Network-motivated Lending Decisions: A Rationale for Forbearance Lending*

Yoshiaki Ogura,[a†] Ryo Okui,[b‡] and Yukiko Umeno Saito[c§]

[a] *School of Political Science and Economics, Waseda University, 1-6-1 Nishi Waseda, Shinjuku-ku, Tokyo 169-8050, Japan*

[b] *Department of Economics and the Institute of Economic Research, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Korea*

[c] *School of Political Science and Economics, Waseda University, 1-6-1 Nishi Waseda, Shinjuku-ku, Tokyo 169-8050, Japan*

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Abstract

We demonstrate theoretically and empirically the presence of forbearance lending by profit-maximizing banks to influential buyers in a supply network. If the financial market is concentrated, then banks can internalize the negative externality of an influential firm's exit. As a result, they may keep refinancing for a loss-making influential firm at an interest rate lower than the prime rate. This mechanism sheds new light on the discussion about bailouts offered to zombie firms. Our empirical study, with a unique dataset containing information about interfirm relationships and main banks, provides evidence for such network-motivated lending decisions.

Keywords: supply network, influence coefficient, forbearance, bailout, zombie.
JEL Classification: C55, D57, G21, G32, L13, L14

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†E-mail address: yogura@waseda.jp.
‡E-mail address: okuiryo@snu.ac.kr.
§E-mail address: saito-yukiko@waseda.jp.

1 Introduction

In economic downturns, we frequently observe financial institutions engage in *forbearance* or *zombie* lending, i.e., repeated refinancing for existing loans for underperforming or nonperforming firms at an extremely low interest rate. Examples in the literature include major Japanese banks during the banking crisis in the late 1990s (e.g., Sekine et al., 2003; Peek and Rosengren, 2005; Caballero et al., 2008), and European banks in the 2012 debt crisis (Acharya et al., 2019).

However, there has not been a consensus regarding the cause of the forbearance. The existing theories have focused on the overinvestment in monitoring within a one-to-one bank–firm relationship (e.g., Dewatripont and Maskin, 1995; Berglöf and Roland, 1997) or gambling for resurrection (e.g., Bruche and Llobet, 2014). The media, however, suggest an economically rational reason for forbearance lending. For example, a news article states: "[A regional] bank avoids hasty bad-loan write-offs with full attention, which can be devastating for the local economy, its revenue base. In the turnaround of borrowing companies, it gives a priority to large borrowers with larger number of customers and suppliers to avoid chain bankruptcy."[1] The existing studies do not address forbearance lending nor provide any counterarguments against it. Understanding the mechanisms of forbearance lending is important because it also provides us with a tool to evaluate the associated welfare loss. The literature on zombie lending; such as studies by Caballero et al. (2008) and Acharya et al. (2019), argues that the growth of healthy firms is hampered as a consequence of credit misallocation caused by forbearance lending. However, the above news article indicates possible benefits of helping other firms in a supply network.

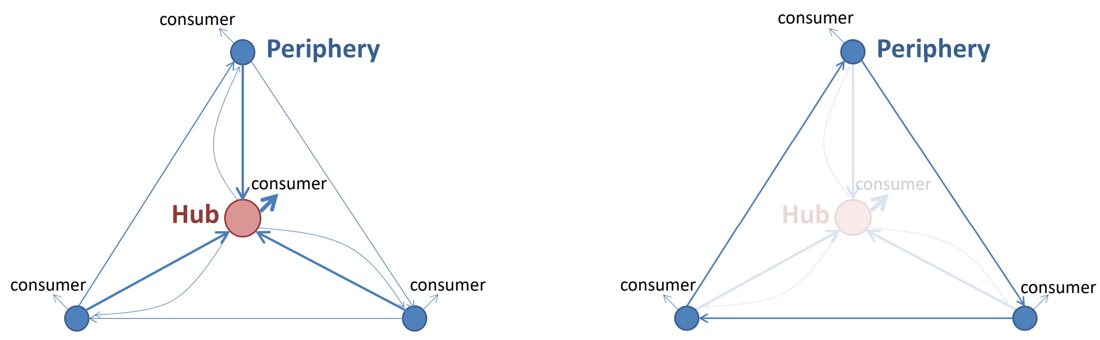
In this paper, we demonstrate theoretically and empirically the possibility that profit-maximizing banks undertake forbearance lending from the perspective of sustaining the supply network among borrowers. We argue that the possibility is higher when recipient firms are influential buyers in a supply network in the sense that its existence creates positive externalities for the total sales and profit of the network. A profit-maximizing bank that is a dominant financier in a region or an industrial group may have an incentive to undertake forbearance lending to internalize this externality. In our empirical analysis we use a unique dataset of interfirm transactions matched with

[1] "Rettou Kin'yu Fairu Tochigi Hatsu," (Archipelago Finance File from Tochigi) Nikkei Finance Newspaper, p. 3, February 28, 2006. Translated by the authors.

the main bank information for each firm. The empirical results are consistent with our theory.

Figure 1: A directed and weighted supply network

(Note) Each node is a hub or peripheral firm. The direction of each arrow represents the direction of product sales and the thickness indicates the amount of sales.



(a) Keeping the hub open (b) Closing the hub

To illustrate the economic problem addressed in this study, we consider the example illustrated in Figure 1. There are two types of companies. One type of company is a hub company that depends heavily on intermediate inputs from other companies (panel (a) of Figure 1). The other type is a peripheral company that is not as dependent on intermediate goods but is a major provider of intermediate inputs. The direction of the arrows in the figure indicates the flow of products, and the thickness indicates the amount of sales. The negative impact of the closure of the hub company propagates throughout the network by reducing sales as shown in panel (b) of the figure. This can even trigger a chain of closures of peripheral companies. If the total profit of this network is positive despite the losses by the hub firm, it can be more profitable to keep the hub firm afloat. A bank is willing to do so if it is a dominant financier for this network, and if it expects a larger profit from supporting the loss-making hub company and recouping the cost from higher interest on the peripheral companies.

We construct a supply network model with a monopolistic bank. Our model provides a measure of the extent of the influence of each firm, which we call the *demand influence coefficient*. Our theory leads to the hypothesis that a loss-making firm with a higher demand influence coefficient is more often a recipient of forbearance lending if a bank is a monopolistic lender or oligopolistic banks collusively behave like a monopolistic lender.

Our empirical study examines these network-motivated lending decisions in two steps. First, we estimate the demand influence coefficients of firms by applying techniques from spatial econometrics. Our dataset is collected from *Tokyo Shoko Research (TSR) Corporate Relationship Database*, which contains the list of corporate customers and suppliers, lending banks, and the basic financial information on about 700,000 firms in Japan at three time points around the 2008 global financial crisis; 2005, 2010, and 2013. The dataset provides information about the supply network across firms. We estimate a spatial autoregressive model to evaluate the degree of interaction among the sales of firms in the network. We then split the network into subnetworks for each bank, which consist of all edges where either a buyer, seller, or both use the bank as their main bank. We compute the demand influence coefficients using these bank-level subnetworks and the estimates from the spatial autoregressive model under the assumption that a main bank observes the supply network among its borrowers and their direct customers and suppliers.

Second, we match the estimated demand influence coefficients with the corporate financial data and examine the effects of the demand influence coefficient on the probability of receiving forbearance lending when in financial distress, i.e., to obtain refinancing at a rate lower than the prime rate when a firm is heavily indebted, and when its operating cash flow is insufficient to cover the prime rate.

We find statistically and economically significant evidence to support our main hypothesis in the postcrisis period of the global financial crisis, after controlling for other firm factors including size, leverage and repayment ability, bank factors, and bank–firm relationship factors. The estimated coefficient indicates that a one-sigma increase in the demand influence coefficient increased the probability of receiving forbearance lending by about 1.4 percentage points in 2008–2010 and 3.3 percentage points in 2011–2013. This is a sizable economic impact given that the probability of distressed firms obtaining forbearance lending is 12.7% on average.

We also find that the demand influence coefficient is higher for firms whose main bank has a larger share in the local loan market or firms that are located in a more concentrated credit market. Banks operating in such credit markets, typically regional banks in the context of Japan, are often a dominant lender in the local market. Thus, these banks are likely to observe the entire supply network within a regional market. It enables them to identify a distressed but influential firm correctly and to recoup the cost of forbearance lending for such an influential firm from peripheral bank-dependent firms. These empirical results are consistent with our theory of network-motivated lending decisions.

Related literature

Our study proposes a novel viewpoint for academic discussion about the mechanism by which a bank engages in forbearance or zombie lending. Existing theories on this subject have focused on the overinvestment in monitoring within a one-to-one bank–firm relationship or gambling for resurrection as we mentioned above. The novelty of our explanation is that it focuses on the fact that a bank lends to thousands of firms interconnected through a supply network and can consider the network effect. Many well-known existing empirical studies have established evidence for the existence of the zombie lending problem among large listed companies. However, several papers find that nonlisted small companies were less likely to have benefited from it although the cause of the difference has not been clarified (Sakai et al., 2010; Hamao et al., 2012).[2] Our theory and empirical study provide a possible explanation for this finding using differences in demand influence coefficients.

Many studies report significant propagation effects of a liquidity shock, a bankruptcy, or other negative shocks through an interfirm transaction network (Hertzel et al., 2008; Boissay and Gropp, 2013; Chen et al., 2013; Calvalho et al., 2016; Barrot and Sauvagnat, 2016; Azizpour et al., 2018). A novel point of our study is that we show the strategic response of banks to this propagation effect,[3] while these studies focus on detecting shock propagation. Moreover, we take into account the higher-order influence of a shock as measured by the demand influence coefficient, not only the impacts on a narrowly defined neighborhood as in the existing studies.

More broadly, several empirical studies report evidence of a spillover effect of bankruptcy or financial distress of firms in a geographical neighborhood (Bernstein et al., 2019) and within a

[2] Similar findings are also reported for Japan; e.g., Fukuda et al. (2007), Hosono (2008), and Ogawa (2008).

[3] Another possible arrangement to avoid the propagation effect is an autonomous bailout by suppliers (see Leitner, 2005; Rogers and Veraart, 2013). This direction is interesting but outside the scope of our paper. Campello and Gao (2017) find that a bank imposes a higher interest rate for borrowers whose sales are concentrated on a certain customer. This is a precautionary response of banks to mitigate shock propagation.

sector including suppliers and buyers for the sector (Giannetti and Saidi, 2019). Giannetti and Saidi (2019) find that a bank with a large lending share to a certain sector tends to increase its lending to the sector when the sector is in distress to avoid the negative externality of fire sales of collateralized assets. Our focus is to identify the demand spillover instead of the fire-sale spillover. Furthermore, thanks to our unique dataset, we examine directly the network in which each firm operates.

In our theoretical model, we formulate the interfirm supply network as an incomplete directed and weighted network of sales among oligopolistic firms offering differentiated products that are intermediate inputs as well as final products. This type of multisector model has already been proposed in macroeconomics to analyze the behavior of the aggregate economy (e.g., Long and Plosser, 1983; Dupor, 1999; Horvath, 2000; Acemoglu et al., 2012; Bigio and La'o, 2016; Baqaee, 2018, among others). We simplify the model of Baqaee (2018) and add the financial sector, which strategically determines the credit allocation. The concept of our demand influence coefficient is different from that of the demand centrality proposed by Baqaee (2018). Ours is designed to measure the externality to the sales of connected firms, whereas demand centrality captures the nominal-term labor intensity of each firm underlying the externality. Another similar yet different measure is the influence coefficient proposed by Acemoglu et al. (2012). The difference is that ours captures the externality as a buyer, instead of a supplier.

Organization of the paper

The remainder of this paper is organized as follows. We introduce a supply network model and derive the equilibrium in Section 2. Our main theoretical results are presented in Section 3, where we show the possibility of network-motivated forbearance lending. We specify the hypotheses to be tested in our empirical study in Section 4. The estimation of the demand influence coefficient is explained in Section 5. Section 6 describes the dataset for our hypothesis test. Section 7 presents the results of our empirical analysis of the relationship between forbearance lending and the demand influence coefficient. Section 8 discusses the welfare implications of our results and possible future research. Section 9 concludes.

2 Theoretical analysis

In this section, we present a theoretical model of a supply network including the financial sector. We derive conditions under which firms are financed by a monopolistic bank. The main implication of the model, network-motivated forbearance lending by a profit-maximizing bank, is discussed in the next section.

2.1 Setup

Our model is a one-period oligopoly model with a banking sector. We assume that the production technology requires intermediate goods and that this creates the supply network. We also assume that firms need to refinance their existing bank loans to keep operating. The role of the banking sector in our model is to refinance them.

The basic setup of the model is as follows. There is a continuum of price-taking households of mass of one with identical utility functions and budget constraints; n firms, indexed by i $(= 1, 2, \ldots, n)$ and a monopolistic bank. There are $(n + 1)$ goods in the economy. These goods are differentiated. Good 0 is a pure intermediate good (i.e., not a consumption good). It is supplied from outside the model's supply network. Good i $(i = 1, 2, \cdots, n)$ is produced and priced by firm i, and can be a consumption good, an intermediate good, or both. The aggregate quantity and the nominal price of good i are denoted by x_i and p_i, respectively. The nominal price of good 0, p_0, is given exogenously. We assume initially that the banking sector is a monopoly in order to elucidate the logic of our model in the simplest manner. An economy with many infinitesimal investors or multiple banks is discussed later.

