Equilibrium Redistribution under Ex-ante Heterogeneity and Income-Dependent Voting*

Bo Hyun Chang
Ministry of Strategy and Finance of Korea

Yongsung Chang
Seoul National University

Sun-Bin Kim
Yonsei University

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Abstract

From a utilitarian point of view, the optimal income tax rate in the U.S. should be much higher than the current one (e.g., Piketty and Saez (2013)). A majority of the population would be in favor of raising taxes, as the income distribution is highly skewed. We show that the political equilibrium is actually close to the current tax rate, once we take into account (i) the ex-ante heterogeneity of earnings (Guvenen, 2009) and (ii) income-dependent voting behavior (Mahler, 2008) across households.

Keywords: Optimal Tax, Ex-ante Heterogeneity, Voting Turnout Rates
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1. Introduction

The standard models have difficulty in justifying the current labor income tax schedule in the U.S., from a utilitarian point of view—a commonly used welfare criteria among economists. The optimal income tax schedule should be much more progressive than the current one (e.g., Piketty and Saez (2013)): the potential benefit from an increase in income tax well exceeds the cost from distorting an efficient allocation of resources. The majority of the population should be in favor of fiscal reform to raise the income tax rate, as the observed cross-sectional income distribution is highly skewed.

While a larger literature (built on an equal-weight utilitarian social welfare function) arrives at a similar conclusion (e.g., Floden and Linde, 2001; Piketty and Saez, 2013; Corbae et al. 2009; and Heathcote and Tsujiyama, 2016), there are a few exceptions. For example, Conesa and Krueger (2006) argue that lowering the marginal tax rate for the rich along with increased deductions for the poor enhances social welfare. However, the income distribution in their model is much more evenly distributed (e.g., 0.39 for the income Gini) than that in the data (e.g., 0.58 in the SCF (Díaz-Giménez et al. 2011) and 0.5 in the OECD database (2015)). Lockwood and Weinzierl (2016), Chang et al. (2018), and Heathcote and Tsujiyama (2016) search for unequal Pareto weights that would justify the current tax rates. Golosov and Tsyvinski (2007) and Chetty and Saez (2010) demonstrate that the existence of private insurance lowers the optimal level of government intervention. Heathcote et al. (2016) introduce endogenous skill investment and the externality associated with public expenditures and argue that the optimal tax scheme is less progressive than the current one.

In this paper we argue that the current tax rate is close to the (political) equilibrium tax rate, once we take into account the ex-ante heterogeneity in household earnings (e.g., Guvenen (2009)) and income-dependent voting behavior, (e.g., Mahler (2008)). An individual household's earnings consist of ex-ante differences in ability and the ex-post realization of shocks. When households' income differences are largely driven by the permanent difference in earnings ability, the potential insurance benefit from the government's tax-and-transfer policy is small. Moreover, when households' voting turnout rates are positively correlated with income level (as documented in Mahler (2008)), the tax rate chosen by the majority vote will be skewed toward the one preferred by high-income households. Based on the simulated voting in our model, we show that the tax rate chosen by the majority is close to the current income tax rate.

We feature three model economies that differ with respect to the *relative* size of ex-ante heterogeneity in the households' income dispersion: (i) no ex-ante heterogeneity (NH), (ii) small ex-ante heterogeneity (SH), and (iii) large ex-ante heterogeneity (LH). By construction, all three economies match the overall income dispersion (Gini coefficients) in the data. The dispersion of permanent components (ex-ante heterogeneity) and that of stochastic components (ex-post heterogeneity) are estimated from the PSID following Guvenen (2009). In the NH model, the household's income distribution is driven by stochastic shocks only. In the SH model, about one-third (31.3%) of the income dispersion is driven by ex-ante heterogeneity (permanent differences in ability). In the LH model, slightly more than half (56.7%) of the income dispersion is driven by the ex-ante heterogeneity.

Due to the veil of ignorance, the optimal income tax rates based on the (equal-weight) utilitarian social welfare function are similar across the three model economies (which exhibit similar overall income distributions by construction): 37.3% (NH), 37.7% (SH), and 36.9% (LH). However, once we take into account the income-dependent voting turnout rates in the data (e.g., Mahler, 2008), the tax rate chosen by the majority of voters differs across the three economies: 35% (NH), 33% (SH), and 27% (LH). The tax rate chosen in the LH model (which is our preferred model economy) is not far from the current average tax rate (23.8%) in the U.S. Since the permanent differences (ex-ante heterogeneity) make up more than half of the earnings dispersion, the insurance benefit from the government tax-and-transfer policy diminishes for high-income households. Mahler (2008) examines the electoral turnout rates and income redistribution among 13 developed countries and shows that turnout rates increase with income level in many countries. According to a meta analysis by Smets and Van Ham (2013), 21 out of 40 studies find a statistically significant relationship between income and turnout rates. Our paper quantitatively shows that the observed pattern in turnout rates can indeed justify the current tax rate as a voting outcome.

Our results are closely related to the large existing literature on the optimal income tax. Piketty and Saez (2013) summarize recent developments in optimal labor-income taxation. According to their optimal linear tax rate formula, based on the Mirrlees (1971) model, both the utilitarian and the median voter tax rates are much higher than the current one in the U.S. Heathcote et al. (2016) present an equilibrium model that features endogenous skill investment, flexible labor supply, and the externality linked to government purchases. Their model suggests that the optimal tax scheme is less progressive than the current one. Corbae et al. (2009) compute the utilitarian optimal tax rates and political outcome by a median voter: the optimal tax rates are higher than those in the data. Corbae et al. (2009) point to voter turnout rates as a possible cause of the gap between the model and the data. Benabou and Ok (2001) present the prospect of upward mobility (POUM) hypothesis: the poor may not support a strong redistribution policy because they may be rich in the future. Alesina et al. (2011) find empirical support for the POUM hypothesis. Charité et al. (2015) report that according to their experiment, subjects' preferences for redistribution diminish significantly when they know their initial endowments than when they don't. Chang et al. (2018) measure the Pareto weights that justify the current tax progressivity across 32 OECD countries through a unified framework and find that Pareto weights vastly differ across countries. Based on the Mirrlees (1971) framework, Lockwood and Weinzierl (2016) infer social weights from U.S. tax policies between 1979 and 2010. Heathcote and Tsujiyama (2016) compare different tax systems given the Pareto weights for the current tax rates. Other studies try to explain the current tax rate in different ways. Golosov and Tsyvinski (2007) and Chetty and Saez (2010) demonstrate that the existence of private insurance lowers the optimal level of government intervention. Weinzierl (2014) presents a survey report that many people prefer the principle of equal sacrifice over conventional utilitarian objectives. Lockwood and Weinzierl (2015) argue that preference heterogeneity can reduce the optimal redistribution under certain conditions.

