (10), taking $p_1$ as given. This yields the first-order condition

$$\frac{1}{p_1} = V_2'(m_1 + p_1q_i^s). \quad (11)$$

Similarly, a buyer chooses $q_i^b$ which solves the first term on the right-hand side of (10) subject to $p_1q_i^b \leq m_1$, taking $p_1$ as given. Notice that we can rule out the case of $p_1q_i^b = m_1$ because $\varepsilon_iu'(0) = \infty$. The optimal condition is

$$\frac{u'(q_i^b)}{p_1} = V_2'(m_1 - p_1q_i^b). \quad (12)$$

**Definition 1** A stationary monetary equilibrium is \( \{ (p_j, q_j, V_j)_{j=1}^3, a, m_{1+1} \} \) that satisfies (1)–(12) and that clears each market \( j \in \{1, 2, 3\} \).

4. Coexistence Result

The following results show that ex-ante return on the bond net of intermediary cost can be negative at the margin depending on (i) the trade opportunity as a buyer ($\rho_b$) and (ii) the likelihood $\delta_h$ of aggregate preference shock that determines amount of transaction.

**Lemma 1** If $\rho_b \leq [(\theta - \gamma)/\theta]$, \( a = (0, m_2) \) is optimal for an agent with $m_2$ after the trade in market 1.

This Lemma suggests that money would not be held at all if the trade opportunity as a buyer is too scarce. Therefore, hereinafter we will focus on the case of $\rho_b > [(\theta - \gamma)/\theta]$. 

7
Proposition 1 Suppose $\rho_b > [(\theta - \gamma)/\theta]$. (i) If $\delta_h < \tilde{\delta}_h$, an agent with $m_2 \geq \tilde{m}_l > p_2 \tilde{q}_{2l}$ chooses $a = (\tilde{m}_l, m_2 - \tilde{m}_l)$, whereas an agent with $m_2 < \tilde{m}_l < p_2 \tilde{q}_{2h}$ chooses $a = (m_2, 0)$. (ii) If $\delta_h \geq \tilde{\delta}_h$, an agent with $m_2 \geq \tilde{m}_h \geq p_2 \bar{q}_{2h}$ chooses $a = (\tilde{m}_h, m_2 - \tilde{m}_h)$, whereas an agent with $m_2 < \tilde{m}_h$ chooses $a = (m_2, 0)$, where $(\tilde{\delta}_h, \tilde{m}_l, \tilde{m}_h)$ is as defined in Appendix.

Proposition 1 implies that ex-ante net return on bond is negative up to $\tilde{m}_h$ if $\varepsilon_h$ is most likely to occur in market 2. Hence, agents are willing to hold sufficient amount of money for the largest possible quantity of trade in the upcoming market 2. On the other hand, if $\varepsilon_l$ is most likely to occur, ex-ante net return on bond exceeding $\tilde{m}_l$ dominates that of money. Hence, no one is willing to hold money more than $\tilde{m}_l$. Proposition 1 also implies that if $\delta_h \geq \tilde{\delta}_h$, all trades in market 2 are made using money carried into the market regardless of the realized aggregate preference shock $\varepsilon_i$. More interesting equilibrium arises with $\delta_h < \tilde{\delta}_h$.

Lemma 2 Suppose $\delta_h < \tilde{\delta}_h$. For $\varepsilon_i = \varepsilon_h$, a buyer with $a = (\tilde{m}_2, g_2)$ chooses $q^b_{2h} = \tilde{q}^b_{2h}$ if $(\tilde{m}_2 + g_2) \geq p_2 \tilde{q}_{2h}$ and $q^b_{2h} = [(\tilde{m}_2 + g_2)/p_2]$ otherwise. For $\varepsilon_i = \varepsilon_l$, a buyer with $a = (\tilde{m}_2, g_2)$ chooses $q^b_{2l} = \tilde{q}^b_{2l} \equiv \tilde{m}_l/p_2$ if $(\tilde{m}_2 + g_2) \geq \tilde{m}_l$ and $q^b_{2l} = [(\tilde{m}_2 + g_2)/p_2]$ otherwise.

Now, in the equilibrium with $\delta_h < \tilde{\delta}_h$, all trades in market 2 are again made using money carried into the market if $\varepsilon_i = \varepsilon_l$ is realized. However, if $\varepsilon_i = \varepsilon_h$ is realized, some fraction of trades in market 2 are carried out by liquidating bonds.

Proposition 2 Suppose $\delta_h < \tilde{\delta}_h$. There exists an equilibrium in which (i) a seller in market 1 holds both money and bonds in market 2, whereas a buyer in market 1 holds money only in market 2; and (ii) a seller in market 1 liquidates bonds for consumption purchase in market 2 if she becomes a buyer and $\varepsilon_h$ is realized.

More specifically, in the equilibrium with $m_1 - p_1 q_1 \equiv m^b_2 < p_2 \tilde{q}_{2l}$ and $m_1 + p_1 q_1 \equiv m^b_2 \geq p_2 \tilde{q}_{2h}$, a seller in market 1 (who carries over relatively large amount of money $(m^b_2)$ to market 2) holds both money and bonds, whereas a buyer in market 1 (who carries over relatively
small amount of money \((m_2^b)\) to market 2) holds money only. This seems to be consistent
with the actual pattern of portfolio holdings depending on the wealth level.

