Discussion Papers
in Economics

Intermediary Cost and Coexistence Puzzle

by

Young Sik Kim and Manjong Lee

Discussion Paper No. 84
May 2012

Institute of Economic Research
Seoul National University

Intermediary Cost and Coexistence Puzzle*

Young Sik Kim† and Manjong Lee‡

Abstract

The coexistence puzzle is explained via an interaction between intermediary cost and uncertainty with regards to consumption trade. If a trade opportunity as a buyer is more likely to arise, ex-ante net return on bond at the margin would be negative up to a certain amount of transaction and, therefore, agents are willing to hold money in the presence of an interest-bearing bond.

Keywords: intermediary cost, interest-bearing asset, coexistence puzzle
JEL classification: E40, E42

1. Introduction

The coexistence puzzle has been one of the long-standing issues for monetary economists at least since Hicks (1935). Gherity (1993) and Burdekin and Weidenmier (2008) report that several types of bonds issued during the U.S. Civil War were circulated at par (face value without interest) as media of exchange until shortly before maturity, but failed to drive money out of circulation.

This paper provides an explanation for the coexistence puzzle based on the interaction between intermediary cost and uncertainty with regards to consumption trade as the key determinant of portfolio choice between money and interest-bearing assets. Specifically, we embed the "shoeleather" or intermediary cost incurred in the purchase of government bonds

*We are grateful to an anonymous referee for the valuable comments and suggestions. The usual disclaimer applies.

†Department of Economics & SIRFE, Seoul National University, 599 Gwanak-no, Gwanak-gu, Seoul 151-746, Korea, Email: kimy@snu.ac.kr, Tel: +82-2-880-6387, Fax: +82-2-886-4231

‡Corresponding Author, Department of Economics, Korea University, Anam-dong, Seongbuk-gu, Seoul 136-701, Korea, Email: manjong@korea.ac.kr, Tel: +82-2-3290-2223, Fax: +82-2-3290-2200

into a general equilibrium model wherein money is essential. A one-period risk-free coupon bond can be freely liquidated at par for consumption-good purchase and intermediary cost is strictly less than exogenous coupon payment at maturity.[1] Therefore, ex-post return on a coupon bond net of intermediary cost is strictly positive.

However, ex-ante return on the bond net of intermediary cost is not always positive. For example, net return on the bond can be negative at the margin if it is used for consumption purchase and its return from early redemption is sufficiently small, which was the case of Civil-War bonds that were circulated at face value without interest. Then people are willing to hold money for consumption purchase even in the presence of an interest-bearing bond. In general, ex-ante return on the bond net of intermediary cost depends on the likelihood that the bond is liquidated for consumption purchase, which is determined by trade opportunity as a buyer and expected amount of transaction. The novelty here is that the commonly-observed intermediary cost combined with uncertainty with regards to consumption transaction turns out to be a fundamental ingredient of the coexistence result.[2]

2. Model

The environment is that of Lagos and Wright (2005) with competitive markets as in Berentsen et al. (2005). Time is discrete and continues forever. In each period, a unit mass of infinitely-lived agents trade in three Walrasian markets for consumption good, called market 1, 2, and 3, which open sequentially. Trading histories are private and agents cannot commit to their future actions, which make a medium of exchange essential. Each agent maximizes discounted expected utility with the discount factor $\beta \in (0, 1)$ between one period and the next.

[1] Based on the observation of a non-trivial fraction of U.S. households with no interest-bearing assets, Mulligan and Sala-i-Martin (1996) and Lucas (2000) argue that such an intermediary cost would be sizable.

[2] Among recent related works using search-theoretic models of money are Aiyagari et al. (1996), Shi (2005), Boel and Camera (2006), Zhu and Wallace (2007), Berentsen and Waller (2008), Marchesiani and Senesi (2009), and Lagos (2010).