The remainder of the paper is organized as follows. In Section 2, we build an incomplete markets model that features both ex-ante and ex-post heterogeneity in earnings. In Section 3, we calibrate our model economy to match the empirical estimates of income profiles in the the U.S. In Section 4, we simulate our model economy to compute the utilitarian optimal tax rates and the tax rate chosen by the majority. Section 5 summarizes the results.

2. Model

The model economy introduces the permanent difference in earnings ability—i.e., *ex-ante* heterogeneity—*a la* Aiyagari (1994) with endogenous labor supply.

Households: There is a continuum (measure one) of households that have identical preferences. An individual household's productivity at time t is z_t. When a household with productivity z_t supplies h_t hours in the market, its labor income is $w_t z_t h_t$, where w_t is the wage rate for the efficiency unit of labor. The household's productivity z_t consists of two components: $z_t = \psi \cdot x_t$, where ψ is a time-invariant deterministic component (e.g., ability) and x_t is a stochastic component (e.g., luck). The stochastic component evolves over time according to a common Markov process with a transition probability distribution function: $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. Households hold assets (claims on production capital) a_t that yield the real rate of return r_t. Both labor and capital incomes are subject to income taxes at the same rate τ. Households receive a lump-sum transfer T_t from the government. A household maximizes its lifetime utility:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{{c_t}^{1-\sigma} - 1}{1-\sigma} - B\frac{{h_t}^{1+1/\gamma}}{1+1/\gamma} \right)$$

subject to

$$c_t + a_{t+1} = (1-\tau)(w_t \psi x_t h_t + r_t a_t) + a_t + T_t,$$

$$a_{t+1} \geq \underline{a}$$

where c_t is consumption. Parameters σ and γ represent relative risk aversion and the Frisch elasticity of labor supply, respectively. Capital markets are incomplete in two senses: (i) physical capital is the only asset available to households to insure against stochastic shocks to their productivity and (ii) households face a borrowing constraint: $a_t \geq \underline{a}$ for all t. Households differ *ex-ante* with respect to their permanent productivity

ψ and differ *ex-post* with respect to their productivity shock x_t and asset holdings a_t. The economy-wide distribution of households is characterized by the probability measure $\mu_t(a_t, x_t, \psi)$.

Firm: A representative firm produces output according to a constant-returns-to-scale Cobb-Douglas production technology using capital, K_t, and effective units of labor, $L_t = \int h_t \psi x_t d\mu$. Capital depreciates at rate δ each period:

$$Y_t = F(L_t, K_t) = {L_t}^{\alpha} {K_t}^{1-\alpha}.$$

Government: The government operates a simple fiscal policy characterized by a flat income tax rate (τ) and a constant lump-sum transfer (T). According to Piketty and Saez (2013), a linear income tax simplifies the analysis but still captures the key equity-efficiency trade-off. For example, progressivity at the top is often counter-balanced by the fact that a substantial fraction of capital income receives preferential tax treatment under most income rules. Overall, all transfers (including various government spending) taken together are fairly close to a demogrant, i.e., they are about constant with income. Hence, the optimal linear tax model with a demogrant transfer is a reasonable first-order approximation of the actual tax system and is useful to understand how the level of taxes and transfers should be set. We also assume that the government spends tax revenues exclusively for lump-sum transfers to households, and balances the budget:

$$T_t = \int \tau \big\{ w_t \psi x_t h_t + r_t a_t \big\} d\mu(a_t, x_t, \psi)$$

Recursive Formulation: It is useful to consider a recursive equilibrium. Let $V(a, x, \psi)$ denote the value function of a household with asset holdings a, productivity shock x, and permanent ability ψ:

$$V(a, x, \psi) = \max_{c,h} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - B \frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta \mathbb{E}\big[V(a', x', \psi) | x, \psi \big] \right\}$$

subject to

$$c + a' = (1-\tau)(w\psi x h + ra) + a + T,$$

$$a' \geq \underline{a}$$

The inter-temporal first-order condition for optimal consumption is:

$$c(a, x, \psi)^{-\sigma} = \beta(1 + (1 - \tau)r)\mathbb{E}\big[c(a', x', \psi)^{-\sigma}\big].$$

The intra-temporal first-order condition for optimal hours worked is:

$$B \cdot h(a, x, \psi)^{1/\gamma} c(a, x, \psi)^{\sigma} = (1 - \tau)w\psi x.$$

Equilibrium: A stationary equilibrium consists of a value function, $V(a, x, \psi)$; a set of decision rules for consumption, asset holdings, and labor supply, $c(a, x, \psi)$, $a'(a, x, \psi)$, $h(a, x, \psi)$; aggregate inputs, K, L; and the invariant distribution of households, $\mu(a, x, \psi)$, such that:

1. Individual households optimize: Given w and r, the individual decision rules $c(a, x, \psi)$, $a'(a, x, \psi)$, $h(a, x, \psi)$ and $V(a, x, \psi)$ solve the Bellman equation.

2. The representative firm maximizes profits:

$$w = \alpha(K/L)^{1-\alpha}$$
$$r + \delta = (1 - \alpha)(K/L)^{-\alpha}$$

3. The goods market clears:

$$\int \big\{a'(a, x, \psi) + c(a, x, \psi)\big\} d\mu = F(L, K) + (1 - \delta)K$$

4. The factor markets clear:

$$L = \int \psi x h(a, x, \psi) d\mu$$
$$K = \int a d\mu$$

5. The government balances the budget:

$$T = \int \tau\{w\psi x h(a, x, \psi) + ra\} d\mu$$

6. Individual and aggregate behaviors are consistent: For all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$

$$\mu(A^0, X^0, \Psi^0) = \int_{A^0, X^0, \Psi^0} \left\{ \int_{\mathcal{A}, \mathcal{X}, \Psi^0} \mathbf{1}_{a'=a'(a,x,\psi)} d\pi_x(x'|x, \psi) d\mu \right\} da' dx' d\psi$$

3. Quantitative Analysis

3.1. Calibration

We construct three model economies that differ with respect to the *relative* size of ex-ante heterogeneity in the income distribution. The first model economy features no ex-ante heterogeneity (denoted by NH). The second and third models feature small (SH) and large ex-ante heterogeneity (LH). While the three model economies differ in the relative size of the ex-ante component of household earnings, we choose the size of stochastic productivity shocks in each model so that all three economies exhibit the same degree of overall dispersion in earnings—i.e., the Gini coefficients of *realized* income distributions in all three models will be identical to that in the data.