In short, ex-ante return on the interest-bearing bond net of intermediary cost at the
margin varies with the likelihood that bond is liquidated for consumption purchase. If the
trade opportunity as a buyer is less likely to arise, ex-ante net return on bond at the margin
is positive and bond dominates money in all respects. On the other hand, if the trade
opportunity as a buyer is more likely to arise, rate-of-return dominance by bond does not
necessarily hold from the viewpoint of its ex-ante return. That is, noting that ex-ante return
on bond net of intermediary cost at the margin is negative up to a certain amount, agents are
willing to hold money for consumption purchase even though bonds can be freely liquidated.

This explanation seems also applicable to the coexistence of money and demand deposit
in modern economies.\(^6\) If one is to spend money sooner or later, she would not bother to
deposit it into bank despite the fact that interest-bearing demand deposit is immediately
available as a means of payment. This is because its expected return would not be large
enough to compensate the intermediary cost.

5. Appendix

Proof of Lemma 1: Notice that \((\partial J/\partial \tilde{m}_2)|_{\tilde{m}_2=0} = \gamma - \rho_s \theta\) where \(\rho_b\) does not appear because
the first term of (6) is not affected at all by this change in portfolio unless \(\tilde{m}_2 \geq p_2 \tilde{q}_b^2\). Hence,
\(a = (0, m_2)\) is indeed optimal if \(\gamma - \rho_s \theta \leq 0\) that can be rewritten as \(\rho_b \leq [(\theta - \gamma)/\theta]\).

Proof of Proposition 1: Consider an agent with \(m_2 \geq p_2 \tilde{q}_b^2\) at the beginning of market
2. For \(\tilde{m}_2 < p_2 \tilde{q}_b^2\), \(I_{\{p_2 \tilde{q}_b^2 \leq \tilde{m}_2 \leq m_2\}} = I_{\{p_2 \tilde{q}_b^2 \leq \tilde{m}_2 \leq m_2\}} = 0\) and hence, from (9), the agent
is willing to hold money at least \(p_2 \tilde{q}_b^2\). Now, let \(\bar{m}_l\) be the solution to \([((\gamma - \rho_s \theta)/\rho_b) = \delta_l\{(\phi + \theta) - \varepsilon_l u' \tilde{q}_b^2(\bar{m}_l)\phi\}\}. With \(X \equiv (\phi + \theta) - \varepsilon_l u' \tilde{q}_b^2(\bar{m}_l)\phi, \bar{m}_l < p_2 \tilde{q}_b^2\) if \([(\gamma - \rho_s \theta)/\rho_b) <

\(^6\)Andolfatto (2006) suggests that this would constitute a present-day version of the coexistence puzzle in
the sense that demand deposits are mostly insured by the government.
\(X \delta_i\), which can be rearranged as \(\delta_h < \{1 - [(\gamma - \rho_s \theta)/X \rho_b]\} = \bar{\delta}_h\). Further, \(\bar{m}_l > p^b_2\bar{q}_2h\) because \(\delta_l[(\phi + \theta) - \epsilon_lu'(\bar{q}_2h)] = 0 < [(\gamma - \rho_s \theta)/\rho_b] = \delta_l\{(\phi + \theta) - \epsilon_lu'[q^b_2(\bar{m}_l)]\}. Then for \(\bar{m}_2 \in [p^b_2\bar{q}_2h, p^b_2\bar{q}_2h]\), \(\mathbb{I}_{\{p^b_2\bar{q}_2h \leq \bar{m}_2 \leq m_2\}} = 0\) and \(\mathbb{I}_{\{p^b_2\bar{q}_2h \leq \bar{m}_2 \leq m_2\}} = 1\) imply the result (i) from (9). For \(\delta_h \geq \bar{\delta}_h\), let \(\bar{m}_h\) be the solution to \([(\gamma - \rho_s \theta)/\rho_b] = (\phi + \theta) - \sum_i \delta_i \epsilon_i u'[q^b_2(\bar{m}_h)]\). Notice that \(\bar{m}_h \geq p^b_2\bar{q}_2h\) because \((\phi + \theta) - \sum_i \delta_i \epsilon_i u'[q^b_2(\bar{m}_h)] = \delta_lX \leq [(\gamma - \rho_s \theta)/\rho_b] = (\phi + \theta) - \sum_i \delta_i \epsilon_i u'[q^b_2(\bar{m}_h)]\). Then for \(\bar{m}_2 \geq p^b_2\bar{q}_2h\), \(\mathbb{I}_{\{p^b_2\bar{q}_2h \leq \bar{m}_2 \leq m_2\}} = \mathbb{I}_{\{p^b_2\bar{q}_2h \leq \bar{m}_2 \leq m_2\}} = 1\) implies the result (ii) from (9).