We assume the following decision timing and settlement scheme.

Time 1. Each firm has an existing loan that must be refinanced. The bank decides which firms will be refinanced. The bank also has an outside investment opportunity. Those firms that do not obtain refinancing exit the economy. It is common knowledge which firms are operating.

Time 2. Firms decide the production level and their demand for inputs. At the same time, households decide the product demand. Each firm sets the price of its product in response to the demand function. All payments are settled by trade credit.

Time 3. Firm profits and outside investment outcomes are realized. All trade credits are cleared. The bank captures all the profits of the firms and obtains the returns to the outside investment opportunity.

Let e_i $(i = 1, 2, \cdots, n)$ equal one if firm i operates and zero otherwise. Good j is not supplied if $e_j = 0$. We assume that $e_0 = 1$, i.e., input supply from outside of the network always exists.

2.1.1 Households

The representative household is utility maximizing, with a constant elasticity of substitution (CES) utility function given by:

$$U = \left(\sum_{j=1}^{n} c_j^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \tag{1}$$

where c_j is the consumption of good j, and θ is the elasticity of substitution. The budget constraint of the representative household is:

$$\sum_{j=1}^{n} c_j p_j \leq R, \tag{2}$$

where R is the nominal income of the household, which is exogenously given. The households also face the availability constraint:

$$(1 - e_j) c_j = 0. \tag{3}$$

This constraint implies that demand has to be zero if the good is not supplied, whereas it can be any nonnegative value otherwise.

2.1.2 Firms

Firms are profit maximizing and have the following CES production function:

$$x_i = \left(\sum_{j=0}^{n} w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1,$$

where θ is the elasticity of input substitution, w_{ij} is the technological importance of the input j for the production of firm i, and x_{ij} is the quantity of inputs supplied by firm j used in production by firm i. We assume that the elasticity of input substitution θ is equal to θ in the utility function to simplify the analysis.

$\{\{w_{ij}\}_{i=1}^{n}\}_{j=0}^{n}$ is the most important determinant of the supply network. We assume that the supply network is rigid in the sense that w_{ij} does not change, even if a supplier of an intermediate product exits.[4] This assumption also means that there is no free entry of new firms into the market. We assume that $\{\{w_{ij}\}_{i=1}^{n}\}_{j=0}^{n}$ satisfies $0 \leq w_{ij} \leq 1$, $\forall\ i, j$; $w_{ii} = 0$; and $0 < \sum_{j=0}^{n} w_{ij} \leq 1$.[5] Note that because some firms may fail to operate, the supply network the firms actually face is $\{\{e_j w_{ij}\}_{i=1}^{n}\}_{j=0}^{n}$.

Each firm must refinance its existing loan of F_i $(i = 1, 2, \cdots, n)$ from the bank to continue its operations. The value of F_i is fixed in real terms but varies across firms.[6] Those firms that can obtain refinancing always operate even when the operation results in a loss because the bank bears their losses.

2.1.3 Financial market

We assume that the financial market is a monopoly for the time being. The bank chooses which firms will be refinanced so as to maximize the total return from these investments. The bank captures the entire profits (and the losses as well) of firms that are financed by the bank. The bank also has an outside opportunity that can yield the real risk-free rate of return ρ (> 0), which we call the prime rate. We assume that the loanable funds of the bank exceed the total demand for funds.

2.2 Equilibrium

We derive the market equilibrium backwardly. That is, we first derive the equilibrium outcome in the input and product markets and then derive the optimal investment decision of the bank.

[4] This assumption is more plausible in industries where the designs of input products or the contents of services are highly customized, information- or skill-intensive, and specific to each user. Automobiles, construction, and some types of retailers/wholesalers dealing in custom-made items are of this type.

[5] The value of $\sum_{j=0}^{n} w_{ij}$ represents the productivity level. Note also that the condition $\sum_{j=0}^{n} w_{ij} \leq 1$ guarantees the existence of the equilibrium price vector (10) as well as the existence of an equilibrium.

[6] An example of such predetermined liabilities is the cost of pensions owed to retired employees. This pension cost must be financed in order for a company to continue operating although it is not directly related to their output level. For example, a news article reports that the Big Three car companies "saddled themselves with a cost structure in flush times that has proved unsustainable as their market share has eroded. They have made great strides of late in shedding legacy pensions and health-care costs, but they took decades to do so." ("The Next Bailout: Detroit," August 22, 2008, *Wall Street Journal*).

2.2.1 Final demand for each product

The final demand function for each product is determined by households' utility maximization after observing the list of operating firms $\{e_i\}_{i=0}^n$. By solving the maximization problem of the utility in (1) with respect to c_i ($i = 1, 2, \ldots, n$) under the budget constraint (2) and the availability constraint (3), we obtain the demand function of household h as follows:

$$c_i = \frac{e_i R}{p_c} \cdot \left(\frac{p_c}{p_i}\right)^{\theta}, \text{ where } p_c \equiv \left(\sum_{j=1}^{n} e_i p_j^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{4}$$

Note that p_c is the consumer price index (CPI).

2.2.2 Intermediate demand for each product

The demand function for each product as an intermediate good is derived by solving the cost-minimization problem of firms given their production levels. The problem for firm i is:

$$\min_{\{x_j\}_{j=0}^n} \sum_{j=0}^{n} p_j x_{ij}, \quad \text{s.t., } x_i = \left(\sum_{j=0}^{n} w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \quad \text{and } (1-e_j)x_{ij} = 0,$$

where the second constraint is the availability constraint. The usual cost minimization gives the demand of firm i for good j:

$$x_{ij} = \left(\frac{p^i}{p_j}\right)^{\theta} e_j w_{ji} x_i, \text{ where } p^i \equiv \left(\sum_{j=0}^{n} e_j w_{ij} p_j^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{5}$$

for $j = 0, 1, \cdots, n$. We note that p^i is the average cost of producing good i because:

$$\sum_{j=0}^{n} p_j x_{ij} = p^i x_i. \tag{6}$$

p^i is understood as the input price index (IPI).

2.2.3 Profit maximization by each firm

Each firm sets its price so as to maximize its profit. We assume that each firm ignores the impact of its pricing strategy on the price indexes (CPI, IPI): thus, firm i maximizes profit under the assumption that $\partial p_c/(\partial p_i) = 0$ and $\partial p^j/(\partial p_i) = 0$. The profit-maximization problem for firm i is:

$$\max_{p_i} \quad (p_i - p^i)x_i,$$

under the demand function for firm i,

$$x_i = c_i + \sum_{j=1}^{n} x_{ji} = \frac{R}{p_c} \cdot \left(\frac{p_c}{p_i}\right)^{\theta} + \sum_{j=1}^{n} \left(\frac{p^j}{p_i}\right)^{\theta} e_j w_{ji} x_j. \tag{7}$$

The first and second terms in the demand function are the final demand (4) and the intermediate demand (5), respectively.

Solving the profit-maximization problem, we obtain the price level and the profit of firm i. The first-order condition provides the price level:

$$p_i = \frac{\theta}{\theta - 1} p^i, \tag{8}$$

for each i $(= 1, 2, \cdots, n)$ that is operating, $e_i = 1$. We use a convention that if $e_i = 0$, then $p_i = \infty$ yet $p_i e_i = 0$. The coefficient $\theta/(\theta - 1)$ is the mark-up. The real profit of firm i deflated by CPI is:

$$\pi_i = \frac{p_i x_i}{\theta p_c}. \tag{9}$$

2.2.4 Equilibrium outcomes of the product market

We now describe the equilibrium level of price, sales, and household income in the product market. These equilibrium outcomes are uniquely determined given $\{e_i\}_{i=1}^{n}$.

The equilibrium price level is obtained by solving the system of simultaneous equations characterized by (8) (raising both sides to the power of $1 - \theta$) and the definition of IPI. Note that the system is linear in $p_i^{1-\theta}$. Therefore, a simple matrix calculation gives:

$$\mathbf{p}_\theta = \left\{ \mathbf{I} - \left(\frac{\theta - 1}{\theta}\right)^{\theta - 1} \mathbf{EWE} \right\}^{-1} \mathbf{E} \mathbf{w_0} \left(\frac{\theta - 1}{\theta p_0}\right)^{\theta - 1}, \tag{10}$$

where $\mathbf{p}_\theta \equiv (p_1^{1-\theta}, p_2^{1-\theta}, \cdots, p_n^{1-\theta})'$, $\mathbf{I}$ is the $n \times n$ identity matrix, $\mathbf{w_0} \equiv (w_{10}, w_{20}, \cdots, w_{n0})'$, $\mathbf{W}$ is the $n \times n$ matrix whose (i, j) element is equal to w_{ij}.[7], and $\mathbf{E}$ is a diagonal matrix whose i-th diagonal element is e_i. The price level is thus uniquely determined.

Next, we derive sales in the equilibrium. The consumption level is determined by (4). Multiplying both sides of (7) by $p_i e_i$ gives the vector of total sales:

$$\mathbf{s} = \mathbf{f} + \mathbf{Qs}, \tag{11}$$

[7]Note that the matrix inverse in (10) is well defined because $((\theta - 1)/\theta)^{\theta - 1} < 1$ and the largest eigenvalue of $\mathbf{W}$ is less than 1 by the assumption that $0 < \sum_{j=0}^{n} w_{ij} \leq 1$.

where $\mathbf{s}$ (total sales) $\equiv (e_1p_1x_1, e_2p_2x_2, \cdots, e_np_nx_n)'$; $\mathbf{Q}$ is the $n \times n$ matrix whose (i, j) element is $q_{ij} \equiv e_i w_{ji} p_i^{1-\theta} {p^j}^{\theta} / p_j$; and $\mathbf{f}$ (sales to consumers) $\equiv (e_1p_1c_1, e_2p_2c_2, \cdots, e_np_nc_n)'$. By the assumptions on w_{ij} and the definition of p^i, the matrix $\mathbf{I} - \mathbf{Q}$ is invertible.[8] Therefore, the sales vector is uniquely determined and is written as:

$$\mathbf{s} = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{f} = \sum_{k=0}^{\infty} \mathbf{Q}^k \mathbf{f}. \tag{12}$$

We note that the sales vector $\mathbf{s}$ is conceptually similar to Bonacich centrality (Bonacich, 1987). Thus, equilibrium in the product market is uniquely determined according to the unique price vector (10). The price level determines sales by (4), and (12).

2.2.5 Financial market

Finally, we discuss the equilibrium of the financial market. The profit-maximization problem of the monopolistic bank is to determine which firms are to be financed. Because the profit of firm i in real terms is $x_i p_i / (\theta p_c)$ and the opportunity cost of refinancing for firm i is $(1 + \rho)F_i$, the bank's profit-maximization problem is:

$$\max_{\mathbf{e}} \ \Pi(\mathbf{e}) - (1 + \rho) \sum_{i=1}^{n} e_i F_i, \tag{13}$$

where

$$\Pi(\mathbf{e}) \equiv \sum_{i=1}^{n} e_i \pi_i(\mathbf{e}),$$

$\mathbf{e}$ is the $n \times 1$ vector of which the i-th element is e_i and $\pi_i(\mathbf{e})$ is the real profit for firm i defined in (9) under $\mathbf{e}$. We note that because the set of values that $\{e_i\}_{i=1}^n$ can take is finite, the bank's problem has a solution, although the solution is not guaranteed to be unique.

We discuss the necessary condition for the maximization of the monopolistic-bank profit. This condition is used later in the empirical test for the network-motivated forbearance lending. Suppose

[8] Let $\|\cdot\|_1$ denote the norm by Bowker (1947), defined as $\|\mathbf{Q}\|_1 \equiv \max_j \sum_{i=1}^n \|Q_{ij}\|$. Let λ be an eigenvalue of $\mathbf{Q}$. It is known that $\|\lambda\| \leq \|\mathbf{Q}\|$. $\|\mathbf{Q}\|_1 < 1$ holds because:

$$\begin{aligned} \sum_{i=1}^{n} q_{ij} &= e_i \left({p^j}^{1-\theta} - w_{j0} p_0^{1-\theta} \right) \frac{{p^j}^{\theta}}{p_j} \\ &= \frac{e_i p^j}{p_j} \left(1 - \frac{w_{j0} p_0^{1-\theta}}{\sum_{i=0}^{n} w_{ji} p_i^{1-\theta}} \right). \end{aligned}$$

The first term, which is the inverse of the mark-up rate, is smaller than one by (8). The second term is also smaller than 1. It follows that $\|\lambda\| \leq \|\mathbf{Q}\|_1 < 1$.

that $\mathbf{e}$ is the maximizer and firm i is financed so that $e_i = 1$. In this case, i's marginal contribution to the bank profit must exceed the cost of refinancing, i.e.:

$$\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-i}) \geq (1+\rho)F_i, \tag{14}$$

where $\mathbf{e}_{-i}$ is the vector whose elements are the same as those of $\mathbf{e}$ except that e_i is set equal to zero. At the maximum, all firms that obtain refinancing must satisfy the condition (14).

The above condition can characterize the profit-maximizing behavior of the monopolistic bank. However, it is a necessary condition and in general not a sufficient condition. Let

$$\hat{\mathbf{e}} = \mathcal{H}(\mathbf{e}), \tag{15}$$

where the i-th element of $\hat{\mathbf{e}}$ $(n \times 1)$ is given by

$$\hat{e}_i = \mathbb{1}\left[\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-i}) \geq (1+\rho)F_i\right].$$

We can prove that there exists a fixed point by applying Tarski's fixed point theorem (Theorem 1 in Tarski, 1955).