Common Parameters The time unit is one year. The labor income share (α) is 0.64, and the annual depreciation rate of capital (δ) is 10%. Both the relative risk aversion (σ) and the labor supply elasticity (γ) are set to 1. Workers are not allowed to borrow in the benchmark: $\underline{a} = 0$. Table 1 summarizes the common parameters of the model economies.

Table 1: Common Parameters

Parameter	Values
Labor income share (α)	0.64
Depreciation rate of capital (δ)	0.10
Relative risk aversion (σ)	1.00
Labor supply elasticity (γ)	1.00
Borrowing constraint ($\underline{a}$)	0.00

Economy-specific Parameters We assume that the time-invariant earnings ability (ex-ante differences in individual productivity) ψ is drawn from a log normal distribution: $\ln \psi \sim N(0, \sigma_\psi^2)$. In the no ex-ante heterogeneity (NH) model, $\sigma_\psi = 0$. The time-varying individual productivity shock (ex-post differences in productivity) x is assumed to follow an AR(1) process in logs: $\ln x' = \rho_x \ln x_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon)$.

For the relative size between ex-ante and ex-post productivity, we follow Guvenen (2009), who decomposes individual earnings into ability and shock. He proposes two specifications: a restricted income profile (RIP) and a heterogeneous income profile (HIP). The SH and LH specifications in our model correspond to RIP and HIP in Guvenen (2009), respectively. The details of the estimation are provided below.[1]

According to these estimates, $\sigma_\psi = 0.301$ under RIP. Given the estimated persistence of the stochastic component in income ($\rho_x = 0.946$), we choose the standard deviation of innovation to productivity shocks ($\sigma_\epsilon = 0.213$) to generate the same before-tax income Gini. The resulting ratio of ex-ante heterogeneity over the dispersion of labor income is $\sigma_\psi/\sigma_z = 31.3\%$: about one-third of the income differences across households are due to permanent differences in ability.[2]

Under HIP, the stochastic process of the productivity shock is now much less persistent ($\rho_x = 0.842$) because a large portion of earnings is captured by the permanent differences in individual ability. The required stochastic shocks to match the overall income Gini is $\sigma_\epsilon = 0.251$, and the value of ex-ante heterogeneity is $\sigma_\psi = 0.619$. As a result, the ratio of ex-ante heterogeneity in the overall income dispersion is $\sigma_\psi/\sigma_z = 56.7\%$.

Finally, for NH, the persistence of the stochastic productivity shock is assumed to be the same as that in SH $\rho_x = 0.946$. We choose $\sigma_\epsilon = 0.23$ to match the before-tax/transfer income Gini coefficient of 0.5 in the data (the value for the U.S. in 2010, according to the 2015 OECD database). Thus, in NH the entire income dispersion is generated by the stochastic shock to productivity, as in Aiyagari (1994).

For each model, the time discount factor, β, is chosen so that the steady-state real interest rate is 4%. Those factors are 0.953 (NH), 0.955 (SH), and 0.96 (LH). Since the uninsurable income risk (ex-post heterogeneity) is the largest in NH, households in NH have the strongest precautionary savings motive. Thus, a small discount factor is required to achieve the same real interest rate. The disutility from working, B, is chosen so that

[1]We use his estimates based on a larger sample instead of his baseline estimates. Since he reports only HIP estimates for a larger sample, we estimate parameters under the RIP specification based on his method and his labor earnings data.

[2]We assume that there are five groups for ψ.

average hours worked in the steady state is 0.323 (OECD, 2015).[3]

The income tax rate, τ_0, is chosen to match the after-tax income Gini coefficient of 0.38 in the U.S. in 2010 (OECD, 2015). The calibrated income tax rate is τ_0, 23.8% in all three economies, which is almost identical to the tax-to-GDP ratio in 2010 (OECD, 2015). We can show that matching the two Ginis generates the same income tax rate in a linear tax system.[4] Table 2 summarizes the economy-specific parameters.

Table 2: Economy-Specific Parameters

Model Economy	NH	SH	LH
Ratio of ex-ante heterogeneity (σ_ψ/σ_z)	0.000	0.313	0.567
SD of permanent component (σ_ψ)	0.000	0.301	0.619
SD of stochastic component (σ_x)	0.711	0.658	0.466
Persistence of shocks (ρ_x)	0.946	0.946	0.842
SD of innovation to shocks (σ_ϵ)	0.230	0.213	0.251
Time discount factor (β)	0.953	0.955	0.960
Disutility from working (B)	5.051	5.092	5.010

Estimating Ex-Ante and Ex-Post Heterogeneity (Guvenen, 2009) In order to decompose the difference in earnings into two types (*ex-ante* and *ex-post*), we assume that individual productivity (earnings or wages) consists of a deterministic age profile and the stochastic shocks around it. We view the worker's age profile as an ex-ante ability and the movements around the profile as the ex-post realization of stochastic shocks (or

[3]This is the average share of time devoted to working. According to the 2015 OECD database, working hours were 1,778 and total discretionary hours were 5,500 in the U.S. in 2010.

[4]Under a linear tax and lump-sum transfer system, the income tax rate τ is the same as the reduction rates in the Gini coefficients through taxes and transfers. Suppose that the Gini coefficient of income y is $G_y = \frac{1}{\mu}\int F(y)(1-F(y))dy$, where μ is average income, and F is the cumulative density function. Let disposable income be $z = (1-\tau)y + T$ (under a linear tax with lump-sum transfer). The Gini coefficient of disposable income is: $G_z = \frac{1}{\mu_z}\int F(z)(1-F(z))dz$, where μ_z is the average disposable income. Since z is a linear function of y, $F(z) = F(y)$. The government uses all tax revenues for transfers so that $\mu = \mu_z$. Then, $G_z = \frac{1}{\mu}\int F(y)(1-F(y))dz$ $= \frac{1}{\mu}\int F(y)(1-F(y))(1-\tau)dy$ (by the chain rule) $= (1-\tau)G_y$.