**Proof of Lemma 2:** Consider a buyer with \(a = (\bar{m}_2, g_2)\) and \(\epsilon_i = \epsilon_h\). Notice that \(\epsilon_h u'(q) < [1 + (\theta/\phi)]\) for \(q \in (\bar{q}_2h, \infty)\) and \(\epsilon_h u'(q) > [1 + (\theta/\phi)]\) for \(q \in [0, \bar{q}_2h]\) by the definition of \(\bar{q}_2h\). Since the marginal cost of increasing consumption beyond \(\bar{q}_2h\) is \(p_2(\phi + \theta) = 1 + (\theta/\phi)\) by Proposition 1-(i), she chooses \(q^b_2 > \bar{q}_2h\) if \((\bar{m}_2 + g_2) \geq p^b_2\bar{q}_2h\) and \(q^b_2 = [(\bar{m}_2 + g_2)/p_2]\) otherwise. For \(\epsilon_i = \epsilon_i\), again by Proposition 1-(i), the marginal cost of increasing consumption beyond \(\bar{q}_2h\) is \(p_2(\phi + \theta) = 1 + (\theta/\phi)\). Then the definition of \(\bar{q}_2h\) and \(\bar{q}_2h < \bar{q}_2h\) give the result.

**Proof of Proposition 2:** It suffices to show that there exists an equilibrium in which \(M - p_1q_1 < p^b_2\bar{q}_2h\) and \(M + p_1q_1 \geq p^b_2\bar{q}_2h\), where we have applied the equilibrium condition \(m_{1,+1} = M\). Since \(\delta_h < \bar{\delta}_h\), by Proposition 1-(i), the seller in market 1 chooses a portfolio \((\bar{m}_l, M + p_1q_1 - \bar{m}_l)\) and the buyer in market 1 chooses a portfolio \((M - p_1q_1, 0)\). Further, in this equilibrium, \(q^b_2\) for the sellers in market 1 equals to \(\bar{q}_2h\) or \(\bar{q}_2h\) contingent on the realization of \(\epsilon_i\) and that for the buyers in market 1 is less than \(\bar{q}_2h\). Then, \(V'_2(M + p_1q_1) = \phi + \theta - \gamma \equiv \phi\) and \(V'_1 = (\phi/2)[u'(q_1) + 1]\), and hence \((1/p_1) = \phi\) from (11) and \(u'(q_1) = [2(\kappa - \beta)/\beta] + 1\). Since \(\phi = \beta V'_1(m_{1,+1})\), where \(\kappa \equiv \phi/\bar{\phi}\). Since \(V'_2(M - p_1q_1) = [\rho_b \delta_h \epsilon_h u'(\bar{q}_2h) + \rho_b \delta_i \epsilon_i u'(\bar{q}_2h) + \rho_s]\phi\), we further have \(u'(q_1) = \kappa[\rho_b \delta_h \epsilon_h u'(\Phi - \phi p_1q_1) + \rho_b \delta_i \epsilon_i u'(\Phi - \phi p_1q_1) + \rho_s]\) from (12), which determines \(\Phi = \phi M\) for a given \(q_1\). Finally, in order for this equilibrium to exist, the real balances for the rich and for the poor in market 2 should satisfy \(\Phi + \phi p_1q_1 \geq \bar{q}_2h\) and \(\Phi - \phi p_1q_1 < \bar{q}_2h\), respectively. Therefore, this equilibrium exists if \(\bar{q}_2h + \phi p_1q_1 > \bar{q}_2h - \phi p_1q_1\) and for the case, the sufficient condition is \(\bar{q}_2h - \phi p_1q_1 \leq \Phi < \bar{q}_2h + \phi p_1q_1\). Notice that if
\[ \varepsilon_h = \varepsilon_l, \quad \bar{q}_{2h}^b = \bar{q}_{2l}^b = q \] and hence \( q - \phi p_1 q_1 \leq \Phi < q + \phi p_1 q_1 \) from \( (q_2^b)_{\text{rich}} = q \leq \Phi + \phi p_1 q_1 \) and \( (q_2^b)_{\text{poor}} = \Phi - \phi p_1 q_1 < q \).

As \( \varepsilon_h \) increases above \( \varepsilon_l \), \( \Phi \) falls faster than \( \bar{q}_{2l}^b + \phi p_1 q_1 \) because \( \partial q_1 / \partial \varepsilon_h < 0 \) and \( (\partial \Phi / \partial q_1) = \phi p_1 + \{ u''(q_1) / [\rho_b \delta_h \varepsilon_h u''(q_2^b) \kappa + \rho_b \delta_i \varepsilon_l u''(q_2^b) \kappa] \} > \phi p_1 \).

This implies that the right-hand inequality of the sufficient condition is preserved. However, the left-hand inequality binds at some \( \varepsilon_h \), say \( \bar{\varepsilon}_h \), and hence this equilibrium exists for \( \varepsilon_h \in (\varepsilon_l, \bar{\varepsilon}_h] \).

**References**


Andolfatto, D., 2006. Revisiting the Legal Restrictions Hypothesis. manuscript, Simon Fraser University.


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