There is one perishable and divisible good which can be potentially produced and consumed by all agents. There are two other objects, called money and bond. Money is durable, divisible, and its stock is exogenously given by $M > 0$. A one-period risk-free bond takes the form of a book-entry coupon bond issued by the government that has a technology for record keeping on intra-day bond transactions, but not on agents' trading histories. The purchase of a bond at a price of one unit of money requires an agent physical effort which incurs disutility $\gamma > 0$ per unit of bond. In the real world, this cost can be interpreted as the time spent in purchasing a bond as well as the transaction fee charged by security dealers. Hence, hereinafter we call it an intermediary cost, although we do not introduce financial intermediary explicitly. Upon request by agents for a book-entry bond liquidation before maturity, the government can cash in a bond at par (i.e., one unit of money) with no coupon payment. This is consistent with the historical facts documented in Gherity (1993) and Burdekin and Weidenmier (2008).[3]

The sequence of events within a period is as follows. When entering market 1 with a given amount of money, each agent receives an individual trading shock such that she will become, with equal probability, either a buyer or a seller of consumption good. Agents get utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of good where $u''(q) < 0 < u'(q)$, $u'(\infty) = 0$, $u(0) = 0$, and $u'(0) = \infty$. Production of $q \in \mathbb{R}_+$ units of good incurs disutility q, which is also the case in market 2 and 3 described below.

With money balance after the trade in market 1, agents move on to market 2 and choose a portfolio of money and bond. Upon demand by agents, the government sells a coupon bond at a price of one unit of money. Then, an agent becomes either a buyer with probability ρ_b or a seller with probability $\rho_s = 1 - \rho_b$. An agent receives utility $\varepsilon_i u(q)$ from consuming q units of good where $\varepsilon_i \in \{\varepsilon_h, \varepsilon_l\}$ with $\varepsilon_h > \varepsilon_l$ represents an aggregate preference shock that is realized together with individual trading shocks (to be a buyer or a seller). In particular,

[3] If a bond takes the form of a discount bond rather than a coupon bond, liquidating a bond can be interpreted as giving up remaining accrued interest.

$\varepsilon_i = \varepsilon_h$ with probability δ_h and $\varepsilon_i = \varepsilon_l$ with probability $\delta_l = 1 - \delta_h$. As a buyer, an agent can freely liquidate a bond at par for consumption purchase, in which case she essentially gives up an interest-bearing coupon. The proceeds of bond sales are used to produce goods by the government which has access to linear technology. Specifically, the government can produce $\theta > \gamma$ units of good per unit of money at no cost.

At the beginning of market 3, the government redeems a bond with a unit of money and θ units of good (exogenous coupon rate) according to its record which is wiped out immediately after redemption. Other than selling and redeeming bonds, there is no activity of the government so that its budget is always in balance. In market 3, all agents can consume, produce, and get utility $U(q)$ from consuming $q \in \mathbb{R}_+$ units of good where $U(\cdot)$ satisfies all the nice properties mentioned above.[4] Agents also choose the balance of money to be carried into the following period.

3. Equilibrium

Let $p_{j,t}$ and $q_{j,t}$ denote price and quantity of good traded in market $j \in \{1, 2, 3\}$ at period t, respectively. We will study equilibria in which real balance of money at the end of period is constant with a fixed stock of money M. For this reason, we will drop the time subscript t and index the next-period variable by $+1$, if there is no risk of confusion.

Let $V_3(m_3, g_3)$ denote the expected value for an agent who enters market 3 with (m_3, g_3) where m_3 and g_3 denote money and government-bond balances brought into market 3, respectively. Let ϕ denote the price of money in terms of good in market 3, $p_3 = 1/\phi$. Then, the problem for an agent entering market 3 with (m_3, g_3) is

$$V_3(m_3, g_3) = \max_{(q_3^b, q_3^s, m_{1,+1})} \left[U(q_3^b) - q_3^s + \beta V_1(m_{1,+1}) \right] \tag{1}$$

[4] As discussed in Berentsen et al. (2005), the different preference in market 3 is simply a technical device to ensure a degenerate distribution at the beginning of each period.

subject to $q_3^b + \phi m_{1,+1} = q_3^s + \phi(m_3 + g_3) + \theta g_3$. Here q_3^b and q_3^s denote consumption and production in market 3, respectively, and $V_1(m_{1,+1})$ is the expected value of entering market 1 in the next period with $m_{1,+1}$. Substituting q_3^s from the constraint, we have

$$V_3(m_3, g_3) = \phi(m_3 + g_3) + \theta g_3 + \max_{(q_3^b, m_{1,+1})} \left[U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_1(m_{1,+1})\right].$$

The first-order conditions and envelope conditions are respectively as follows:

$$U'(q_3^b) = 1 \tag{2}$$

$$\phi \geq \beta V_1'(m_{1,+1}) \text{ “=” if } m_{1,+1} > 0 \tag{3}$$

$$V_{3,1}'(m_3, g_3) = \phi; \quad V_{3,2}'(m_3, g_3) = \phi + \theta. \tag{4}$$

As in Lagos and Wright (2005), all agents consume $q_3^b = q_3^* = \arg\max[U(q_3^b) - q_3^b]$ regardless of (m_3, g_3) and exit market 3 with an identical balance of money.