luck). Specifically, we use the econometric procedure developed by Guvenen (2009), who proposed two hypotheses regarding the shape of the income profile: a restricted income profile (RIP) and a heterogeneous income profile (HIP). The RIP assumes that workers have a common age-earnings profile except for an intercept. The HIP allows for individual differences in the slope (or wage growth) as well as the intercept. Suppose that the log earnings of worker i at time t with j (after controlling for a common age profile) is $\hat{z}^i_{j,t}$. His earnings consist of a deterministic individual-specific age profile $\hat{\psi}^i_j$, a (persistent) stochastic shock $\hat{x}^i_{j,t}$, and i.i.d. measurement errors $\eta^i_{j,t}$:[5]

$$\begin{aligned}
\hat{z}^i_{j,t} &= \hat{\psi}^i_j + \hat{x}^i_{j,t} + \xi_t \eta^i_{j,t} \\
\hat{\psi}^i_j &= \theta^i + \phi^i j \\
\hat{x}^i_{j,t} &= \rho_x x^i_{j-1,t-1} + \zeta_t \epsilon^i_{j,t}
\end{aligned}$$

According to HIP, the individual-specific deterministic age profile ($\hat{\psi}^i_j$) consists of an intercept θ^i and a slope ϕ^i. The stochastic income shock ($\hat{x}^i_{j,t}$) follows an AR(1) process with persistence ρ_x and innovation $\epsilon^i_{j,t}$. Any time effect on measurement error and innovation to the stochastic shock are reflected in ξ_t and ζ_t, respectively. RIP corresponds to the case where $\phi^i = 0$ for all i (i.e., no difference in the slope of the age profile).

The labor earnings data are obtained from Guvenen (2009) and are originally from the PSID 1968-1993. The main sample consists of male heads of household between ages 20 and 64. While Guvenen's baseline estimates restrict the samples to individuals whose labor-market experience is at least 20 years (1,270 individuals), we use the estimates based on a larger sample—all workers whose labor market experience is 4 years or more (3,858 individuals). Table 3 compares our estimates based on the larger sample to those in Guvenen (2009).[6]

For RIP, while Guvenen's baseline estimate of the persistence, $\rho_x = 0.988$, is somewhat higher than those in the literature (e.g., Floden and Linde, 2001; Heathcote et al., 2008),

[5]More exactly, $\hat{z}^i_{j,t}$ are the residuals from the regression of individual income on the polynomials of labor market experience–i.e., unexplained income components by a common trend of market experience.

[6]Guvenen (2009) provides the estimates of HIP only for a larger sample. Thus, the estimates of RIP for a large sample is based on our estimates.

our estimate based on the larger sample, $\rho_x = 0.946$, is close to those in the literature.

For HIP, the cross-sectional variance of the intercept of the age profile based on the larger sample, $\sigma^2_\theta = 0.072$, is much bigger than that (0.022) in the baseline estimate of Guvenen (2009) because the larger sample includes younger workers. The variance of the slope parameter $100 \cdot \sigma^2_\phi = 0.043$ is similar to that (0.038) in the baseline case. The correlation between the intercept and the slope, $corr(\theta, \phi)$, is slightly more negative in the larger sample (-0.33 vs. -0.23 in the baseline). The persistence of the stochastic component is similar in both cases ($\rho_x = 0.842$ in the large sample and 0.821 in the baseline case).

Table 3: Estimates of Earnings Process

Model	σ^2_θ	$100 \cdot \sigma^2_\phi$	$corr(\theta, \phi)$	ρ_x	σ^2_ϵ	σ^2_η	Ratio ($\frac{E[\sigma_\psi]}{E[\sigma_\psi]+\sigma_x}$)
	Estimates in Guvenen (2009)						
RIP	0.058	-	-	0.988	0.015	0.061	0.233
HIP	0.022	0.038	-0.23	0.821	0.029	0.047	0.573
	Estimates Based on Larger Sample						
RIP	0.077	-	-	0.946	0.039	0.009	0.313
HIP	0.072	0.043	-0.33	0.842	0.032	0.044	0.567

Note: Data are taken from Guvenen (2009) and are based on PSID 1968-1993. The baseline estimates are based on male heads of household between ages 20 and 64 whose labor market experience is at least 20 years (1,270 individuals). The estimates from the larger sample are based on all workers whose labor market experience is at least 4 years (3,858 individuals).

The dispersion of ex-ante heterogeneity is measured by the average standard deviation of $\hat{\psi}$: $E[\sigma_\psi] = E\left[\sqrt{\sigma^2_\theta + 2\sigma_{\theta,\phi} j + \sigma^2_\phi j^2}\right]$.[7] The dispersion of ex-post heterogeneity is measured by the standard deviation of $\hat{x}$: $\sigma_x = \sigma_\epsilon / \sqrt{(1 - \rho^2_x)}$. Then, the relative importance of ex-ante heterogeneity in the total dispersion of the cross-sectional income distribution is $\frac{E[\sigma_\psi]}{E[\sigma_\psi]+\sigma_x}$. This ratio is 0.31 and 0.57, respectively in RIP and HIP, according to the large sample.

[7] We consider the market experience j from 1 to 35, when we calculate the average standard deviation of $\hat{\psi}$ ($E[\sigma_\psi]$)

For our quantitative analysis below, the estimated income process from RIP (based on the larger sample) is used for the model with a small ex-ante heterogeneity (SH) and that from HIP is used for the model with a large ex-ante heterogeneity (LH). The model with no ex-ante heterogeneity (NH) corresponds to the case where the entire dispersion of the cross-sectional income distribution is driven completely by stochastic shocks (x), as in Aiyagari (1994).

3.2. Income and Wealth Distributions

Table 4 reports the steady-state wealth Gini and the relative incomes across the five income quintiles in the data (SCF) and the three model economies. By construction, the steady-state interest rate (4%), average hours worked (0.323), before-tax income Gini (0.5), and after-tax income Gini (0.38) are identical across all three model economies. Since the income Gini coefficients are identical, relative incomes across income quintiles are also very similar. For example, the relative income (before taxes and transfers) in the first income quintile is 18% in NH, 17% in SH, and 17% in LH. Relative incomes from the SCF are more dispersed because the before-tax Gini coefficient in the SCF (0.575) is slightly larger than our target (0.5 from the OECD database). The wealth Ginis of NH (0.765) and SH (0.770) are somewhat larger than that of LH (0.69) because of the stronger precautionary savings motive and also because of more persistent productivity shocks ($\rho_x = 0.946$ vs. 0.842).

Table 4: Income and Wealth Distribution

	Data (SCF)	NH	SH	LH
Wealth Gini	0.834	0.765	0.770	0.690
Relative Income				
1st quintile	0.140	0.176	0.172	0.165
2nd	0.335	0.379	0.390	0.375
3rd	0.565	0.651	0.644	0.653
4th	0.915	1.089	1.086	1.115
5th	3.045	2.705	2.709	2.692

Note: The SCF statistics are based on Díaz-Giménez et al. (2011).