Proposition 1 *If $1 < \theta \leq 2$, there exists a fixed point for the recursive map (15). The maximizer of the monopolistic-bank profit is a fixed point.*

The proof is shown in Online Appendix 1.2. Proposition 1 assures that the maximum exists and is included in the set of fixed points in the recursive mapping (15). The supermodularity of the bank profit, which is shown in the proof of Lemma 7 in Online Appendix 1.2, ensures that those that are dropped in the firm-by-firm screening never revive in the set-by-set screening. Here "set-by-set" screening means that the profit comparison in the mapping (15) is conducted with respect to a set of multiple firms instead of each single firm. This result implies the necessity of being a fixed point. However, it is not sufficient. To see this point, consider two firms, i and j. It is possible that the following three inequalities are satisfied simultaneously:

$$\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-(i,j)}) < (1+\rho)(F_i + F_j),$$

$$\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-i}) \geq (1+\rho)F_i,$$

$$\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-j}) \geq (1+\rho)F_j.$$

The first inequality implies the set of firms i and j cannot obtain refinancing in the maximum, but the second and third inequalities imply these firms can be refinanced if they are evaluated separately. This example shows that the condition in Proposition 1 is not sufficient. Nonetheless, we note that the main implication of the model discussed in the next section depends only on the necessary condition presented in Proposition 1.

3 Network-motivated forbearance lending for an influential firm

Given the above equilibrium, we now discuss the main result of this paper: the possibility that a profit-maximizing bank may rationally undertake forbearance lending. We also show that firms with a strong influence on aggregate profit through the supply network are likely to be the target of forbearance lending. We first define the demand influence coefficients of firms.

3.1 Demand influence coefficients

The vector of the demand influence coefficients $\mathbf{v}$ is defined by:

$$\mathbf{v}' \equiv \mathbf{1}'(\mathbf{I}-\mathbf{Q})^{-\mathbf{1}} = \mathbf{1}'\sum_{k=0}^{\infty}\mathbf{Q}^k. \tag{16}$$

The i-th element of $\mathbf{v}$, denoted by v_i, is what we call the *demand influence coefficient* of firm i. It measures the centrality of a firm as a buyer. Note that our demand influence coefficient is different from the influence coefficient proposed by Acemoglu et al. (2012) because theirs represents the centrality of a firm as a supplier.

The demand influence coefficient represents the influence of firm i on aggregate profit. This observation comes from the fact that, by equation (9), the aggregate profit is θ^{-1} times

$$\mathbf{1}'\mathbf{s} = \mathbf{1}'(\mathbf{I}-\mathbf{Q})^{-\mathbf{1}}\mathbf{f} = \mathbf{v}'\mathbf{f}. \tag{17}$$

In words, the demand influence coefficient v_i indicates the magnitude of the influence on the total network sales of the change in the sales of firm i to households, i.e.:

$$v_i = \frac{\Delta\text{Total sales of the network}}{\Delta\text{Sales of firm } i \text{ to households.}}.$$

Higher values of v_i indicate that a negative shock to the final sales of firm i is more damaging to aggregate profit than a negative shock to firm j with $v_j < v_i$. It is important to recognize that the demand influence coefficient considers not only the first-order impact (i.e., the impact to the adjacent neighbors), but also all the higher-order impacts. In the summation in (16), $\mathbf{Q}$ captures the first-order impact, the second-order impact is determined by $\mathbf{Q}^2$, and so forth.

3.2 Network-motivated forbearance lending

We now discuss the possibility that the profit-maximizing monopolistic bank may undertake forbearance lending to a loss-making, but influential, firm. Our argument is based on externalities working through the supply network. The level of sales to households by a firm with a higher demand influence coefficient has a greater impact on the levels of sales of the other firms and, thereby, on the total profit of the network. The monopolistic bank can consider this a positive externality to maximize the total profit.

We first define forbearance lending in the context of our model.

Definition 1 (Forbearance lending) *We say a bank undertakes* ***forbearance lending*** *if it extends a loan to firm z even if that firm's real economic profit is negative; namely,*

$$e_z = 1 \text{ and } \pi_z(\mathbf{e}) - (1+\rho)F_z < 0. \tag{18}$$

As discussed in Section 2.2.5, the necessary condition for the monopolistic bank to extend a loan to firm z is the inequality (14) with respect to firm z. As

$$\Pi(\mathbf{e}) = \sum_{i=1}^{n} e_i \pi_i(\mathbf{e}) = \sum_{i=1}^{n} \frac{e_i v_i p_i c_i}{\theta p_c}$$

by equation (17), this necessary condition is expressed as follows.

$$\underbrace{\frac{v_z p_z c_z - p_z x_z}{\theta p_c}}_{\text{propagation effect of z}} \tag{19}$$

$$+ \underbrace{\sum_{i \neq z} e_i \left(\frac{v_i p_i c_i}{\theta p_c} - \frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} \right)}_{\text{changes in relative prices and influences of others}} \tag{20}$$

$$> \underbrace{-\left(\frac{p_z x_z}{\theta p_c} - (1+\rho) F_z \right)}_{\text{cost to support } z} > 0, \tag{21}$$

where the tildes indicate the values under $\mathbf{e_{-z}}$. The left-hand side of the inequality captures the externalities to the system-wide profit from keeping firm z open. The right-hand side (21) is the direct cost to keep firm z open.

We also observe that the interest rates for firms receiving forbearance loans must be less than the prime rate ρ and may even be negative (debt waiver). The contracted interest rate for firm z is

$$\frac{x_z p_z}{\theta p_c F_z} - 1. \tag{22}$$

Under condition (18), this interest rate is less than ρ. The cost of extending a loan to a loss-making firm at a rate less than the prime rate is covered by the higher interest payments from loans to other firms profiting by selling their products to the loss-making firm.

The following proposition summarizes the discussions above.[9]

Proposition 2 (Network-motivated forbearance lending) *Suppose that for firm z, $\pi_z(\mathbf{e}) - (1+\rho)F_z < 0$ holds. The monopolistic bank can maximize its profit by undertaking forbearance lending to firm z when the inequality consisting of (19), (20), and (21) holds. In that case, the interest rate of a loan to firm z is below the prime rate, and at least one firm faces an interest rate over the prime rate.*

We now examine the condition in (19)–(21) more closely to derive a testable implication. The term in (19) is the positive externality of firm z to the other firms. This term is always positive as long as at least two firms including firm z and another firm that supplies firm z operate at the optimum for the bank. The higher is the demand influence coefficient v_z of firm z at the optimum, the larger is the propagation effect to the sales and profits of the other firms, and so the more beneficial it is for the bank to support firm z. It thus suggests that firm z is more likely to be the target of forbearance lending if it is more "influential" than the others; i.e., when v_z is larger than v_i $(i \neq z)$.

However, the sign of the term (20) is ambiguous, which is the effect of firm z on the demand influence coefficients of the other firms, the relative price of their product, and the CPI. The effect on the demand influence coefficients is ambiguous. An additional operating firm reduces the prices

[9] A numerical example of this proposition is presented in Online Appendix 2.

of the products for which the additional good is used as an input because of the love-of-variety assumption of the production function. This effect also reduces CPI. The reduction in CPI increases the relative price of each product, and also increases the real income of households. A more precise explanation is given in the note after the proof of Proposition 3 in Online Appendix 1.3.

Nonetheless, we show that the more influence a firm has, the more likely it is a target for forbearance lending if the bank ignores the impact of its decision on the product prices. The proof is given in Online Appendix 1.3.

Proposition 3 *For a given set of operating firms $S = \{i|e_i = 1, i \neq j\}$, the monopolistic bank is more likely to provide a loan to firm z with a larger demand influence coefficient v_z if the bank assumes that its decision does not change product prices p_i $(i = 1, 2, \cdots, n)$.*

It is noteworthy that there could be the case where the directions of the inequalities in (21) are opposite. In this case, firm z is forced to exit despite being profitable.[10] That case is also interesting, but we focus on the case described in the above propositions, which are testable with our dataset.

3.3 Decentralized financial market

In this section, we examine the case in which the financial market is decentralized. We start the analysis of the decentralized financial market with the extreme case of many infinitesimal investors as the benchmark case. After that, we show the possibility that forbearance lending emerges in the decentralized financial market in the form of tacit collusion or coalition. We also conjecture that so-called relationship banking, which yields monopolistic power to the relational lender, generates the possibility for lenders to recoup the costs of forbearance lending.

3.3.1 Many infinitesimal investors

We assume that a large number of infinitesimal investors noncooperatively decide which firm they refinance. They can get a return on their share of return is equal to the share of funding. Each of them has a very small amount of available funds and cannot cooperate with the others. We

[10] This case is similar to the anticompetitive effect of common ownership (Azar et al., 2018). Our theory is clearly different from the hypothesis in that study because we focus on the common lender rather than the common owner.

maintain the assumption that available funds exceed funds demand in aggregate. Each investor invests in firm i if and only if:

$$\pi_i(\mathbf{e}) - (1+\rho)F_i \geq 0$$

for a given $\mathbf{e}$. In other words, they never undertake forbearance lending. Let us consider the following recursive map, such that:

$$\bar{\mathbf{e}} = \mathcal{G}(\mathbf{e}), \tag{23}$$

where the i-th element of $\bar{\mathbf{e}}$ $(n \times 1)$ is given by

$$\bar{e}_i = \mathbb{1}\left[\pi_i(\mathbf{e}) \geq (1+\rho)F_i\right].$$

Starting from a certain initial value of $\mathbf{e}$, the investors repeat this procedure until they obtain a fixed point $\mathbf{e}^\dagger = \mathcal{G}(\mathbf{e}^\dagger)$. We can show that such a fixed point exists by Tarski's fixed point theorem (Theorem 1 in Tarski, 1955). The detail of the proof is presented in Online Appendix 1.4.

Proposition 4 *If $1 < \theta \leq 2$, there exists a fixed point $\mathbf{e}^\dagger = \mathcal{G}(\mathbf{e}^\dagger)$ for the recursive map (23), i.e., a competitive equilibrium in the decentralized financial market. Forbearance lending never emerges in the equilibrium.*

3.3.2 Tacit coalition

When the financial market is oligopolistic, banks may be able to collude and engage in forbearance lending that is profitable and improves the profit of the network. Let us consider an arrangement where each of the competing banks holds a share of the optimal portfolio that a monopolistic bank would hold. If one of them declines to extend a loan to an influential but unprofitable firm z, this can lead to the closure of firm z and a resulting cascade of firm closures. If each bank's profit is larger in the former case than in the latter, then each bank has an incentive to keep the coalition intact.

3.3.3 Relational lending

The other possible situation in which rational forbearance lending is likely to emerge in an oligopolistic financial market is related to so-called relational lending. It is widely recognized that a bank can earn a quasi-rent from lending relationships, which eventually generates the information advantage over rival banks (Sharpe, 1990; Rajan, 1992), or the ability to provide relation-specific value-adding services by making use of this information advantage (Boot and Thakor, 2000). Many empirical studies provide evidence of this possibility in the financing of small businesses, which are presumably peripheral in the supply network described here (Degryse and van Cayseele, 2000; Ioannidou and Ongena, 2010). This implies that relationship banking enhances the ability of a bank to recoup the cost of a forbearance loan from peripheral borrowers. Thus, banks that maintain lending relationships with a large enough part of a supply network around an influential firm that receives a forbearance loan, and that expect higher excess returns from the relational lending to this part of the network, have a greater incentive to engage in network-motivated forbearance lending.

4 Hypothesis setting for the empirical study

The primary purpose of our empirical study is to test Proposition 3. Under the assumption in the proposition that the monopolistic bank or collusive banks ignore the impact of their lending decisions on product prices, the term (20) is zero among the three components of the condition for forbearance lending. Thus, the condition is simplified to:

$$\frac{v_z p_z c_z}{\theta p_c} \geq (1+\rho)F_z.$$

Dividing both sides by $(1+\rho)F_z/x_z$, and taking the logarithm gives the following expression.

$$\ln(v_z) + \ln\left(\frac{c_z}{x_z}\right) - \ln(p_c\theta) + \ln\left(\frac{p_z x_z}{(1+\rho)F_z}\right) \geq 0. \tag{24}$$

However, the condition for an infinitesimal investor to lend is

$$\pi_z \equiv \frac{p_z x_z}{\theta p_c} \geq (1+\rho)F_z.$$

Dividing both sides by $(1+\rho)F_z$ gives an expression similar to the previous one.

$$-\ln(p_c\theta) + \ln\left(\frac{p_z x_z}{(1+\rho)F_z}\right) \geq 0. \tag{25}$$

Equations (24) and (25) imply that the probability of firm z obtaining a loan is increasing in its demand influence coefficient, ceteris paribus, if the bank takes into account the network. Thus, our baseline test is to estimate the following linear probability model for distressed firms to obtain forbearance lending, which we will define based on our dataset later, and examine whether the demand influence coefficient $\ln v_i$ is positive.

$$\mathbb{1}\left[forbear_i\right] = \beta_0 + \beta_1 \ln v_i + \beta_2 \ln\left(\frac{p_i x_i}{(1+\rho)F_i}\right) + \beta_3' z_i + \iota_i + \epsilon_i, \tag{26}$$

where $\mathbb{1}[\cdot]$ is the indicator function, z_i is a vector of firm and main-bank characteristics, ι_i is the sectoral factor to control for the ratio of sales to households c_i/x_i, and ϵ_i is the error term.