We next turn to market 2. Let $V_2(m_2)$ be the expected value for an agent entering market 2 with m_2. We let $\Gamma(m_2)$ be a set of feasible portfolios for an agent with m_2, defined by $\Gamma(m_2) = \{\mathbf{a} = (\tilde{m}_2, g_2) \in \mathbb{R}_+^2 : \tilde{m}_2 + g_2 \leq m_2\}$. Then the portfolio-choice problem is

$$V_2(m_2) = \max_{\mathbf{a} \in \Gamma(m_2)} J(\mathbf{a}) \tag{5}$$

where $J(\mathbf{a})$, the expected payoff from choosing $\mathbf{a} = (\tilde{m}_2, g_2)$ prior to the realization of individual trading shocks and aggregate preference shocks in market 2, satisfies

$$\begin{aligned} J(\mathbf{a}) = \rho_b & \left\{ \sum_{i \in \{h,l\}} \delta_i \max_{q_{2i}^b} \left[\varepsilon_i u(q_{2i}^b) + V_3\left((\tilde{m}_2 - p_2 q_{2i}^b)\mathbb{I}^{\mathbf{c}}, g_2 - \left(p_2 q_{2i}^b - \tilde{m}_2\right) \mathbb{I}_{\{\tilde{m}_2 < p_2 q_{2i}^b \leq m_2\}}\right)\right] \right\} \\ & + \rho_s \left\{ \max_{q_2^s} \left[-q_2^s + V_3\left((\tilde{m}_2 + p_2 q_2^s), g_2\right)\right] \right\} - \gamma g_2. \end{aligned} \tag{6}$$

Here $\mathbb{I}_{\{\boldsymbol{\chi}\}} = 1$ if and only if $\boldsymbol{\chi}$ is true and $\mathbb{I}^{\mathbf{c}} = 1 - \mathbb{I}$. Noting that, upon request by agents, the government can redeem a book-entry coupon bond at the face value before maturity, $p_2 q^b_{2i} \in (\tilde{m}_2, m_2]$ implies that $(p_2 q^b_{2i} - \tilde{m}_2)$ amount of bonds is cashed in at par without coupon payment.

Conditional on the realization of ε_i and individual trading shocks, an agent chooses q^b_{2i} and q^s_2, taking p_2 as given. More specifically, as a seller, an agent solves the second term of (6), which yields the first-order condition

$$p_2 = (1/\phi) = p_3. \tag{7}$$

That is, whatever demand arises, sellers are willing to supply goods at a price $p_2 = p_3$. Similarly, as a buyer, an agent solves the first term of (6) and chooses q^b_{2i} which satisfies

$$\frac{\varepsilon_i u'(q^b_{2i})}{p_2} \geq \phi \mathbb{I}^{\mathbf{c}} + (\phi + \theta)\mathbb{I}_{\{\tilde{m}_2 < p_2 q^b_{2i} \leq m_2\}} \tag{8}$$

where the equality holds when the constraint ($p_2 q^b_{2i} \leq \tilde{m}_2$ for $\mathbb{I} = 0$ and $p_2 q^b_{2i} \leq m_2$ for $\mathbb{I} = 1$) is inactive. Let $\bar{q}^b_{2i}$ be the solution to (8) with equality and $\mathbb{I}_{\{\tilde{m}_2 < p_2 q^b_{2i} \leq m_2\}} = 1$. Then, an agent with m_2 chooses $\tilde{m}_2$ which satisfies

$$(\gamma - \rho_s \theta) \geq \rho_b \sum_{i \in \{h,l\}} \delta_i \left\{ (\phi + \theta) - \varepsilon_i u'[q^b_2(\tilde{m}_2)]\phi \right\} \mathbb{I}_{\{p_2 \bar{q}^b_{2i} \leq \tilde{m}_2 \leq m_2\}} \tag{9}$$

where the equality holds when the constraint ($\tilde{m}_2 \leq m_2$) is inactive. This implies that a higher intermediary cost γ makes the bond less attractive and hence $\tilde{m}_2$ will increase: i.e., $g_2 = m_2 - \tilde{m}_2$ will decrease.[5]