4. Tax Reform and Political Outcome

4.1. Utilitarian Optimal Tax

Starting from the current steady state, where the income tax rate is $\tau_0 = 23.8\%$, we look for the optimal tax rate τ^* that maximizes the (equal-weight) utilitarian social welfare (as in Aiyagari and McGrattan (1998)):

$$\mathcal{W}(\tau^*, \tau_0) = \int V(a_0, x_0, \psi; \tau^*, \tau_0)\, d\mu(a_0, x_0, \psi; \tau_0),$$

where $V(a_0, x_0, \psi; \tau^*, \tau_0)$ is the discounted sum of the lifetime utility of a household with asset holdings a_0, stochastic productivity x_0, and deterministic ability ψ, under the current tax rate τ_0. The steady-state distribution of households is denoted by $\mu(a_0, x_0, \psi; \tau_0)$. More specifically:

$$V(a_0, x_0, \psi; \tau^*, \tau_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c(a_t, x_t, \psi; \tau^*, \tau_0)^{1-\sigma} - 1}{1-\sigma} - B\frac{h(a_t, x_t, \psi; \tau^*, \tau_0)^{1+1/\gamma}}{1+1/\gamma} \right\}$$

This is an unexpected once-and-for-all change from τ_0 to τ^* (permanent tax reform). In computing welfare, we take into account the transition periods from the current to a new steady state. A detailed computational algorithm for the optimal tax rate is provided in Appendix A.2.

Table 5 summarizes the optimal tax rates and corresponding new steady states. In all three model economies, the optimal tax rates are much higher than the current rate: 37.3% (NH), 37.7% (SH) and 36.9% (LH). Due to the veil of ignorance, the optimal tax rates are similar in all three economies (which exhibit similar income distributions by construction) regardless of the composition of ex-ante and ex-post heterogeneity. This result (a very high optimal tax rate) is consistent with a majority of the literature based on a utilitarian social welfare function (for example, Piketty and Saez, 2013; Heathcote and Tsujiyama, 2016; and Chang et al., 2018).

A fiscal reform always generates winners and losers. Thus, there is no guarantee that the utilitarian optimal tax rate will be chosen as a political outcome. While the examination of a complicated political process to select a policy is beyond the scope of this paper, we can still ask a simple question of whether a proposed fiscal reform will

Table 5: Optimal Tax Rates under Utilitarian Welfare Function

Model	NH	SH	LH
Current τ	0.238	0.238	0.238
Optimal τ^*	0.373	0.377	0.369
Interest rates	0.054	0.054	0.052
Hours worked	0.279	0.277	0.281
Before-tax Gini	0.525	0.527	0.524
After-tax Gini	0.329	0.328	0.331
Wealth Gini	0.781	0.787	0.714
Approval Rate	0.576	0.556	0.538

attain a majority vote. Table 5 shows the approval rates for these optimal tax reforms: 57.6% (NH), 55.6% (SH), and 53.8% (LH). As the relative size of ex-ante heterogeneity becomes large—i.e., the household's income is largely determined by permanent earnings ability (as opposed to by stochastic shocks), the approval rate for a higher tax declines because a potential insurance benefit from a higher tax diminishes. Nevertheless, in all three economies, the majority of the population is in favor of raising the tax rate to the utilitarian optimal level.

4.2. Successive Binary Votes

One may argue that the political equilibrium should be the tax rate that maximizes the welfare of the median household—the so-called median voter theorem (e.g., Romer (1975) and Roberts (1977)). However, the median voter theorem cannot be easily applied in our model because the median household is hard to identify. Households differ along multiple dimensions (e.g., asset holdings, permanent ability, and ex-post realization of the productivity shock) and the state of households changes over time. Instead, we look for a politically feasible tax rate (that is close to the utilitarian optimum) based on a series of successive binary voting as follows.

Starting with a newly proposed tax rate $\tau = \tau_0 + 1\%$, which is 1 percentage point higher (or lower) than the current tax rate τ_0, we simulate a binary voting between the current ($\tau_0 = 23.8\%$) and proposed tax rates ($\tau = 24.8\%$). If the proposed tax rate is approved by the majority, we immediately propose another tax rate ($\tau = \tau_0 + 2\%$) that is one more percentage point higher (or lower) than the current one and simulate a binary voting between the previous winner ($\tau = 24.8\%$) and the new contender (e.g., $\tau = 25.8\%$) and so forth. We call a final winner of this successive binary voting, " Majority τ^M." Figure 1 illustrates the approval rates for a series of proposed tax rates τ's. As anticipated, starting from the current tax rate, the approval rate for a tax reform that increases 1 percentage point is well above 50%. For example, the proposal to increase the income tax rate by 1 percentage point from the current rate ($\tau = 23.8\%$) receives an approval rate of 73% in NH. The same proposal receives an approval rate of 66% and 57% in SH and LH, respectively. As the tax rate goes higher, the approval rate (for an incremental tax rate increase) decreases. The highest tax rate with a majority vote is slightly lower than—but actually close to—the utilitarian optimal tax rate in all three models. For example, in NH $\tau^M = 36.8\%$, only 0.5 percentage point lower than the optimal tax rate $\tau^* = 37.3\%$. The tax chosen by this successive voting in LH (33.8%) is 3 percentage points lower than the optimal rate of 36.9%. As the ex-ante heterogeneity becomes large, the potential insurance benefit from a heavy tax-and-transfer policy diminishes.[8]

Table 6: Tax Reform by a Successive Majority Vote

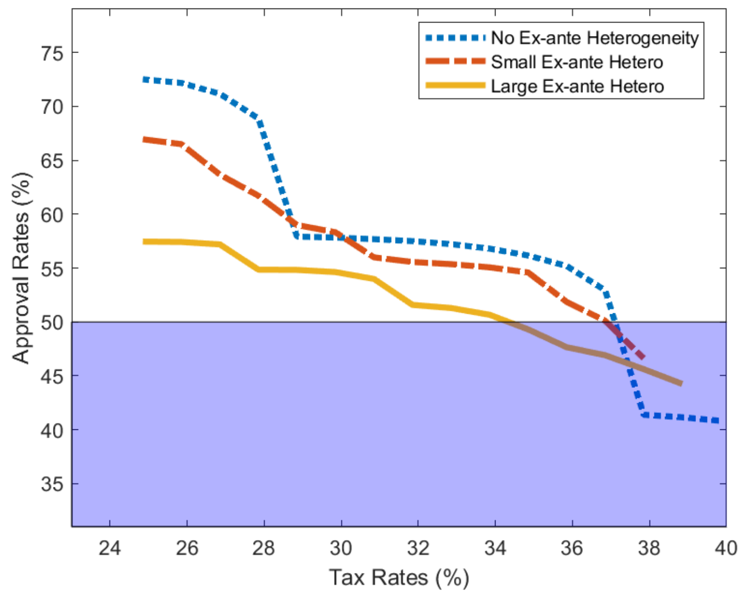
Tax Rate	NH	SH	LH
Current: τ_0	0.238	0.238	0.238
Utilitarian Optimal: τ^*	0.373	0.377	0.369
Simple Majority: τ^M	0.368	0.368	0.338
Effective Majority: τ^{EM}	0.348	0.328	0.268