Hypothesis 1 (Test for network-motivation)

$\beta_1 > 0$ *for distressed firms.*

5 Estimating the demand influence coefficient

In this section, we describe the estimation method for the demand influence coefficients with the datasets provided by the TSR. The dataset contains information about which firms are connected but does not provide the price levels of the products nor the amount of the trade between firms. We first estimate the importance of each connection using a spatial autoregressive model of the supply network. We then compute the demand influence coefficient of each firm by using the estimates from it.

5.1 TSR Corporate Relationship and Corporate Information Database

We use the Corporate Relationship Database (*TSR Kigyo Sokan File*) provided by the TSR (hereafter, we call this dataset the *TSR Relationship Data*) to estimate the demand influence coefficient of each firm. This database contains the names and IDs of important corporate clients and suppliers to companies, with up to 24 clients/suppliers recorded for each company, including those in the process of a bankruptcy.[11] The degree of each firm can be larger than 24 by making use of both sellers' lists and buyers' lists because a firm can be specified as an important buyer by more than 24 firms. The limitation of this dataset is that the actual amount of each interfirm transaction is not recorded; thus, we estimate the weight of each interfirm link using the sales information from the method described in the next section.

[11] We retain bankrupt companies if their sales are reported because they are still operating with the aim of revival.

The dataset is matched with the Corporate Information Database, which contains basic information from the firms' financial statements from the prior three years, including sales and profits, and other characteristics such as the names and IDs of the largest three lending banks,[12] the head office address, and four-digit industry classification, based on the Japanese Standard Industrial Classification (JSIC).

Our sample is comprised of firms listed in the TSR Corporate Relationship Database for which positive sales in Year and Year–1 are recorded. We use data for the years 2005, 2010, and 2013. After dropping observations whose latest accounting year is too far in the past,[13] we have 639,459, 766,327, and 733,749 observations for the years 2005, 2010, and 2013, respectively. In addition, the 2013 series is accompanied with the detailed financial information for every firms in the database. We make use of the purchasing expense information to control for sectoral difference in the intermediate input dependence in the estimation of the demand influence coefficient.

5.2 Spatial autoregressive model

We estimate the spatial autoregressive model (11). Note that while $\mathbf{s}$ (total sales) is in our dataset, $\mathbf{Q}$ and $\mathbf{f}$ (sales to households) are not. To eliminate the firm fixed effects, we use the differential version of (11):

$$\mathbf{\Delta s} = \hat{\mathbf{Q}}\mathbf{\Delta s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon, \tag{27}$$

where Δ is the difference operator for accounting periods, $\mathbf{Ind}$ are industry dummies (three-digit JSIC), $\mathbf{Pre}$ are prefecture dummies,[14], and γ are vectors of coefficients. $\mathbf{Ind}$ and $\mathbf{Pre}$ are proxies for the change in sales to households $\mathbf{\Delta f}$.

We approximate the unobservable $\hat{\mathbf{Q}}$ as follows:

$$\hat{\mathbf{Q}} = (\gamma_0 \mathbf{I} + \gamma_1 \mathbf{I_{mfg}} + \gamma_2 \mathbf{I_{ws}})\mathbf{GM}, \tag{28}$$

where γ are the coefficients to be estimated, $\mathbf{I_{mfg}}$ and $\mathbf{I_{ws}}$ ($n \times n$) are diagonal matrices, of which the i-th diagonal element is the manufacturing or wholesale sector dummy of the i-th firm, $\mathbf{G}$ is the

[12] Only the largest lender is available in the database for 2010.

[13] We retain those whose latest report is in the 12 months from October 2004 to September 2005 for the 2005 series, from January 2010 to December 2010 for the 2010 series, and May 2013 to April 2014 for the 2013 series.

[14] The industries are categorized according to the three-digit Japan Standard Industry Classification (401 sectors in the 2005 series, 419 sectors in other series). There are 47 prefectures in Japan.

adjacency matrix of a sales network where the (i, j) element is equal to 1 if firm i purchases from firm j or zero otherwise, and $\mathbf{M}$ $(n \times n)$ is a diagonal matrix, of which the i-th diagonal element is the sector (two-digit level) average of the ratio of purchasing expenses over total sales of the sector to which firm i belongs in the fiscal year ending in 2013. The coefficients γ_1 and γ_2 capture the average importance of the seller's products as inputs for buyers, which presumably varies with the seller's sector. We include only the manufacturing and the wholesale dummies.[15] $\mathbf{M}$ captures the extent of a buyer's dependence on intermediate goods. By postmultiplying this matrix, we can set more weight for a firm that depends more on intermediate inputs.

Under these assumptions, we have the following spatial autoregressive model:

$$\mathbf{\Delta s} = (\gamma_0 \mathbf{I} + \gamma_1 \mathbf{I_{mfg}} + \gamma_2 \mathbf{I_{ws}})\mathbf{GM\Delta s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon. \tag{29}$$

We estimate this model by ordinary least squares (OLS) for the years 2005, 2010, and 2013 using the TSR data.[16] Table 1 presents the results. The estimated γ_0, γ_1, and γ_2 are all positive and statistically significant, although the values are small.

5.3 Demand influence coefficient

In computing the demand influence coefficient from the above estimates, we construct the supply networks of borrowers of each bank and their direct customers and suppliers under the assumption that it is the range of a network that is observable for a bank. Each firm may obtain multiple v's because it participates in the networks to which its customers and suppliers belong. We take the demand influence coefficient in the network of the main bank of the firm in such a case. A unique main bank is identified as the bank with the highest outstanding amount of loans to a company.

Let $\mathbf{G}^{(b)}$ be the adjacency matrix of the supply network for bank b. Let $\mathbf{v}^{(b)}$ be the vector of demand influence coefficients in it. By following the definition (16), the demand influence coefficient for the supply network at bank b is estimated as follows:

$$\mathbf{v}^{(b)\prime} = \mathbf{m}' \sum_{k=0}^{100} \{(\hat{\gamma}_0 \mathbf{I} + \hat{\gamma}_1 \mathbf{I_{mfg}} + \hat{\gamma}_2 \mathbf{I_{ws}})\mathbf{G^{(b)}M}\}^k, \tag{30}$$

[15] In a preliminary analysis, all sector dummies are included but do not appear to influence the results.

[16] For spatial models, the OLS estimator is not guaranteed to be consistent (Kelejian and Prucha, 1998). The reason for this is that the regressor $\mathbf{GM\Delta s}$ and the error term ϵ are likely to be correlated by construction. However, in our case, the bias resulting from this correlation turns out to be negligible. Online Appendix 2 provides a more detailed discussion.

Table 1: Estimation results of the spatial autoregressive model

(Notes) Estimated by OLS. The prefecture factor is controlled by 46 prefecture dummies (Hokkaido is the base prefecture). The industrial factor is controlled by the 401 (1) or 419 (2,3) sector dummies, which indicate one of the small (three-digit) classifications of the JSIC, revised in 2002. The constant term and estimated coefficients of these dummies are not reported. The definitions of the manufacturing sector and wholesale sector are the same as in the notes of Table 4. The sector average of purchasing expenses/sales is calculated for each middle (two-digit) classification for 2013. The middle classification is identical for 2013 and 2010. The classification in 2005 is manually matched with that in 2013. The matching is listed in the notes for Table 4. For classes that are newly generated by splitting the existing class, we apply the observation-weighted average of such new classes in 2013 to a corresponding class in 2005 (e.g., sectors "72 and 74" in 2013 to sector "80" in the 2005 series, "76 and 77" in 2013 to "70" in the 2005 series). *** p<0.01, ** p<0.05, * p<0.1 in t-test (two-tail) for H0: coef = 0.

	(1) 2005 series		(2) 2010 series		(3) 2013 series	
γ_0	1.22E-03	***	2.55E-03	***	2.20E-03	***
	(2.20E-04)		(5.78E-04)		(2.92E-04)	
γ_1	3.84E-03	***	4.64E-03	***	8.10E-03	***
	(3.44E-04)		(7.78E-04)		(3.95E-04)	
γ_2	6.99E-03	***	7.01E-03	***	7.56E-03	***
	(3.83E-04)		(8.88E-04)		(4.55E-04)	
Adj. R^2	0.201		0.042		0.041	
#sector(3d)	401		419		419	
#pref.dum.	46		46		46	
N	639,459		766,327		733,749	

where $\mathbf{m}$ is the column vector whose i-th element is a dummy variable that equals one if bank b is the main bank for firm i. $\hat{\gamma}$ are the estimated coefficients in (29). Note that in the theoretical model that assumes a monopolistic bank, we sum up all the column elements to calculate the demand influence coefficient. Here, we sum only the elements that are relevant to bank b because the network at a bank includes not only firms that use the bank as the main bank but also their suppliers and buyers who have different main banks.

We estimate the parameters using the economy-wide supply network instead of the bank-level supply networks for two reasons. First, for some banks, it is difficult to obtain reliable estimates because of the lack of sample size. Second, and more importantly, it is more appropriate to use the information about the economy-wide supply network to estimate γ because the product market is not segmented by main banks.

Table 2: Descriptive statistics of the demand influence coefficient

Series	N	mean	sd	min	p1	p5	p10
2005	639,017	1.001	0.004	1.000	1.000	1.000	1.000
2010	764,706	1.001	0.004	1.000	1.000	1.000	1.000
2013	727,604	1.001	0.006	1.000	1.000	1.000	1.000
Series	p25	p50	p75	p90	p95	p99	max
2005	1.000	1.000	1.000	1.001	1.003	1.008	2.248
2010	1.000	1.000	1.000	1.001	1.003	1.009	2.145
2013	1.000	1.000	1.000	1.001	1.004	1.012	2.686

5.4 Descriptive statistics of the demand influence coefficient

Table 2 shows the descriptive statistics of the demand influence coefficients v, which are estimated by (30). The distribution of the demand influence coefficient is highly skewed to the left. Even at the 99th percentile, the factor is very close to 1. Most of the variation is concentrated in the top one percent. This is plausible given the fact that large companies, which are more likely to be a major procurer from multiple suppliers, account for less than 1% of the total number of companies in Japan.[17]

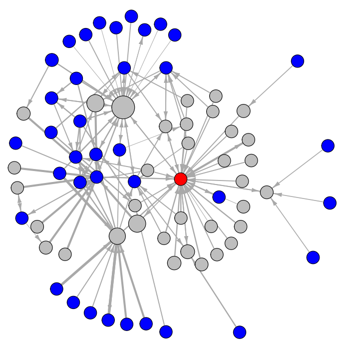
Another important feature of the estimated demand influence coefficient is the very low time-series variation. The correlation coefficients among the demand influence coefficients estimated for 2005, 2010, and 2013 are in the range of 0.767–0.905. This implies that models with firm fixed effects are not suitable for testing our hypothesis because the demand influence coefficients have little variation over time. Thus, we estimate the linear probability model (26) with the cross-section data for each year by controlling for firm characteristics as much as possible and also by including time and main bank fixed effects.

Figure 2 is an example of an actual network among borrowers of a bank and their direct suppliers and customers. The size of each node indicates the level of the demand influence coefficient v,

[17] *Establishment and Enterprise Census* (Ministry of Internal Affairs and Communications) in 2004 reports the number of small and medium-sized enterprises (SMEs) is 1,508,194, excluding sole proprietorship, in the nonagricultural sectors. The number of large enterprises is 11,793 in nonagricultural sectors. The latter accounts for only 0.8% of the total number of enterprises. An enterprise is classified as an SME if it has 300 full-time employees or fewer, or its capital is 300 million JPY (about three million USD) or less. These thresholds are adjusted for three sectors: 100 employees and 100 million JPY for the wholesale sector, 50 employees and 50 million JPY for the retail and restaurant sectors and 100 employees and 50 million JPY for the service sector.

Figure 2: Example of the network around a firm receiving a forbearance loan

(Notes) Plot the distance-2 neighbor reachable to a firm receiving a forbearance loan (2005 network, by igraph in R). The size of the nodes indicates the value of v_{2005}. The red node at the center is the forbearance lending firm ($v_{2005} = 1.0016$). The blue nodes are suppliers using the same main bank. The gray nodes are other suppliers.



estimated using the 2005 data. The red node at the center is the firm that received forbearance lending. The figure shows the distance-2 neighborhood reachable by this firm. The blue nodes are suppliers whose main bank is the same as the firm receiving a forbearance loan. The gray nodes are other suppliers. The firm receiving a forbearance loan is an important customer for many blue nodes directly and indirectly.

6 Data for the hypothesis test

In this section, we explain the dataset used for the hypothesis test and the definitions of key variables.

6.1 TSR Financial Information Database

We match the estimated demand influence coefficients with the detailed financial data. The financial data are collected from the TSR Financial Information Database (*TSR Zaimu Joho File*). Most firms in the dataset are privately-owned SMEs. We obtained an incomplete panel dataset of firms for each accounting year ending in the years from 2006 to 2016, which are chosen using stratified random sampling with respect to employment-size class, capital-size class, and industry classification as of

2000. We also obtained data on up to 10 banks that have a lending relationship with each firm in this detailed dataset.

Our sample focuses on firms whose main bank is a commercial bank, such as a major bank and a regional bank, or a cooperative bank.[18] For many firms whose main bank does not belong to these classes, a government-controlled bank is the largest lender. We drop these firms because the objective of these banks is not always maximization of their own profit. We also dropped firms in the following sectors: utilities; finance/insurance; postal services; cooperatives, political, economic, cultural, and religious organizations, and other public services.