[5]The left-hand side of (9) represents the expected net marginal return from holding money in terms of the intermediary cost saving, net of the forgone coupon payment in case she becomes a seller. The right-hand side of it represents the expected net marginal (opportunity) cost of holding money in terms of the forgone expected marginal return from holding bond, net of the marginal liquidity return from holding money. This is in line with the asset-pricing equations in Lagos (2011) in which money and equity shares can be used as means of payment. In our model, equity share is replaced by a one-period book-entry coupon bond that the government issues upon demand by agents at its face value with an exogenous coupon rate.

Finally, $V_1(m_1)$, the expected value for an agent entering market 1 with m_1, satisfies the following Bellman equation:

$$V_1(m_1) = \frac{1}{2}\left\{\max_{q_1^b}\left[u(q_1^b) + V_2(m_1 - p_1 q_1^b)\right]\right\} + \frac{1}{2}\left\{\max_{q_1^s}\left[V_2(m_1 + p_1 q_1^s) - q_1^s\right]\right\}. \tag{10}$$

As in market 2, a seller chooses q_1^s which solves the second term on the right-hand side of (10), taking p_1 as given. This yields the first-order condition

$$\frac{1}{p_1} = V_2'(m_1 + p_1 q_1^s). \tag{11}$$

Similarly, a buyer chooses q_1^b which solves the first term on the right-hand side of (10) subject to $p_1 q_1^b \leq m_1$, taking p_1 as given. Notice that we can rule out the case of $p_1 q_1^b = m_1$ because $\varepsilon_i u'(0) = \infty$. The optimal condition is

$$\frac{u'(q_1^b)}{p_1} = V_2'(m_1 - p_1 q_1^b). \tag{12}$$

Definition 1 *A stationary monetary equilibrium is* $\left\{(p_j, q_j, V_j)_{j=1}^3, \mathbf{a}, m_{1,+1}\right\}$ *that satisfies (1)–(12) and that clears each market* $j \in \{1, 2, 3\}$.

4. Coexistence Result

The following results show that ex-ante return on the bond net of intermediary cost can be negative at the margin depending on (i) the trade opportunity as a buyer (ρ_b) and (ii) the likelihood δ_h of aggregate preference shock that determines amount of transaction.

Lemma 1 *If* $\rho_b \leq [(\theta - \gamma)/\theta]$, $\mathbf{a} = (0, m_2)$ *is optimal for an agent with* m_2 *after the trade in market 1.*

This Lemma suggests that money would not be held at all if the trade opportunity as a buyer is too scarce. Therefore, hereinafter we will focus on the case of $\rho_b > [(\theta - \gamma)/\theta]$.

Proposition 1 *Suppose* $\rho_b > [(\theta - \gamma)/\theta]$. *(i) If* $\delta_h < \bar{\delta}_h$*, an agent with* $m_2 \geq \bar{m}_l > p_2\bar{q}^b_{2l}$ *chooses* $\mathbf{a} = (\bar{m}_l, m_2 - \bar{m}_l)$*, whereas an agent with* $m_2 < \bar{m}_l < p_2\bar{q}^b_{2h}$ *chooses* $\mathbf{a} = (m_2, 0)$. *(ii) If* $\delta_h \geq \bar{\delta}_h$*, an agent with* $m_2 \geq \bar{m}_h \geq p_2\bar{q}^b_{2h}$ *chooses* $\mathbf{a} = (\bar{m}_h, m_2 - \bar{m}_h)$*, whereas an agent with* $m_2 < \bar{m}_h$ *chooses* $\mathbf{a} = (m_2, 0)$*, where* $(\bar{\delta}_h, \bar{m}_l, \bar{m}_h)$ *is as defined in Appendix.*

Proposition 1 implies that ex-ante net return on bond is negative up to $\bar{m}_h$ if ε_h is most likely to occur in market 2. Hence, agents are willing to hold sufficient amount of money for the largest possible quantity of trade in the upcoming market 2. On the other hand, if ε_l is most likely to occur, ex-ante net return on bond exceeding $\bar{m}_l$ dominates that of money. Hence, no one is willing to hold money more than $\bar{m}_l$. Proposition 1 also implies that if $\delta_h \geq \bar{\delta}_h$, all trades in market 2 are made using money carried into the market regardless of the realized aggregate preference shock ε_i. More interesting equilibrium arises with $\delta_h < \bar{\delta}_h$.