4.3. Income-Dependent Voting Turnout

We have shown that the tax rate chosen by either the utilitarian social planner or the successive majority voting is far from the currently observed rate, regardless of the composition of the income process. If the majority of the population can improve upon the

[8]Alesina et al. (2011) and Charité et al. (2015) report empirical evidence for the correlation between ex-ante heterogeneity and preferences for redistribution.

Figure 1: Approval Rates for Tax Reforms



tax reform, why hasn't a society adopted it? First, the "optimality" depends on the specification of the social welfare function. It is not obvious whether each government's goal is to maximize the equal-weight utilitarian welfare function, despite its popular use in quantitative macroeconomic analysis. There are many other alternative criteria. One may argue that it is desirable for a society to maximize the welfare of the poorest members instead of the average (i.e., Rawlsian). Society's choice for redistribution is also affected by various factors such as the externality of public expenditures (Heathcote et al., 2016), profession (Lockwood et al., 2016), the preference heterogeneity (Lockwood and Weinzierl, 2015), the reference point (Charité et al., 2015), benefit-based taxation (Weinzierl, 2016), or the equal sacrifice rule (Weinzierl, 2014). Moreover, the process under which policies are actually determined is much more complicated than a simple majority rule. For example, the political equilibrium under a multi-party system can be different from that chosen by the median. These questions are immensely important but beyond the scope of this paper. Here, we suggest a rather simpler reconciliation: the income-dependent turnout rates in voting. It is well known that in the U.S. low-income households are much less likely to participate in voting than are high-income households.

Table 7: Turnout Rates by Income Quintiles in the U.S.

Income Quintile	1st	2nd	3rd	4th	5th
Turnout Rates	50.6	55.4	66.0	72.6	86.7

Source: Mahler (2008)

According to Mahler (2008), the voting turnout rate of the lowest income quintile is only 50.6%, while that of the highest income quintile is 86.7% (Table 7).[9]

We incorporate this income-dependent turnout rate profile in Mahler (2008) into our model. More specifically, we assume that the turnout rate $TR(z)$ depends on productivity z as:

$$TR(z) = \overline{TR} \cdot \exp(\omega) \cdot z$$

where ω and $\overline{TR}$ are constants. Two parameters ω (slope) and $\overline{TR}$ (average) are chosen to match the income quintile-turnout rates profile from Mahler (2008) in Table 7. The left panel of Figure 2 shows the turnout rates by income quintiles from the model and the data (Mahler, 2008). The calibrated turnout rates in the three models are very similar (as we tie the turnout rate to realized income z). The turnout rate profiles in the model are slightly flatter than that in the data. Thus, our calibration is conservative. This is because we are constrained in choosing ω so that $TR(z_{max})$ cannot exceed 100%. The right panel of Figure 2 illustrates the turnout rates across five ex-ante productivity (ψ) groups. [10]

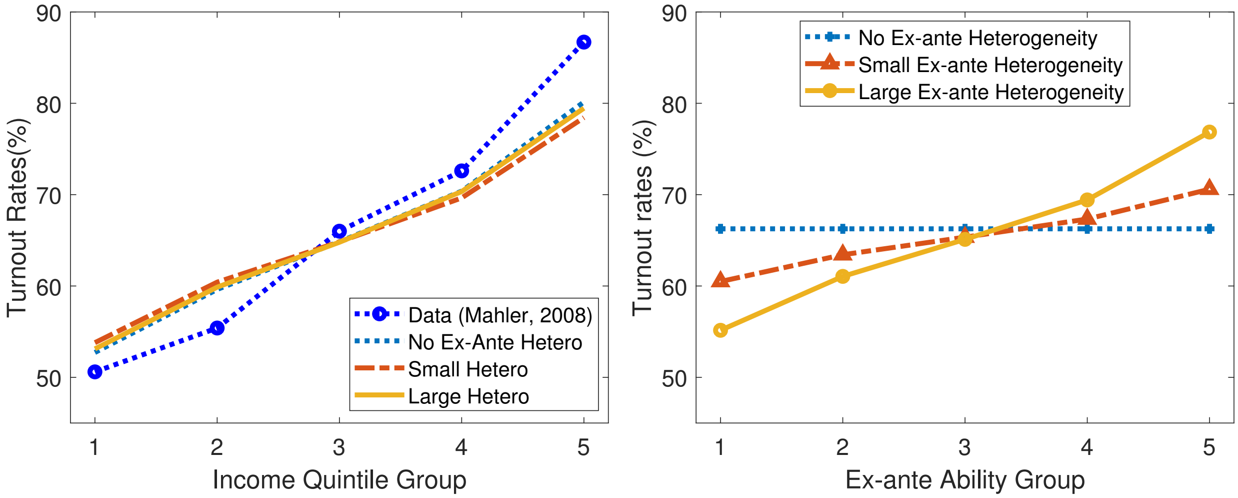
We repeat the successive-voting simulation taking into account the upward-sloping profile of turnout rates. Figure 3 compares the approval rates under this income-dependent turnout rate (which we call "Income-Dependent Vote") to those under the assumption

[9] The voter turnout rates by income from Mahler (2008) are based on the Comparative Study of Electoral Systems (CSES), which are post-election surveys conducted across countries in 1996 and 2000. The Mahler numbers are based on the 1996 survey. According to Institute for Democracy and Electoral Assistance (IDEA), the voter turnout rate (votes cast divided by registered voters) for the presidential election was 82% and that of Parliament was 67% in 1996.

[10] The chosen values of ω are 0.204 (NH), 0.18 (SH), 0.191 (LH), respectively, and those of $\overline{TR}$ are 0.682 (NH), 0.674 (SH), and 0.662 (LH), respectively.