6.2 Definition of distressed firms and forbearance lending

We define distressed firms and forbearance lending using detailed financial data. For the former, we follow the recent literature that examines zombie lending in the Japanese economy, which defines the distressed firms as those who "cannot pay minimum interest from its own cashflow, and heavily indebted" (Fukuda and Nakamura, 2011; Imai, 2016). The corresponding definition in our dataset for firm i in year t is:

$$\text{Distress}_{it} = \begin{cases} 1 & \text{if } \sum_{k=1}^{2}(EBIT_{it-k} - R^*_{it-k}) < 0, \\ & \quad \text{and } (\textit{external debt}_{it-1})/\textit{asset}_{it-1} \geq 0.5, \\ 0 & \text{otherwise,} \end{cases} \tag{31}$$

where $EBIT$ is the sum of operating income and depreciation, *external debt* is the sum of short-term and long-term loans from financial institutions and bonds, *asset* is total assets, and R^* is the hypothetical interest expense under the prime rate, which is defined as:

$$\begin{aligned} R^*_{it-k} \equiv & rs_{t-k} BS_{it-k-1} + \left(\frac{1}{5} \sum_{m=0}^{4} rl_{t-k-m} \right) BL_{it-k-1} \\ & + \min(rcb_{t-k-4}, \cdots, rcb_{t-k}) \cdot Bond_{it-k-1}, \end{aligned} \tag{32}$$

[18] Major banks include Mizuho, Mizuho Corporate, Mitsubishi UFJ, Sumitomo Mitsui, Risona, Saitama Risona, Shinsei, Aozora, Mitsubishi UFJ Trust, Chuo-Mitsui Trust, Sumitomo Trust, and Mizuho Trust. Regional banks include those belonging to the Regional Banks Association of Japan, or the Second Association of Regional Banks. Cooperative banks include Shinkin banks. We do not include Shinkumi banks, which are smaller than Shinkin banks, into our dataset because we lack their financial data and the number of firms whose main bank is Shinkumi is very small in our dataset.

where rs is the one-year TIBOR (Tokyo interbank offered rate),[19] rl is the long-term prime rate, rcb is the minimum coupon rate of bonds including convertible bonds in each year, BS and BL are the outstanding amounts of short-term and long-term loans from financial institutions, and $Bond$ is the outstanding amount of bonds including convertible bonds. In addition, if a firm is filing for bankruptcy, we treat it as a distressed firm, irrespective of the above conditions. We consider two-year spans for the interest coverage condition in the first line of (31) to address the higher volatility of the performance of SMEs. As for the long-term prime rate, we take the five-year average to take into account the average maturity of long-term loans.

We also follow the existing literature for the definition of forbearance or zombie lending. Namely, forbearance lending is the extension or refinancing of a loan at a rate less than the hypothetical prime rate despite the firm being distressed, i.e.:

$$forbear_{it} = \begin{cases} 1 & \text{if } external\ debt_{it} - (external\ debt_{it-1} - BS_{it-1} - BL1_{it-1}) > 0, \text{ and} \\ & \quad interest\ expense_{it} < R^*_{it}, \text{ and} \\ & \quad distress_{it} = 1, \\ 0 & \text{otherwise,} \end{cases} \tag{33}$$

where $BL1$ is the outstanding amount of long-term loans and bonds due in one year, and *interest expense* is the actual interest payment reported in the income statement. We subtract short-term loans due within a year to take into account the refinancing of the short-term loan (Fukuda and Nakamura, 2011).

6.3 Control variables for the hypothesis test

We collected firm and bank characteristics data, which are required as control variables for the estimation of the linear probability model (26), from the TSR Financial Information Database and the TSR Corporate Information Database. The first set of control variables describes firm characteristics. The second set characterizes the relationship with a main bank and the regional competitive environment. The third set describes the main-bank characteristics. We also include the sector and regional dummies.

The firm characteristics include $\ln(sale/R^*)$, i.e., the ratio of total sales to the interest cost

[19] Existing studies use the short-term prime rate, published by the Bank of Japan. We use the TIBOR because the short-term prime rate has not been updated since January 2009, and since August 2011 the TIBOR has been higher than the long-term prime rate, which is updated frequently. It is also reasonable to use the TIBOR because most Japanese banks use it as the reference rate in loan pricing.

calculated by the hypothetical prime rate, which is the proxy for the term $\ln(p_i x_i/(1+\rho)F)$ in the baseline model (26). The lending decision depends not only on the current performance but also on future prospects. We use the real growth rate of sales deflated by the CPI, $\ln(1+\Delta real\ sale)$, to control for this factor. In a similar vein, we include the ratio of tangible assets over total assets $\ln(1+collateral)$ to control for the amount of assets that can be used as collateral. To control for reputation as a safe company with a long history, we include firm age, $\ln(age)$. To control for the too-big-to-fail motivation, we include a group of size measures, such as the number of employees, $\ln(\#employee)$, and total asset $\ln(asset(t-1))$. We also include the ratio of interest-bearing external debts over total assets *leverage*, to control for the level of financial damage, which presumably affects the lending decision by banks.

The characteristics of bank–firm relationships are introduced to control for the factors related to relationship banking, especially flexible renegotiation (e.g., Chemmanur and Fulghieri, 1994) and implicit insurance (e.g., Bolton et al., 2016). The control variables for this purpose include a dummy variable indicating whether a firm has switched main banks from the previous year, *switch*, and the number of lending banks, *#bank*. The existing empirical studies show that the motivation for relationship banking depends on the number of competing banks or the degree of lending competition (e.g., Boot and Thakor, 2000; Dinç, 2000). We include the Herfindahl index of the banking sector, *hi*, to address this possibility. In addition, our theory suggests that a bank that is dominant in a local market is more likely to observe the full structure of the local network and obtain more benefits from sustaining the network. We include the branch share of the main bank in a local market, *share*, to control for this possibility.

The final set of control variables relate to bank characteristics. The existing studies show that larger banks are not competent in providing relationship banking because of the distance between the information producer and the decision authority (e.g., Stein, 2002; Berger et al., 2005). To control for this possibility, we include the book value of total assets of the main bank $ln(MB\ asset)$. In addition, to control for the risk-absorbing capacity of the main bank, we include the gross capital ratio, *MB cap ratio*, which is defined as the ratio of the book-value net assets to total assets. We also control unobservable bank characteristics by the main-bank fixed effect for several estimations with a larger sample.

Table 3: Ratio of distressed firms and firm receiving forbearance loans

(Note) Columns (i)–(iii) indicate the number of sample firms.

	(i)total	(ii)distress=1	(iii)forbear=1	%(ii)/(i)	%(iii)/(ii)	%(iii)/(i)
2008	8,183	801	81	9.8	10.1	1.0
2009	8,192	941	126	11.5	13.4	1.5
2010	8,880	1,200	194	13.5	16.1	2.2
2011	8,586	1,004	166	11.7	16.5	1.9
2012	8,577	907	125	10.6	13.8	1.5
2013	8,741	934	125	10.6	13.4	1.4
2014	8,804	836	96	9.5	11.5	1.1
2015	8,349	699	64	8.4	9.2	0.8
2016	7,265	524	38	7.3	7.4	0.5
Total	78,754	8,299	1,056	10.6	12.7	1.3

6.4 Data matching and screening

From the panel dataset collected from the TSR Financial Information Database, we identified and dropped outliers whose $sale(t-1)/R^*(t)$ or $\Delta real\ sale(t)$ were in the top or bottom 1% in each year, or whose $collateral(t-1)$ was in the top 1% in each year. We also dropped those whose main bank merged within each data window that we will define below to avoid matching the demand influence coefficient measured by the network before the bank merger.[20] We retained those that switched main banks. We replaced the demand influence coefficient with the one calculated for the network of the new main bank for these observations. The remaining observations are the baseline sample (78,754 firm × year).

Our dataset used to estimate the demand influence coefficients includes only three years: 2005, 2010, and 2013, which we denote by v_{2005}, v_{2010}, and v_{2013}. To match these data to the panel data of the financial information from 2008 to 2016, we split the financial panel data into three periods (1) 2008–2010, (2) 2011–2013, and (3) 2014–2016, and merged each of them with v_{2005}, v_{2010}, and v_{2013}, respectively after collapsing each of them into a cross-section dataset.[21] We assigned a lagged demand influence coefficient for each data window to focus on the effect of the demand influence

[20] We calculated the demand influence coefficient for the 2005 data after incorporating bank mergers up to December 2007. Likewise, mergers up to December 2009 and those up to April 2013 are reflected in the calculation of the demand influence coefficients using the 2010 data and using the 2013 data, respectively.

[21] We examined the robustness of the results by estimating a model using annual cross-sectional data for each year. The results are consistent with those presented here but less precise because of the smaller sample sizes.

coefficient on the forbearance lending decision. In the collapsed data, we set the dependent variable $\mathbb{1}[forbear]$ equal to one if the firm received a forbearance loan in any year within the window, or zero otherwise. As for the other variables, we retained the values from the first year when the firm became distressed in any subsequent window. We dropped firms that had never been distressed.

6.5 Characteristics of forbearance firms

In the baseline sample, around 10% of firms were distressed, as defined in Section 6.2 (Table 3). The peak of the ratio was at about 13.5% in 2010, following the start of the global financial crisis. Among the distressed firms, we found that about 7.3% to 16.5% firms received forbearance loans in each year. A peak appeared again in 2010 and 2011. The ratio of forms receiving forbearance loans to the total number of firms ranged from 0.5% to 2.2%. This ratio is lower than the ratio of Japanese zombie firms in the existing literature (Fukuda and Nakamura, 2011; Imai, 2016), which reports about 4–10% in the 2000s. The difference partly reflects the fact that our sample includes more small firms than other samples; 90% of our sample consists of SMEs, while almost no SMEs are included in Fukuda and Nakamura (2011), and 40% are SMEs in Imai (2016). Another cause is that we use the TIBOR instead of the short-term prime rate for the reason described in Footnote 18.

Table 4 reports the ratio of distressed firms in each sector. The distressed-firm ratio is high, especially in the construction sector. It is also somewhat higher in the real estate and retail sectors. Table 5 reports the ratio of firms receiving forbearance loans over the entire sample in each industrial sector. The ratio is higher in construction, services, and other sectors.

Table 6 reports the growth of assets, number of employees, and sales by firms receiving forbearance loans in comparison with those distressed but not receiving forbearance loans. In the year before the forbearance loan, the number of employees decreased to a lesser extent in firms that would eventually receive forbearance loans than in the others. The difference is statistically significant at the 5% significance level. However, we do not observe any significant difference between the firms in terms of reduction in sales or assets. In the year of the forbearance loans, there is no significant difference in the sales reduction, but the number of employees and assets decreased to a lesser extent in firms with forbearance loans than those without. The differences are statistically

Table 4: Ratio of distressed firms (%) by sector

(Notes) Ratio of firms that received a forbearance loan to total number of sample firms. The industrial classification is based on the TSR middle (two-digit) classification, which mostly corresponds to JSIC. JSIC 11th edition (March 2002) is applied before 2010. JSIC 12th edition (November 2007, and the corresponding revision of the TSR classification since January 2011) is after 2011. The definition of each large class is as follows (the definition in the 12th ed. is indicated in parentheses if it differs from the 11th ed.); Communication: 37–41, construction: 06–08, transportation: 42–48 (42–49), manufacturing: 09–32, real estate: 68–69, retail: 55–60 (56–61), wholesale: 49–54 (50–55), services and others: all remaining classes.

	2008	2009	2010	2011	2012	2013	2014	2015	2016
Communication	4.7	8.1	9.6	8.9	7.3	7.4	4.5	2.5	5.5
Construction	15.2	16.8	19.7	18.6	17.0	17.0	14.6	12.8	9.5
Transportation	10.3	9.5	11.6	10.9	8.5	8.0	8.2	7.8	7.3
Manufacturing	7.3	9.4	12.3	9.3	7.8	9.0	8.0	6.9	6.8
Real estate	13.1	13.7	12.4	7.2	10.1	7.7	6.7	4.9	4.9
Retail	10.9	11.8	12.8	10.8	12.0	11.1	7.7	7.5	6.2
Wholesale	7.7	9.1	10.7	9.3	8.4	8.4	8.4	8.0	7.3
Services and others	9.2	12.0	11.9	12.2	9.8	6.6	7.1	5.7	4.7

Table 5: Forbearance ratio (%) by sector

(Note) Ratio of firms that received a forbearance loan to the total number of sample firms.