Lemma 2 *Suppose* $\delta_h < \bar{\delta}_h$. *For* $\varepsilon_i = \varepsilon_h$*, a buyer with* $\mathbf{a} = (\tilde{m}_2, g_2)$ *chooses* $q^b_{2h} = \bar{q}^b_{2h}$ *if* $(\tilde{m}_2 + g_2) \geq p_2\bar{q}^b_{2h}$ *and* $q^b_{2h} = [(\tilde{m}_2 + g_2)/p_2]$ *otherwise. For* $\varepsilon_i = \varepsilon_l$*, a buyer with* $\mathbf{a} = (\tilde{m}_2, g_2)$ *chooses* $q^b_{2l} = \hat{q}^b_{2l} \equiv \bar{m}_l/p_2$ *if* $(\tilde{m}_2 + g_2) \geq \bar{m}_l$ *and* $q^b_{2l} = [(\tilde{m}_2 + g_2)/p_2]$ *otherwise.*

Now, in the equilibrium with $\delta_h < \bar{\delta}_h$, all trades in market 2 are again made using money carried into the market if $\varepsilon_i = \varepsilon_l$ is realized. However, if $\varepsilon_i = \varepsilon_h$ is realized, some fraction of trades in market 2 are carried out by liquidating bonds.

Proposition 2 *Suppose* $\delta_h < \bar{\delta}_h$. *There exists an equilibrium in which (i) a seller in market 1 holds both money and bonds in market 2, whereas a buyer in market 1 holds money only in market 2; and (ii) a seller in market 1 liquidates bonds for consumption purchase in market 2 if she becomes a buyer and* ε_h *is realized.*

More specifically, in the equilibrium with $m_1 - p_1q_1 \equiv m^b_2 < p_2\bar{q}^b_{2l}$ and $m_1 + p_1q_1 \equiv m^s_2 \geq p_2\bar{q}^b_{2h}$, a seller in market 1 (who carries over relatively large amount of money (m^s_2) to market 2) holds both money and bonds, whereas a buyer in market 1 (who carries over relatively small amount of money (m_2^b) to market 2) holds money only. This seems to be consistent with the actual pattern of portfolio holdings depending on the wealth level.

In short, ex-ante return on the interest-bearing bond net of intermediary cost at the margin varies with the likelihood that bond is liquidated for consumption purchase. If the trade opportunity as a buyer is less likely to arise, ex-ante net return on bond at the margin is positive and bond dominates money in all respects. On the other hand, if the trade opportunity as a buyer is more likely to arise, rate-of-return dominance by bond does not necessarily hold from the viewpoint of its ex-ante return. That is, noting that ex-ante return on bond net of intermediary cost at the margin is negative up to a certain amount, agents are willing to hold money for consumption purchase even though bonds can be freely liquidated.

This explanation seems also applicable to the coexistence of money and demand deposit in modern economies.[6] If one is to spend money sooner or later, she would not bother to deposit it into bank despite the fact that interest-bearing demand deposit is immediately available as a means of payment. This is because its expected return would not be large enough to compensate the intermediary cost.

5. Appendix

Proof of Lemma 1: Notice that $(\partial J/\partial \tilde{m}_2)|_{\tilde{m}_2=0} = \gamma - \rho_s\theta$ where ρ_b does not appear because the first term of (6) is not affected at all by this change in portfolio unless $\tilde{m}_2 \geq p_2\bar{q}_{2l}^b$. Hence, $\mathbf{a} = (0, m_2)$ is indeed optimal if $\gamma - \rho_s\theta \leq 0$ that can be rewritten as $\rho_b \leq [(\theta - \gamma)/\theta]$.