Figure 2: Voting Turnout Rates: Data vs. Models

(a) By Income Quintile (b) By Ex-ante Ability Group



that all households vote (labeled as "Simple Vote"). With the income-dependent turnout rates, fiscal reform to raise the tax rate obtains much lower approval rates than under the simple vote assumption. The differences are not large in NH; the tax rate chosen under the effective majority voting, reflecting income-dependent turnout rates, $\tau^{EM} = 34.8\%$, is only 2 percentage points lower than that chosen by the simple majority ($\tau^{M} = 36.8\%$). In SH, $\tau^{EM} = 32.8\%$ is 4 percentage points lower than $\tau^{M} = 36.8\%$. In LH, $\tau^{EM} = 26.8\%$ is now 7 percentage points lower than $\tau^{M} = 33.8\%$. The tax rate chosen by the majority (when the income-dependent turnout rate is taken into account) is now much closer to the current rate. As ex-ante productivity becomes more important in total earnings, the potential insurance benefit from a high tax-and-transfer policy diminishes.

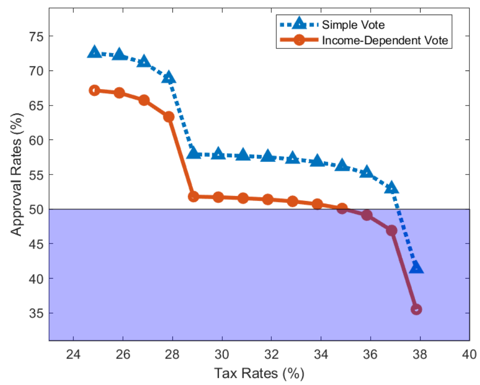
With a large ex-ante heterogeneity, voting behavior is sharply divided among the population. Table 8 reports the approval rates for the chosen tax rates (τ^{EM}) across five income quintiles and those across five ex-ante productivity groups. In LH *all* households in the bottom two ex-ante productivity groups support this reform, while *all* households in the two highest ex-ante productivity groups oppose the tax reform.

Table 8: Approval Rates by Income Quintile or Ex-Ante Ability

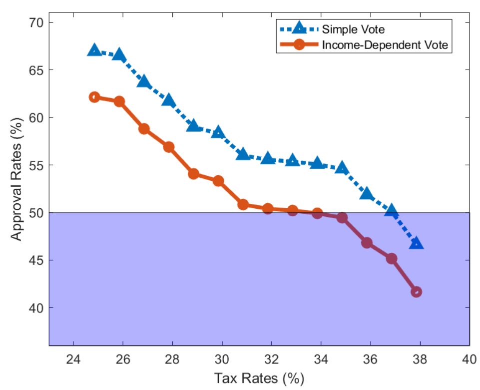
	NH	SH	LH
Utilitarian Optimal: τ^*	0.373	0.377	0.369
Simple Majority: τ^M	0.368	0.368	0.338
Effective Majority: τ^{EM}	0.348	0.328	0.268
Approval Rates			
- By Income Quintile			
1st	1.000	1.000	0.993
2nd	1.000	0.968	0.842
3rd	0.825	0.658	0.674
4th	0.058	0.278	0.333
5th	0.008	0.045	0.030
- By Ex-Ante Ability			
1st	0.562	0.922	1.000
2nd	0.562	0.727	1.000
3rd	0.562	0.574	0.853
4th	0.562	0.402	0.006
5th	0.562	0.144	0.000

Figure 3: Approval Rates for New Tax Reforms

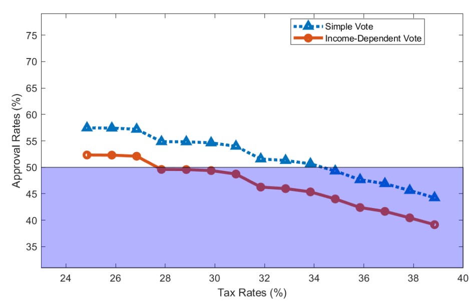
(a) No Ex-ante Heterogeneity



(b) Small Ex-ante Heterogeneity



(c) Large Ex-ante Heterogeneity



5. Summary

According to the utilitarian social welfare function or the median voter theorem (widely used criteria in the optimal taxation literature), the current tax rate of labor income in the U.S. is far below its optimal (see Piketty and Saez, 2013, for example). In this class of models, the insurance benefit from tax dominates the cost from distorting labor supply.

We argue that the interaction between ex-ante heterogeneity and income-dependent voting behavior helps us to understand why the current tax rate is much lower than the utilitarian optimum. In our model, individual earnings consist of permanent ability (ex-ante heterogeneity) and stochastic shocks (ex-post heterogeneity) whose decomposition is estimated from the panel data following Guvenen (2009). When household income is determined primarily by permanent ability (rather than luck), the potential benefit from social insurance (through the government tax-and-transfer) decreases significantly for a high-income group. When the voting behavior of the model is calibrated to match the observed income-dependent turnout rates reported in Mahler (2008), the tax rate chosen by the majority drops to 27%, close to the current average income tax rate in the U.S.

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Appendix A: Computational Procedures

A.1. Steady-State Equilibrium

The distribution of households, $\mu(a, x, \psi)$, is time-invariant in the steady state, as are factor prices. We modify the algorithm suggested by José-Víctor Ríos-Rull (1999) in finding a time-invariant distribution μ. Computing the steady-state equilibrium amounts to finding the value functions, associated decision rules, and time-invariant measure of households. The current income tax rate τ_0 is chosen to match the after-tax Gini coefficient in the data. We simultaneously search for (i) the discount factor β that clears the capital market at the given annual real interest rate of 4%; (ii) the standard deviation of idiosyncratic productivity, σ_η, that matches the before-tax Gini coefficient; and (iii) the disutility parameter B to match the average hours worked, 0.323. The details are as follows:

1. Choose the grid points for asset holdings (a), ex-ante productivity (ψ) and idiosyncratic productivity ($z = \psi x$). The number of grids is denoted by N_a, N_ψ and N_z, respectively. We use $N_a = 490$, $N_\psi = 5$, and $N_z = 75$. Note that we set $N_x = 15$ for stochastic productivity in each ex-ante productivity so that the total number of grids for idiosyncratic productivity is $N_z = 75$. The asset holding a_t is in the range of [0, 90]. The grid points of assets are not equally spaced. We assign more grids in the lower asset range to better approximate the savings decisions of the households near the borrowing constraint.