	2008	2009	2010	2011	2012	2013	2014	2015	2016
Communication	0.5	0.5	1.3	0.5	1.0	1.4	0.5	1.0	0.5
Construction	1.3	2.3	2.9	3.1	2.0	2.1	1.8	1.1	1.0
Transportation	1.1	0.7	2.1	2.1	1.2	1.9	0.6	0.7	0.3
Manufacturing	0.6	1.1	1.7	1.2	1.1	1.1	0.8	0.6	0.3
Real estate	0.9	4.1	1.0	1.5	1.0	1.5	0.5	0.0	0.0
Retail	0.9	1.3	2.2	2.3	2.4	1.3	1.1	0.6	0.3
Wholesale	0.9	1.0	1.6	1.4	1.2	1.2	0.7	0.8	0.6
Services and others	1.8	2.7	4.3	3.2	1.1	0.9	1.5	0.8	0.3

Table 6: Growth of distressed firms with or without forbearance loans

(Note) Sample mean and standard errors among distressed firms without forbearance loans (i) and those with forbearance loans (ii) in the period from 2008 to 2016. *** p<0.01, ** p<0.05, * p<0.1 in t-test (two-tail) of mean difference between (i) and (ii).

	(i)Forbear(t)=0			(ii)Forbear(t)=1			
	N	mean	s.e.	N	mean	s.e.	
Δasset(t-1)/asset(t-2)	5,547	-2.13	0.43	816	-2.94	0.98	
Δ#employee(t-1)/#employee(t-2)	5,534	-1.77	0.24	813	0.08	1.16	**
Δreal sale(t-1)/real sale(t-2)	5,547	-3.88	0.38	816	-4.94	1.00	
Δasset(t)/asset(t-1)	6,892	-1.65	0.29	1,025	1.12	0.99	***
Δ#employee(t)/#employee(t-1)	6,872	-1.77	0.23	1,020	-0.31	1.02	**
Δreal sale(t)/real sale(t-1)	6,892	-2.21	0.27	1,025	-1.85	0.72	
Δasset(t+1)/asset(t)	5,101	-0.20	0.36	802	1.09	0.89	
Δ#employee(t+1)/#employee(t)	5,083	-1.37	0.28	797	-0.76	0.59	
Δreal sale(t+1)/real sale(t)	5,101	2.01	0.43	802	4.11	1.09	*

significant at the 1% significance level. The firms receiving forbearance loans undertake restructuring and downsizing to a lesser extent. In the year following the forbearance loans, sales growth is higher for firms with forbearance loans than those without. The difference is statistically significant at the 10% significance level, i.e., firms receiving forbearance loans recover quickly. Table 8 shows the descriptive statistics of the independent variables for each subsample split by whether or not a firm obtains a forbearance loan. The precise definition of each variable is given in Table 7.

The demand influence coefficient varies from 1 to about 1.2 in our dataset for the hypothesis test. The variation is mainly found above the 90th percentile as in the entire sample in Table 2. Employee size indicates that more than 90% of our sample distressed firms are SMEs under the Japanese standard classification where the cut-off point with respect to the number of employees is 300 workers in the manufacturing sector or 100 workers in the retail sector.

The right end of Panel (ii) in Table 8 indicates the statistical significance of the mean difference of each variable between the firms receiving forbearance loans and those distressed but not in receipt of a forbearance loan. Consistent with our theory, the demand influence coefficient in 2010 is significantly larger for the firms receiving forbearance loans. We also find those with a forbearance loan are significantly larger in terms of sales and assets. The comparison table also indicates that those more heavily indebted are more likely to be a target of a forbearance loan. These points

Table 7: Variable definition

Variable	Definition
sale/R^*	Net sales(t-1) / hypothetical interest payment at prime rate (t). Rates for the denominator are defined as follows. Short-term rate is the monthly average of the one-year TIBOR (Japanese Bankers Association). Long-term rate is the monthly average of the long-term prime rate (Bank of Japan). Bond rate is the monthly minimum of the coupon rate of JPY-denominated straight and convertible bonds issued by Japanese firms (Nikkei NEEDS).
sale	Net sales, mil. JPY.
asset	Total assets, mil. JPY.
age	Years since incorporation.
employee	Number of employees, persons.
leverage	External debt / asset. External debt is the sum of long-term and short-term loans and bonds.
collateral	Tangible asset / external debt. External debt is the same as the previous definition.
Δreal sale	Real growth rate of net sales from the previous year. Deflated by CPI (general, nationwide, 2005 basis).
switch	1 if the main bank switched since the previous year, 0 otherwise.
merge	1 if the main bank merged with another bank in year t, t-1, or t-2, 0 otherwise.
hi	Herfindahl index of bank branches in the telephone area code where a firm is located. Telephone area codes are as of May 2013 (No later changes are reported). Branch information as of October in each year is collected from CD-ROM of Nihon Kin'yu Meikan, published by Nihon Kin'yu Tsushinsha. Areas that are geographically distant but have the same code are treated as a separate market (area code = 055, 04, and 0980). The index includes both commercial banks and Shinkin banks if the stated capital of the firm is less than 900 mil. JPY. Shinkin banks are excluded otherwise. Headquarters and regular branches are included. Other types of branches, such as subbranches and internet branches, are excluded from the calculation.
share	Branch share of the main bank in the prefecture where the firm is located. The same set of branches as that for hi are used.
#bank	Number of banks that a firm borrows from.
MB asset	Total assets of the main bank, bil. JPY. Collected from nonconsolidated balance sheets from Nikkei NEEDS, and augmented by the database for unlisted banks on the Japanese Bankers Association website. https://www.zenginkyo.or.jp/abstract/stats/year2-02/.
MB cap ratio	Gross capital ratio (net assets)/(total assets) of the main bank. Collected from nonconsolidated balance sheets from Nikkei NEEDS. Augmented by the database on the above-mentioned JBA website for unlisted banks.

Table 8: Descriptive statistics

(Note) Descriptive statistics are calculated from distressed firms in 2008–2016. *** p<0.01, ** p<0.05, * p<0.1 in t-test (two-tail) of mean difference between (i) and (ii).

(i) forbear = 0

	N	mean	sd	min	p10	med	p90	max	
v_{2005}	6356	1.001	0.003	1.000	1.000	1.000	1.003	1.064	
v_{2010}	5950	1.001	0.004	1.000	1.000	1.000	1.004	1.109	
v_{2013}	5056	1.002	0.006	1.000	1.000	1.000	1.004	1.116	
sale/R^*	6831	292.1	1009.0	21.8	61.3	155.2	468.1	51272.8	
sale	6831	2393.8	41329.8	2.8	95.6	453.0	3063.1	2773390.0	
asset	6831	1661.5	21725.8	0.7	53.7	363.9	2578.2	1289082.0	
age	6816	41.9	17.2	3.5	19.3	41.8	64.4	134.0	
employee	6818	46.6	214.1	1	5	17	92	14544	
leverage	6831	1.208	1.089	0.361	0.777	1.013	1.635	38.387	
collateral	6798	0.852	28.416	0.000	0.032	0.392	0.915	2341.514	
Δreal sale	6831	-0.022	0.223	-0.585	-0.290	-0.036	0.256	0.958	
switch	6831	0.027	0.162	0	0	0	0	1	
hi	6819	0.196	0.104	0.050	0.052	0.191	0.320	1.000	
share	6796	0.170	0.123	0.001	0.027	0.155	0.351	0.663	
#bank	6831	4.156	2.100	1	2	4	7	10	
MB asset	6831	22188.9	47174.9	57.1	498.0	3587.8	107478.2	200261.9	
MB cap ratio	6831	0.051	0.016	-0.060	0.033	0.050	0.071	0.209	

(ii) forbear = 1

	N	mean	sd	min	p10	med	p90	max	
v_{2005}	944	1.001	0.005	1.000	1.000	1.000	1.004	1.120	
v_{2010}	899	1.002	0.007	1.000	1.000	1.000	1.004	1.179	**
v_{2013}	758	1.002	0.007	1.000	1.000	1.000	1.005	1.146	
sale/R^*	1015	260.3	605.6	22.0	38.2	138.1	493.4	13304.3	
sale	1015	4692.7	26112.7	5.0	38.0	385.4	7671.1	667990.0	*
asset	1015	7069.3	36774.5	1.1	22.4	337.8	7568.2	504656.0	***
age	1010	40.9	16.4	12.0	19.8	39.9	63.1	102.7	*
employee	1013	56.3	150.2	1	3	15	132	3200	
leverage	1015	1.338	1.190	0.493	0.726	0.985	2.126	14.666	***
collateral	1015	0.433	0.600	0.000	0.004	0.287	0.961	11.898	
Δreal sale	1015	-0.019	0.231	-0.594	-0.301	-0.026	0.265	0.914	
switch	1015	0.036	0.188	0	0	0	0	1	*
hi	1014	0.178	0.098	0.050	0.051	0.171	0.309	0.556	***
share	1010	0.155	0.124	0.001	0.016	0.110	0.348	0.663	***
#bank	1015	3.988	2.203	1	2	4	7	10	**
MB asset	1015	27915.1	50400.2	60.0	576.7	4584.3	125910.0	200261.9	***
MB cap ratio	1015	0.049	0.016	-0.060	0.032	0.048	0.068	0.227	***

suggest the importance of controlling for size and leverage to identify the too-connected-to-fail from the too-big-to-fail firms.

The rows for *hi* and *share* also indicate that a forbearance loan is more likely in a slightly more competitive market. The rows *MB size* and *MB cap ratio* indicate that larger and more financially sound banks are more likely to extend forbearance loans.

7 Results

Our baseline results of the OLS estimation (26) with the time-aggregated dataset comprising the distressed firms are listed in Table 9. We estimate the model as the within estimate after subtracting the main-bank average from both sides to delete the main-bank fixed effect. The model includes the regional, sectoral, and first-distress-year dummies. Standard errors are clustered at the main-bank level.[22] The first column of Table 9 presents the results for the data window from 2008 to 2010, where the demand influence coefficient is estimated using the 2005 data. The second column shows the results for the window from 2011 to 2013, where the demand influence coefficient is estimated using the 2010 data. The third column shows the results for the window from 2014 to 2016, where the demand influence coefficient is estimated using the 2013 data.

The coefficient of $\ln(v)$ is positive in the first two columns for 2008–10 and 2011–2013, around the period of the global financial crisis. However, only the second column, following the crisis, is statistically significant at the 1% level. These results indicate that network-motivated forbearance lending is observed in the postcrisis period.

The estimated coefficient implies that a 0.4 percentage point increase (about one sigma) from one in the demand influence coefficient increases the probability of obtaining a forbearance loan by 1.4 percentage points in 2008–10 and 3.3 percentage points in 2011–2013. These results are economically significant because the sample mean of the forbearance lending probability is 12.7%.

The coefficient of $\ln(v)$ becomes very small and insignificant in the period from 2014 to 2016. The slow but steady economic recovery with extremely low interest rates in Japan after the introduction

[22] Our regression potentially suffers from the underestimation of the standard errors because of the error-in-variable problem as the demand influence coefficient is an estimate. However, we can show that this problem is negligible in our case, thanks to the very large sample size for estimation of the demand influence coefficient (See Online Appendix 3 for more details).

of the quantitative and qualitative easing by the Bank of Japan in April 2013 reduced the number of distressed or severely damaged firms that needed a forbearance loan.

Among the control variables, $sale/R^*$ has a negative and significant coefficient. This indicates that more severely damaged firms are more likely to be a target of forbearance lending. We can interpret the positive and significant coefficient of *leverage* likewise. The coefficient of *#banks* is negative and statistically significant in the window of 2011–2013. This result may reflect the difficulty of coordination among multiple lenders, as is predicted by Bolton and Scharfstein (1996). The negative coefficient of *MB cap ratio* indicates that those with a high capital ratio tend to avoid forbearance lending or have less need for such lending.

We also obtain consistent results from the analysis with the annual cross-section data, and the sample selection model to treat the determinants of the distress probability explicitly. These results are presented in Online Appendix 5.

8 Discussion

We discuss the interpretation of our analysis and its limitation from the perspective of the market structure of the lending market and the economic welfare.

8.1 Market structure in the lending market and forbearance lending

The above empirical finding about the positive impact of the demand influence coefficient on the forbearance probability has an interesting implication about the market structure of the lending market. We investigate the features of firms with higher demand influence coefficients by regressing the demand influence coefficients in each year on other variables in the corresponding year with samples that only include distressed firms. Table 10 lists the estimated coefficients for each year. The most interesting and notable coefficient is that of the branch share (share) of the main bank and the Herfindahl index (hi). These coefficients are consistently positive and statistically significant for all years. This shows that the demand influence coefficient is higher for firms whose main bank has a large market share or for firms that are located in a more concentrated lending market. It is a plausible result because it is easier for a bank to cover the entire local network when it is a dominant lender than otherwise, and in a concentrated market than in a competitive market. The

Table 9: Baseline result with time-aggregated data

(Notes) Estimated by OLS with the main-bank fixed effect. The dependent variable is forbear (0,1). Time-aggregated data for each period are used. The sample includes firms that have been distressed (distress = 1) in any year during each period. Control variables are those as of the first year of the distress of each firm in each period. Column 1 uses v for 2005, column 2 for 2010, and column 3 for 2013. The year dummy is the dummy indicating the first year of distress of the firm (two dummies). The industry dummy is based on the two-digit classification. Region dummies include Tohoku, Kanto, Koshin'etsu, Tokai, Hokuriku, Kansai, Chugoku, Shikoku, Kyushu, (nine dummies, base: Hokkaido). The estimates of the constant term and the coefficients of these dummies are not reported. Main-bank-clustered standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

	(1)	(2)	(3)
	$v = v_{2005}$	v_{2010}	v_{2013}
VARIABLES	Period: 2008-10	2011-13	2014-16
ln(v)	3.613	8.248***	0.215
	(3.143)	(2.727)	(1.568)
ln(sale/R^*)	-0.026**	-0.036**	-0.005
	(0.013)	(0.014)	(0.019)
ln(1+Δ real sale)	0.004	0.012	-0.022
	(0.045)	(0.055)	(0.068)
ln(1 + collateral(t-1))	-0.032	-0.038	0.117*
	(0.037)	(0.049)	(0.070)
ln(age)	-0.013	0.011	0.007
	(0.023)	(0.025)	(0.036)
ln(#employee(t-1))	-0.019	-0.000	0.016
	(0.014)	(0.020)	(0.027)
ln(asset(t-1))	0.002	-0.014	-0.027
	(0.011)	(0.015)	(0.022)
leverage(t-1)	0.068**	0.024	0.019
	(0.027)	(0.016)	(0.016)
switch	0.024	0.205*	-0.008
	(0.056)	(0.123)	(0.101)
#bank	-0.008	-0.015**	-0.010
	(0.005)	(0.006)	(0.007)
share	-0.085	-0.223	-0.169
	(0.130)	(0.193)	(0.238)
hi	-0.027	-0.046	0.096
	(0.175)	(0.152)	(0.189)
ln(MB asset)	0.106	0.033	-0.864
	(0.474)	(0.283)	(0.770)
MB cap ratio	-3.489***	0.835	-2.419
	(0.966)	(5.732)	(12.282)
N	1,752	1,518	974
Adj.R^2	0.086	0.055	0.036
#group(banks)	251	243	208

network that a bank can observe tends to be fragmented when the bank is not a dominant one or the market is highly competitive. The demand influence coefficient tends to be lower for such fragmented networks.