Proof of Proposition 1: Consider an agent with $m_2 \geq p_2\bar{q}_{2l}^b$ at the beginning of market 2. For $\tilde{m}_2 < p_2\bar{q}_{2l}^b$, $\mathbb{I}_{\{p_2\bar{q}_{2h}^b \leq \tilde{m}_2 \leq m_2\}} = \mathbb{I}_{\{p_2\bar{q}_{2l}^b \leq \tilde{m}_2 \leq m_2\}} = 0$ and hence, from (9), the agent is willing to hold money at least $p_2\bar{q}_{2l}^b$. Now, let $\bar{m}_l$ be the solution to $[(\gamma - \rho_s\theta)/\rho_b] = \delta_l\{(\phi + \theta) - \varepsilon_l u'[q_2^b(\bar{m}_l)]\phi\}$. With $\mathbb{X} \equiv (\phi + \theta) - \varepsilon_l u'\left(\bar{q}_{2h}^b\right)\phi$, $\bar{m}_l < p_2\bar{q}_{2h}^b$ if $[(\gamma - \rho_s\theta)/\rho_b] <$

[6] Andolfatto (2006) suggests that this would constitute a present-day version of the coexistence puzzle in the sense that demand deposits are mostly insured by the government.

$\mathbb{X}\delta_l$, which can be rearranged as $\delta_h < \{1 - [(\gamma - \rho_s\theta)/\mathbb{X}\rho_b]\} \equiv \bar{\delta}_h$. Further, $\bar{m}_l > p_2\bar{q}^b_{2l}$ because $\delta_l[(\phi + \theta) - \varepsilon_l u'(\bar{q}^b_{2l})\phi] = 0 < [(\gamma - \rho_s\theta)/\rho_b] = \delta_l\{(\phi + \theta) - \varepsilon_l u'[q^b_2(\bar{m}_l)]\phi\}$. Then for $\tilde{m}_2 \in [p_2\bar{q}^b_{2l}, p_2\bar{q}^b_{2h})$, $\mathbb{I}_{\{p_2\bar{q}^b_{2h} \leq \tilde{m}_2 \leq m_2\}} = 0$ and $\mathbb{I}_{\{p_2\bar{q}^b_{2l} \leq \tilde{m}_2 \leq m_2\}} = 1$ imply the result *(i)* from (9). For $\delta_h \geq \bar{\delta}_h$, let $\bar{m}_h$ be the solution to $[(\gamma - \rho_s\theta)/\rho_b] = (\phi + \theta) - \sum_i \delta_i\varepsilon_i u'[q^b_2(\bar{m}_h)]\phi$. Notice that $\bar{m}_h \geq p_2\bar{q}^b_{2h}$ because $(\phi + \theta) - \sum_i \delta_i\varepsilon_i u'(\bar{q}^b_{2h})\phi = \delta_l[(\phi + \theta) - \varepsilon_l u'(\bar{q}^b_{2h})\phi] = \delta_l\mathbb{X} \leq [(\gamma - \rho_s\theta)/\rho_b] = (\phi + \theta) - \sum_i \delta_i\varepsilon_i u'[q^b_2(\bar{m}_h)]\phi$. Then for $\tilde{m}_2 \geq p_2\bar{q}^b_{2h}$, $\mathbb{I}_{\{p_2\bar{q}^b_{2h} \leq \tilde{m}_2 \leq m_2\}} = \mathbb{I}_{\{p_2\bar{q}^b_{2l} \leq \tilde{m}_2 \leq m_2\}} = 1$ implies the result *(ii)* from (9).

Proof of Lemma 2: Consider a buyer with $\mathbf{a} = (\tilde{m}_2, g_2)$ and $\varepsilon_i = \varepsilon_h$. Notice that $\varepsilon_h u'(q) < [1 + (\theta/\phi)]$ for $q \in (\bar{q}^b_{2h}, \infty)$ and $\varepsilon_h u'(q) > [1 + (\theta/\phi)]$ for $q \in [0, \bar{q}^b_{2h})$ by the definition of $\bar{q}^b_{2h}$. Since the marginal cost of increasing consumption beyond $\bar{q}^b_{2h}$ is $p_2(\phi + \theta) = 1 + (\theta/\phi)$ by Proposition 1-(i), she chooses $q^b_{2h} = \bar{q}^b_{2h}$ if $(\tilde{m}_2 + g_2) \geq p_2\bar{q}^b_{2h}$ and $q^b_{2h} = [(\tilde{m}_2 + g_2)/p_2]$ otherwise. For $\varepsilon_i = \varepsilon_l$, again by Proposition 1-(i), the marginal cost of increasing consumption beyond $\hat{q}^b_{2l}$ is $p_2(\phi + \theta) = 1 + (\theta/\phi)$. Then the definition of $\bar{q}^b_{2l}$ and $\bar{q}^b_{2l} < \hat{q}^b_{2l}$ give the result.