2. Pick initial values of β, B, and σ_η. The dispersion of ability, σ_ψ, is set by ex-ante heterogeneous ratio ($\frac{\sigma_\psi}{\sigma_\psi+\sigma_\eta/\sqrt{(1-\rho_x^2)}}$). We construct five ability groups ψ_g (where the lower and upper bounds are set to $\pm 2\sigma_\psi$): -1.42σ_ψ, -0.55σ_ψ, 0, 0.55σ_ψ and 1.42 σ_ψ, respectively. For the idiosyncratic productivity shock, we construct five grid vectors of length N_x around each ψ_g. Elements in each vector, denoted by ln z_j's, are equally spaced on the interval $[\psi_g - 3\sigma_\eta/\sqrt{1-\rho_x^2},\ \psi_g + 3\sigma_\eta/\sqrt{1-\rho_x^2}]$. Then, we approximate the transition matrix of the idiosyncratic productivity using George Tauchen's (1986) algorithm.

3. Start with an initial amount of government transfers T. Given β, B, σ_η, τ, and T, we solve the individual value functions V at each grid point for individual states. In this step, we also obtain the optimal decision rules for asset holdings $a'(a_i, x_j, \psi_g)$ and labor supply $h(a_i, x_j, \psi_g)$. This step involves the following procedure:

   (a) Initialize value functions $V_0(a_i, x_j, \psi_g)$ for all $i = 1, 2, \cdots, N_a$, $j = 1, 2, \cdots, N_x$, and $g = 1, 2, \cdots N_\psi$

   (b) Update value functions by evaluating the discretized versions:

   $$V_1(a_i, x_j, \psi_g) = \max \Big\{ u\big((1-\tau_0)(wh(a_i, x_j, \psi_g)\psi x_j + ra_i) + a_i + T - a', h(a_i, x_j, \psi_g)\big) + \beta \sum_{j'=1}^{N_x} V_0(a', x'_j, \psi_g))\pi_x(x_{j'}|x_j, \psi_g) \Big\},$$

   where $\pi_x(x_{j'}|x_j)$ is the transition probability of x, which is approximated using Tauchen's algorithm.

   (c) If V_1 and V_0 are close enough for all grid points, then we have found the value functions. Otherwise, set $V_0 = V_1$, and go back to step 3(b).

4. Using $a'(a_i, x_j, \psi_g)$ and $\pi_x(x_{j'}, x_j)$ obtained from step 3, we obtain the time-invariant measures $\mu^*(a_i, x_j, \psi_g)$ as follows:

   (a) Initialize the measure $\mu_0(a_i, x_j, \psi_g)$.

   (b) Update the measure by evaluating the discretized version of a law of motion for each ψ_g:

   $$\mu_1(a_{i'}, x_{j'}, \psi_g) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbf{1}_{a_{i'} = a'(a_i, x_j, \psi_g)} \mu_0(a_i, x_j, \psi_g)\pi_x(x_{j'}|x_j).$$

   (c) If μ_1 and μ_0 are close enough in all grid points, then we have found the time-invariant measure. Otherwise, replace μ_0 with μ_1 and go back to step 4(b).

5. Using decision rules and invariant measures, check the balance of the government budget. Total tax revenues are:

   $$Rev = \int_{a,x,\psi} \tau_0(w\psi xh + ra)d\mu(a, x, \psi).$$

If Rev is close enough to T, then we have obtained the amount of government transfers. Otherwise, choose a new T and go back to step 3.

6. We calculate the real interest rate, Gini coefficient, individual hours worked using μ^* and decision rules. If the calculated real interest rate, average hours worked, and before-tax Gini coefficient are close to the assumed ones, we have found the steady state. Otherwise, we choose a new β, B, and σ_η, and go back to step 2.

A.2. Optimal Tax Rates and Voting Outcome

In calculating the welfare consequences of a tax reform, we include the utilities during the transition from an initial to a new steady state. Computing the transition equilibrium amounts to finding the value functions, the associated decision rules, and the distribution of households in each period. The details are as follows:

1. Compute the initial steady state under the current tax rate (τ). Use the algorithm for the steady-state equilibrium.

2. Choose a new tax rate and compute all transition paths as follows:

   (a) Compute the final steady state under a new tax rate. Use the algorithm for the steady-state equilibrium.

   (b) Assume that the transition is completed after $T-1$ periods, and that the economy is in the initial steady state at time 1 and in the final steady state at T. Choose a T big enough so that the transition path is unaltered by increasing T.

   (c) Guess the capital-labor ratios $\{K_t/E_t\}_{t=2}^{T-1}$ and compute the associated $\{r_t, w_t\}_{t=2}^{T-1}$.

   (d) Guess the path of government transfers $\{T\}_{t=2}^{T-1}$. Note that the amounts of government transfers are all different in each period, since decision rules and measures are different. Going backward, compute the value functions and policy functions for all transition periods by using $V_T(\cdot)$ from the final steady state. Using the initial steady-state distribution μ_1 and the decision rules, find the measures of all periods $\{\mu_t\}_{t=2}^{T-1}$.

(e) Based on the decision rules and measures, compute the aggregate variables and total tax revenues. If the total tax revenue is close to the assumed transfers, we obtain the amount of transfers. Otherwise, choose a new path of government transfers and go back to 2(d).

(f) Compute the paths of aggregated capital and effective labor and compare them with the assumed paths. If they are close enough in each period, we find the transition paths. Otherwise, update $\{K_t/E_t\}_{t=2}^{T-1}$ and go back to 2(c).

3. Choose the tax rate that yields the highest social welfare, which is the sum of individual values. This is the optimal tax rate under the utilitarian criteria.

4. For the voting outcome, propose a new tax rate that is 1 percentage point higher (or lower) than the current one ($\tau_1 = \tau_0 + 0.01$) . Using the above procedure, compute the individual value (including the transition from the current steady state to a new steady state under the proposed tax rate). If individual values under new tax reform are higher than the values under the current tax rate, i.e. $V(a, x, \psi|\tau_1, \tau_0) > V(a, x, \psi|\tau_0, \tau_0)$, then this individual is assumed to vote for the new tax reform. If a majority prefer this reform, increase (or decrease) the new tax rate by 1 percentage point ($\tau_2 = \tau_1 + 0.01$) further and compute individual values. If $V(a, x, \psi|\tau_2, \tau_0) > V(a, x, \psi|\tau_1, \tau_0)$, then this individual votes for the second reform. Continue to propose a new 1-percentage-point higher (or lower) tax rate until the majority rejects a proposed tax reform. When the majority rejects, the last winner is the highest (or lowest) tax rate chosen by a successive-majority voting.