This result is consistent with our theory that network-motivated forbearance lending emerges in the monopolistic banking case, whereas it does not in an infinitesimal investor case. It is also consistent with the conjecture and the empirical findings in the US by Giannetti and Saidi (2019) that a bank with a larger share in a certain sector is more willing to extend rescue loans when that sector is in distress.

8.2 Welfare implications

Regarding the welfare impact of forbearance lending, we can show that it improves welfare in our specific theoretical model. Social welfare in this case is the aggregate indirect utility of households minus the total cost of producing final products after netting out the intermediate inputs within the network. We can show that the total real profit of firms in the network equals the social welfare.

$$\frac{\sum_{i=1}^{n} e_i(p_i x_i - p^i x_i)}{p_c} - (1+\rho)\sum_{i=1}^{n} e_i F_i = \frac{R}{p_c} - \frac{p_0 \sum_{i=1}^{n} e_i x_{i0}}{p_c} - (1+\rho)\sum_{i=1}^{n} e_i F_i.$$

The first term on the right-hand side is consumer surplus, the second is the variable cost of production, and the third is the refinancing cost. Thus, the profit-maximizing bank behavior also maximizes social welfare in our setup.

However, we need to raise several caveats about the welfare implications because they rely on special features of our model. In particular, the following three features are relevant: 1) the network is fixed; 2) fund demand is fixed; and 3) the moral hazard of an influential firm is ignored.

First, our analysis is a short-run analysis where we assume away possible new links. We assume that the technological importance of inputs w_{ij}, which are the primary determinants of the demand influence coefficient of each firm, is fixed. We do not allow for entry from outside the given supply network. The analysis of Caballero et al. (2008) assumes the opposite extreme: that there are always potential entrants who are more efficient than the incumbent firms. That assumption is suitable for a long-run welfare analysis because the supply network will be flexible in the long run. Empirical studies are needed to clarify the rigidity of the supply network.[23] Nonetheless, we conjecture that

[23] The known empirical results on this point are mixed. For example, Fukao and Kwon (2006) and Nishimura et al.

Table 10: Determinants of the demand influence coefficient

(Notes) Estimated by OLS. The dependent variable is $ln(v)$ and column 1 shows the results for 2005, column 2 shows the results for 2010, and column 3 shows the results for 2013. The sample is from an accounting year ending in 2008 (column 1), 2009 (column 2), and 2014 (column 3), respectively. Estimates of the constant term and the coefficients of the prefecture dummies (46 dummies) are not presented. Main-bank-clustered standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

	(1)	(2)	(3)
	$ln(v_{2005})$	$ln(v_{2010})$	$ln(v_{2013})$
	fy = 2008	fy = 2011	fy = 2014
ln(sale/R^*)	0.0003***	0.0005***	0.0004
	(0.0001)	(0.0002)	(0.0003)
ln(1+Δ real sale)	-0.0011**	0.0001	-0.0016***
	(0.0004)	(0.0004)	(0.0006)
ln(1 + collateral(t-1))	-0.0012**	-0.0011*	-0.0019**
	(0.0006)	(0.0006)	(0.0008)
ln(age)	-0.0003	-0.0000	-0.0017*
	(0.0003)	(0.0003)	(0.0009)
ln(#employee(t-1))	0.0003	0.0006***	0.0004
	(0.0002)	(0.0002)	(0.0003)
ln(asset(t-1))	0.0023***	0.0025***	0.0046***
	(0.0006)	(0.0007)	(0.0013)
leverage(t-1)	0.0016***	0.0019***	0.0033***
	(0.0004)	(0.0004)	(0.0011)
switch	-0.0010***	-0.0017***	-0.0014
	(0.0003)	(0.0006)	(0.0009)
#bank	-0.0003*	-0.0003	-0.0005**
	(0.0002)	(0.0002)	(0.0002)
hi	0.0037***	0.0063***	0.0089***
	(0.0013)	(0.0015)	(0.0021)
share	0.0116***	0.0145***	0.0136***
	(0.0019)	(0.0022)	(0.0023)
ln(MB asset)	0.0002	0.0002	0.0004
	(0.0001)	(0.0001)	(0.0003)
MB cap ratio	-0.0071	0.0118	0.0308
	(0.0067)	(0.0119)	(0.0195)
communication	-0.0011***	-0.0018***	-0.0024***
	(0.0003)	(0.0006)	(0.0008)
construction	0.0012***	0.0009	0.0038***
	(0.0003)	(0.0006)	(0.0009)
transport	-0.0015***	-0.0030***	-0.0034***
	(0.0003)	(0.0004)	(0.0005)
manufacturing	0.0014***	0.0008**	0.0024***
	(0.0004)	(0.0004)	(0.0007)

(cont.)

Table 10: (cont.)

real estate	-0.0009	-0.0005	-0.0030***
	(0.0010)	(0.0011)	(0.0011)
retail	0.0068***	0.0074***	0.0117***
	(0.0012)	(0.0009)	(0.0016)
wholesale	0.0061***	0.0074***	0.0137***
	(0.0011)	(0.0013)	(0.0032)
prefecture dum.	yes	yes	yes
N	7,823	7,418	7,676
Adj. R^2	0.1533	0.2062	0.2140
#cluster (banks)	308	307	308

whether such efficient potential entrants exist depends on the type of industry and the economic environment at each point in time. For example, if an industry requires the accumulation of relation-specific information and design to improve productivity or product quality, which is known to occur, for example, in the automotive and financial sectors, then potential entrants are not likely to be more efficient than are incumbent firms. However, if an industry produces commodities that are less differentiated and do not require relation-specific investments, potential entrants could be more efficient than incumbent firms. Thus, the applicability of our results for welfare analysis will differ across economies and sectors.

Second, in our setting, the capital cost of external finance is not related to the output level. The usual route from monopolistic capital cost to welfare loss is through the reduction of output caused by high capital costs. However, we have shut down this channel of welfare loss by making sales independent of capital costs to make the analysis tractable. If this channel is considered, the ability of the bank to recoup the forbearance loan cost by imposing higher interest rates on loans to peripheral firms could be limited. This might reduce the likelihood of forbearance lending, and the welfare consequences of forbearance lending will be ambiguous under this scenario.

Third, we do not explicitly include the possible moral hazard for an influential company that

(2005) find that in Japan in the 1990s, less-efficient companies increased their market shares and were less likely to exit. In contrast, Sakai et al. (2010), using extensive small business lending microdata in Japan, find that those that exited were less profitable than those that survived in the 1990s. Calvalho et al. (2016) show that if a firm has suppliers or corporate customers that were damaged by a great earthquake, it will search and find a new supplier or a new customer. However, the economic significance of such adjustment in a circumstance without such an extreme shock could be smaller.

is considered too connected to fail. Such a company can expect a bailout by a bank or a government, and this assurance is likely to provide perverse incentives for managers and shareholders. Our estimation of the sample selection model shows that the demand influence coefficient has a positive correlation with the distress probability, although it is not statistically significant. This result suggests the possibility of such a moral hazard because of the network-motivated forbearance lending.

These points are beyond the scope of this paper but raise important questions for future research.

9 Concluding remarks

We have shown that a bank acting as the dominant financier to an interfirm network is motivated to strategically provide forbearance lending to an influential buyer. Our empirical study provides evidence for this motivation by statistically verifying that those with larger externalities, measured by a demand influence coefficient, are more likely to obtain forbearance lending in a situation of economic distress after controlling for the observable factors of firms, main banks, and firm–bank relationships. We also find that the demand influence coefficient tends to be larger in more concentrated lending markets, which implies those banks in concentrated markets are more willing to extend a forbearance loan.

Our findings suggest that the terms of a financial contract can be affected by the importance of the contracting firm relative to other firms within the loan portfolio of a bank. This implies that we need to look at the relative position of each firm within the bank's portfolio as well as the characteristics of individual firms and individual banks in examining the economic efficiency of loan contracts and the financial market. Evaluating the welfare implication of forbearance lending while taking into account both the influence of a hub company and the possibility of new entrants remains a challenging future research subject.

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For Online Publication
Appendix to "Network-motivated Lending Decisions: A Rationale for Forbearance Lending"

Yoshiaki Ogura,[a*] Ryo Okui,[b†] and Yukiko Umeno Saito[c‡]

[a]*School of Political Science and Economics, Waseda University,*
1-6-1 Nishi Waseda, Shinjuku-ku, Tokyo 169-8050, Japan
[b]*Department of Economics an the Institute of Economic Research, Seoul National University,*
1 Gwanak-ro, Gwanak-gu, Seoul 08826, Korea
[c]*School of Political Science and Economics, Waseda University*
1-6-1 Nishi Waseda, Shinjuku-ku, Tokyo 169-8050, Japan

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Abstract

This online appendix includes supplemental materials for "Network-motivated Lending Decisions: A Rationale for Forbearance Lending" by Ogura, Okui, and Saito. Online Appendix Online Appendix 1: contains the proofs of the propositions. Online Appendix Online Appendix 2: presents a numerical example for the theoretical model. Online Appendix Online Appendix 3: derives the bias property of the OLS estimator applied to the spatial autoregressive model. Online Appendix Online Appendix 4: presents the formula for standard errors for the OLS estimator, which are adjusted to address the problem that the demand influence coefficients are estimated regressors. Online Appendix Online Appendix 5: presents additional empirical results to examine the robustness of the empirical results in the main text.

[*]E-mail address: yogura@waseda.jp.
[†]E-mail address: okuiryo@snu.ac.kr.
[‡]E-mail address: saito-yukiko@waseda.jp.

Online Appendix 1: Proofs of propositions

A1.1 Full solution for the monopolistic-bank case and useful lemmas

The real profit of the monopolistic bank, i.e., the total profit from all operating firms in the entire network, before subtracting the costs of refinancing $\sum_{i\in S}(1+\rho)F_i$, is:

$$\Pi(\mathbf{e}) = \sum_{i\in S} \pi_i(\mathbf{e}) = \frac{\mathbf{v}'\mathbf{f}}{p_c\theta}, \tag{A.1}$$

where $\pi_i(\mathbf{e})$ is the profit of firm i before repayment of $(1+\rho)F_i$, and $\mathbf{e} \equiv (e_1, e_2, \cdots, e_n)'$.