Proof of Proposition 2: It suffices to show that there exists an equilibrium in which $M - p_1q_1 < p_2\bar{q}^b_{2l}$ and $M + p_1q_1 \geq p_2\bar{q}^b_{2h}$ where we have applied the equilibrium condition $m_{1,+1} = M$. Since $\delta_h < \bar{\delta}_h$, by Proposition 1-($i$), the seller in market 1 chooses a portfolio $(\bar{m}_l, M + p_1q_1 - \bar{m}_l)$ and the buyer in market 1 chooses a portfolio $(M - p_1q_1, 0)$. Further, in this equilibrium, q^b_2 for the sellers in market 1 equals to $\hat{q}^b_{2l}$ or $\bar{q}^b_{2h}$ contingent on the realization of ε_i and that for the buyers in market 1 is less than $\bar{q}^b_{2l}$. Then, $V'_2(M + p_1q_1) = \phi + \theta - \gamma \equiv \tilde{\phi}$ and $V'_1 = (\tilde{\phi}/2)[u'(q_1) + 1]$, and hence $(1/p_1) = \tilde{\phi}$ from (11) and $u'(q_1) = [2(\kappa - \beta)/\beta] + 1$ from $\phi = \beta V'_1(m_{1,+1})$, where $\kappa \equiv \phi/\tilde{\phi}$. Since $V'_2(M - p_1q_1) = [\rho_b\delta_h\varepsilon_h u'(q^b_2) + \rho_b\delta_l\varepsilon_l u'(q^b_2) + \rho_s]\phi$, we further have $u'(q_1) = \kappa[\rho_b\delta_h\varepsilon_h u'(\Phi - \phi p_1q_1) + \rho_b\delta_l\varepsilon_l u'(\Phi - \phi p_1q_1) + \rho_s]$ from (12), which determines $\Phi = \phi M$ for a given q_1. Finally, in order for this equilibrium to exist, the real balances for the rich and for the poor in market 2 should satisfy $\Phi + \phi p_1q_1 \geq \bar{q}^b_{2h}$ and $\Phi - \phi p_1q_1 < \bar{q}^b_{2l}$, respectively. Therefore, this equilibrium exists if $\bar{q}^b_{2l} + \phi p_1q_1 > \bar{q}^b_{2h} - \phi p_1q_1$ and for the case, the sufficient condition is $\bar{q}^b_{2h} - \phi p_1q_1 \leq \Phi < \bar{q}^b_{2l} + \phi p_1q_1$. Notice that if

$\varepsilon_h = \varepsilon_l$, $\bar{q}^b_{2h} = \bar{q}^b_{2l} = \bar{q}$ and hence $\bar{q} - \phi p_1 q_1 \leq \Phi < \bar{q} + \phi p_1 q_1$ from $(q^b_2)_{\text{rich}} = \bar{q} \leq \Phi + \phi p_1 q_1$ and $(q^b_2)_{\text{poor}} = \Phi - \phi p_1 q_1 < \bar{q}$. As ε_h increases above ε_l, Φ falls faster than $\bar{q}^b_{2l} + \phi p_1 q_1$ because $\partial q_1 / \partial \varepsilon_h < 0$ and $(\partial \Phi / \partial q_1) = \phi p_1 + \{u''(q_1)/[\rho_b \delta_h \varepsilon_h u''(q^b_2)\kappa + \rho_b \delta_l \varepsilon_l u''(q^b_2)\kappa]\} > \phi p_1$. This implies that the right-hand inequality of the sufficient condition is preserved. However, the left-hand inequality binds at some ε_h, say $\bar{\varepsilon}_h$, and hence this equilibrium exists for $\varepsilon_h \in (\varepsilon_l, \bar{\varepsilon}_h]$.

References

Aiyagari, R., Wallace N., Wright, R., 1996. Coexistence of Money and Interest-Bearing Securities. Journal of Monetary Economics 37, 397-419.

Andolfatto, D., 2006. Revisiting the Legal Restrictions Hypothesis. manuscript, Simon Fraser University.

Berentsen, A., Camera, G., Waller, C., 2005. The Distribution of Money Balances and the Nonneutrality of Money. International Economic Review 46, 465-487.

Berentsen, A., Waller, C., 2008. Outside Bonds Versus Inside Bonds. University of Zurich working paper 372.

Boel, P., Camera, G., 2006. Efficient Monetary Allocations and the Illiquidity of Bonds. Journal of Monetary Economics 53, 1693-1715.

Burdekin, R., Weidenmier, M., 2008. Can Interest-Bearing Money Circulate? A Small-Denomination Arkansan Experiment, 1861-63. Journal of Money, Credit and Banking 40, 233-241.

Gherity, J., 1993. Interest-bearing Currency: Evidence from the Civil War Experience. Journal of Money, Credit and Banking 25, 125-131.

Hicks, J., 1935. A Suggestion for Simplifying the Theory of Money. Economica, N.S.2, 1-19.

Lagos, R., 2010. Moneyspots: Extraneous Attributes and the Coexistence of Money and Interest-Bearing Nominal Bonds. manuscript, New York University.

Lagos, R., 2011. Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy. Journal of Money, Credit and Banking 43, 521-552.

Lagos, R., Wright, R., 2005. A Unified Framework for Monetary Theory and Policy Analysis. Journal of Political Economy 113, 463-484.

Lucas, R., 2000. Inflation and Welfare. Econometrica 68, 247-274.

Marchesiani, A., Senesi, P., 2009. Money and Nominal Bonds. Macroeconomic Dynamics 13, 189-199.

Mulligan, C., Sala-i-Martin, X., 1996. Adoption of Financial Technologies: Technologies and Implications for Money Demand and Monetary Policy. NBER Working Paper 5504.

Shi, S., 2005. Nominal Bonds and Interest Rates. International Economic Review 46, 579-612.

Zhu, T., Wallace, N., 2007. Pairwise Trade and Coexistence of Money and Higher-Return Assets. Journal of Economic Theory 133, 524-535.

Discussion Papers in Economics
Seoul National University

For a listing of papers 1-70 please contact us by e-mail ecores@snu.ac.kr

71. Youngsub Chun and Toru Hokari, "On the Coincidence of the Shapley Value and the Nucleolus in Queueing Problems," October 2006; *Seoul Journal of Economics* 20 (2007), 223-237.

72. Bong Chan Koh and Youngsub Chun, "Population Sustainability of Social and Economic Networks," October 2006.

73. Bong Chan Koh and Youngsub Chun, "A Decentralized Algorithm with Individual Endowments for the Probabilistic Serial Mechanism," October 2006.

74. Sunghoon Hong and Youngsub Chun, "Efficiency and Stability in a Model of Wireless Communication Networks," July 2007.

75. Youngsub Chun and Eun Jeong Heo, "Queueing Problems with Two Parallel Servers," November 2007.

76. Byung-Yeon Kim and Youngho Kang, "The Informal Economy and the Growth of Small Enterprises in Russia," September 2008.

77. Byung-Yeon Kim, "Informal Economy Activities and Entrepreneurship: Evidence from RLMS," September 2008.

78. Youngsub Chun and Boram Park, "Population Solidarity, Population Fair-Ranking, and the Egalitarian Value," April 2010.

79. Youngsub Chun and Boram Park, "Fair-Ranking Properties of a Core Selection and the Shapley Value," August 2010.

80. Donghyu Yang, " Regional Integration, Collective Security, and Trade Networks: West German and Japanese Economies under Allied Occupation," May 2012.

81. Chulhee Lee and Jinkook Lee, " Employment Status, Quality of Matching, and Retirement in Korea: Evidence from Korean Longitudinal Study of Aging," May 2012.

82. Chulhee Lee, "Industrial Characteristics and Employment of Older Manufacturing Workers in the Early-Twentieth-Century United States," May 2012.

83. Chulhee Lee, "Military Service and Economic Mobility: Evidence from the American Civil War," May 2012.