The demand influence coefficient in the main text is:

$$\mathbf{v}' = \mathbf{1}' \sum_{k=0}^{\infty} \mathbf{Q}^k. \tag{A.2}$$

By substituting the f.o.c. of firm i, (10) in the main text into p^j in the definition of the (i,j)th element of $\mathbf{Q}$, $q_{ij} \equiv e_i w_{ji} p_i^{1-\theta} {p^j}^{\theta}/p_j$, we obtain the following expression.

$$\mathbf{Q} = \left(\frac{\theta-1}{\theta}\right)^{\theta} \mathbf{P}_\theta \mathbf{E}\mathbf{W}'\hat{\mathbf{P}}_\theta, \tag{A.3}$$

where $\mathbf{P}_\theta$ is an $n \times n$ diagonal matrix of which the ith diagonal element is $p_i^{1-\theta}$ (we assume that $p_i^{1-\theta} = 0$ if $e_i = 0$), $\mathbf{E}$ is an $n \times n$ diagonal matrix of which the i th diagonal element is e_i, i.e., $\mathbf{e} = \mathbf{E}\mathbf{1}$, and $\hat{\mathbf{P}}_\theta$ is an $n \times n$ diagonal matrix of which the ith diagonal element is $p_i^{\theta-1}$ if $e_i = 1$ or 0 otherwise. Note that $\hat{\mathbf{P}}_\theta \mathbf{P}_\theta = \mathbf{P}_\theta \hat{\mathbf{P}}_\theta = \mathbf{E}$. Substituting (A.3) into the expression (A.2) gives:

$$\mathbf{v}' = \mathbf{1}'\mathbf{P}_\theta \mathbf{A}^{-1}\hat{\mathbf{P}}_\theta, \text{ where } \mathbf{A} \equiv \mathbf{I} - \left(\frac{\theta-1}{\theta}\right)^{\theta} \mathbf{E}\mathbf{W}'. \tag{A.4}$$

The price vector is given by the following expression.

$$\mathbf{p}_\theta' \equiv \mathbf{1}'\mathbf{P}_\theta = \mathbf{w_0}'\mathbf{E}\mathbf{B}^{-1} \left(\frac{\theta-1}{\theta p_0}\right)^{\theta-1}, \tag{A.5}$$

$$\text{where } \mathbf{B} \equiv \mathbf{I} - \left(\frac{\theta-1}{\theta}\right)^{\theta-1} \mathbf{E}\mathbf{W}'\mathbf{E}. \tag{A.6}$$

The vector of the final demand for each firm $(n \times 1)$ is:

$$\mathbf{f} = Rp_c^{\theta-1}\mathbf{P}_\theta \mathbf{e}. \tag{A.7}$$

Substituting (A.4) and (A.7) into (A.1) gives the following expression.

$$\Pi(\mathbf{e}) = \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \mathbf{A}^{-1}\mathbf{e}. \tag{A.8}$$

p_c is expressed by:

$$p_c = (\mathbf{1}'\mathbf{P}_\theta \mathbf{1})^{\frac{1}{1-\theta}} = (\mathbf{w_0'}\mathbf{E}\mathbf{B}^{-1}\mathbf{1})^{\frac{1}{1-\theta}} \cdot \frac{p_0 \theta}{\theta - 1}, \tag{A.9}$$

We list useful results as lemmas. Let $\mathbf{E}_1$ be the $n \times n$ diagonal matrix whose i-th diagonal element is $e_i = 1$ for $i \in S$, $e_i = 0$ for $i \in S^c \backslash \{z\}$, and $e_z = 1$. Let $\mathbf{E}_0$ be the $n \times n$ diagonal matrix whose i-th diagonal element is $e_i = 1$ for $i \in S$, $e_i = 0$ for $i \in S^c \backslash \{z\}$, and $e_z = 1$. Let p_{ce_z} be the value of p_c under $\mathbf{E}_{e_z}$. Similarly, let $\mathbf{P}_{\theta e_z}$ and $\mathbf{p}_{\theta e_z}$ be the value of $\mathbf{P}_\theta$ and $\mathbf{p}_\theta$ under $\mathbf{E}_{e_z}$, respectively. First, we show that the price of each product and CPI are nonincreasing in the number of operating firms. This holds because of the love-of-variety assumption in our product function. A higher variety of inputs improves the productivity and reduces product prices.

Lemma 1 $p_i^{1-\theta}$ *($i = 1, 2, \cdots, n$) is nondecreasing in the number of operating firms. Namely, each product price is nonincreasing in the number of operating firms.*

Proof. From (A.5), we have:

$$\begin{aligned} \mathbf{p'_{\theta 1}} - \mathbf{p'_{\theta 0}} &\propto \mathbf{w'_0}(\mathbf{E_1}\mathbf{B_1^{-1}} - \mathbf{E_0}\mathbf{B_0^{-1}}) \\ &= \mathbf{w'_0}\{\mathbf{E_1}\mathbf{B_1^{-1}}(\mathbf{B_0} - \mathbf{B_1})\mathbf{B_0^{-1}} + (\mathbf{E_1} - \mathbf{E_0})\mathbf{B_0^{-1}}.\} \geq \mathbf{0} \end{aligned} \tag{A.10}$$

"$\geq \mathbf{0}$" indicates that every element of a matrix is nonnegative. This holds because all elements in $\mathbf{B_{e_z}}, \mathbf{B_{e_z}^{-1}}, \mathbf{E_1} - \mathbf{E_0}$ and $\mathbf{w_0}$ are nonnegative by definition. As

$$\begin{aligned} \mathbf{B_0} - \mathbf{B_1} &\propto \mathbf{E_1}\mathbf{W}\mathbf{E_1} - \mathbf{E_0}\mathbf{W}\mathbf{E_0} \\ &= (\mathbf{E_1} - \mathbf{E_0})\mathbf{W}\mathbf{E_1} + \mathbf{E_0}\mathbf{W}(\mathbf{E_1} - \mathbf{E_0}) \geq \mathbf{0}, \end{aligned} \tag{A.11}$$

every element of the first term in the bracket is also nonnegative. Thus,

$$\mathbf{p'_{\theta 1}} \geq \mathbf{p'_{\theta 0}}. \qquad \square \tag{A.12}$$

Lemma 2 p_c *is nonincreasing in the number of operating firms.*

Proof. From (A.9),

$$
\begin{aligned}
p_{c1}^{1-\theta} - p_{c0}^{1-\theta} &\propto \mathbf{1}'(\mathbf{E_1}\mathbf{B_1^{-1}}\mathbf{E_1} - \mathbf{E_0}\mathbf{B_0^{-1}}\mathbf{E_0})\mathbf{w_0} \\
&= \mathbf{1}'\{(\mathbf{E_1} - \mathbf{E_0})\mathbf{B_1^{-1}}\mathbf{E_1} + \mathbf{E_0}\mathbf{B_1^{-1}}(\mathbf{B_0} - \mathbf{B_1})\mathbf{B_0^{-1}}\mathbf{E_1} + \mathbf{E_0}\mathbf{B_0^{-1}}(\mathbf{E_1} - \mathbf{E_0})\}\mathbf{w_0} \\
&> 0. \qquad \text{(A.13)}
\end{aligned}
$$

The last inequality comes from the fact that all elements in $\mathbf{B_{e_z}}, \mathbf{E_1} - \mathbf{E_0}$ and $\mathbf{w_0}$ are nonnegative by definition, and all elements in $\mathbf{B_0} - \mathbf{B_1}$ are nonnegative by (A.11). This implies $p_{c1} < p_{c0}$ because $\theta > 1$. □

Lemma 3 *$v_i p_i^{1-\theta}$ $(i = 1, 2, \cdots, n)$, i.e., each element of the vector $\mathbf{1}'\mathbf{P}_\theta\mathbf{A}^{-1}\mathbf{E}$ is nondecreasing in the number of operating firms.*

Proof.

$$
\begin{aligned}
\mathbf{1}'\mathbf{P}_{\theta 1}\mathbf{A_1^{-1}}\mathbf{E_1} - \mathbf{1}'\mathbf{P}_{\theta 0}\mathbf{A_0^{-1}}\mathbf{E_0} =& \mathbf{1}'(\mathbf{P}_{\theta 1} - \mathbf{P}_{\theta 0})\mathbf{A_1^{-1}}\mathbf{E_1} \\
&+ \mathbf{1}'\mathbf{P}_{\theta 0}\{\mathbf{A_1^{-1}}(\mathbf{A_0} - \mathbf{A_1})\mathbf{A_0^{-1}}\mathbf{E_1} + \mathbf{A_0^{-1}}(\mathbf{E_1} - \mathbf{E_0})\}, \qquad \text{(A.14)}
\end{aligned}
$$

where

$$
\mathbf{A_0} - \mathbf{A_1} = \left(\frac{\theta - 1}{\theta}\right)^{\theta} (\mathbf{E_1} - \mathbf{E_0})\mathbf{W}' \geq \mathbf{0}. \qquad \text{(A.15)}
$$

Every element of the first term is nonnegative by Lemma 1 and because every element of $\mathbf{A_{e_z}}, \mathbf{A_{e_z}^{-1}}$ and $\mathbf{P}_{\theta \mathbf{e_z}}$ is nonnegative. Every element of the second term is also nonnegative because all elements of $\mathbf{E_1} - \mathbf{E_0}, \mathbf{P}_{\theta \mathbf{e_z}}$, $\mathbf{A_{e_z}}$, and $\mathbf{A_{e_z}^{-1}}$ are nonnegative and $\theta > 1$. □

Lemma 4 *The profit before repayment of an operating firm i, $\pi_i(\mathbf{e})$ $(i = 1, 2, \cdots, n)$, is nondecreasing in the number of operating firms if $1 < \theta \leq 2$.*

Proof. From (A.8), the row vector $\pi(\mathbf{e})$ of which the i th element is $\pi_i(\mathbf{e})$ $(i = 1, 2, \cdots, n)$ is given by:

$$
\pi(\mathbf{e}) = \frac{R p_c^{\theta-2}}{\theta}\mathbf{E}\mathbf{A}'^{-1}\mathbf{p}_\theta. \qquad \text{(A.16)}
$$

Let us denote $\mathbf{E_0 1}$ and $\mathbf{E_1 1}$ by $\mathbf{e_0}$ and $\mathbf{e_1}$, respectively. It is sufficient to show that every element of the following vector is nonnegative.

$$\begin{aligned}\pi(\mathbf{e_1}) - \pi(\mathbf{e_0}) =& \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{E_1 A_1'^{-1} p_{\theta 1}} - \frac{Rp_{c0}^{\theta-2}}{\theta}\mathbf{E_0 A_0'^{-1} p_{\theta 0}} \\ =& \frac{Rp_{c1}^{\theta-2}}{\theta}\{\mathbf{E_1 A_1'^{-1} p_{\theta 1}} - \mathbf{E_0 A_0'^{-1} p_{\theta 0}}\}, \qquad \text{(A.17)} \\ &+ \frac{R}{\theta}(p_{c1}^{\theta-2} - p_{c0}^{\theta-2})\mathbf{E_0 A_0'^{-1} p_{\theta 0}}. \qquad \text{(A.18)}\end{aligned}$$

Every element of the first term (A.17) is nonnegative by Lemma 3. The second term is positive if $1 < \theta < 2$, zero if $\theta = 2$, and negative if $\theta > 2$ by Lemma 2. Thus, both terms are nonnegative if $1 < \theta \leq 2$. □

Lemma 5 *The monopolistic-bank profit before subtracting the costs of refinancing,* $\Pi(\mathbf{e})$*, is increasing in the number of operating firms if* $1 < \theta \leq 2$.

Proof. By the definition (A.1) and Lemma 4, $\Pi(\mathbf{e})$ is increasing in the number of operating firms if $1 < \theta \leq 2$. □

Lemma 6 *The set of* $\mathbf{e}$ *equipped with a partial order, where the order for each pair of* $\mathbf{e}$ *and* $\hat{\mathbf{e}} \in \{0,1\}^n$ *is defined by* $\mathbf{e} \geq \hat{\mathbf{e}}$ *if* $e_i \geq \hat{e}_i$ *for any* $i = 1, 2, \cdots, n$*, is a complete lattice.*

Proof. Let us define the lattice operations for $\mathbf{e}, \hat{\mathbf{e}} \in \{0,1\}^n$,

$$\mathbf{e} \vee \hat{\mathbf{e}} \equiv (\max[e_1, \hat{e}_1], \max[e_2, \hat{e}_2], \cdots, \max[e_n, \hat{e}_n]),$$

$$\mathbf{e} \wedge \hat{\mathbf{e}} \equiv (\min[e_1, \hat{e}_1], \min[e_2, \hat{e}_2], \cdots, \min[e_n, \hat{e}_n]).$$

There exist $\mathbf{e} \vee \hat{\mathbf{e}}$ and $\mathbf{e} \wedge \hat{\mathbf{e}} \in \{0,1\}^n$ for any pair of $\mathbf{e}$ and $\hat{\mathbf{e}} \in \{0,1\}^n$. Thus, the set of $\mathbf{e}$ with the partial order is a lattice. Moreover, it is a complete lattice because there exist the supremum $\vee\mathcal{S}$ and the infimum $\wedge\mathcal{S}$ for any subset $\mathcal{S} \subseteq \{0,1\}^n$. □

A1.2 Existence of the solution for profit maximization by the monopolistic bank.

Let us consider the following recursive algorithm to find a local maximum of the monopolistic-bank profit. We start with an initial value of $\mathbf{e} = \mathbf{1}$. The bank decides to refinance $(1+\rho)F_i$ for an operating firm i ($e_i = 1$) if its marginal contribution to the bank profit exceeds the refinancing cost, i.e.,

$$\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-i}) \geq (1+\rho)F_i, \tag{A.19}$$

where $\mathbf{e}_{-i}$ is the vector whose element is the same as the initial value $\mathbf{e}$ except that the i th element e_i is set at zero. The bank sets $e_i = 1$ if this inequality holds or 0 otherwise. The bank conducts this firm-by-firm evaluation. We denote the resulting updated vector by $\hat{\mathbf{e}}$. The bank repeats this procedure. We can express this procedure by the following recursive map:

$$\hat{\mathbf{e}} = \mathcal{H}(\mathbf{e}), \tag{A.20}$$

where the i-th element of $\hat{\mathbf{e}}$ ($n \times 1$) is given by:

$$\hat{e}_i = \mathbb{1}\left[\Pi(\mathbf{e}) - \Pi(\mathbf{e}_{-\mathbf{1}}) - (1+\rho)F_i \geq 0\right]. \tag{A.21}$$

The fixed point of this recursive form is a local maximum. The next lemma is a required component for establishing the existence of such a fixed point.

Lemma 7 *The profit difference* $\Pi(\mathbf{e_1}) - \Pi(\mathbf{e_0})$ *resulting from switching* e_i *(*$i = 1, 2, \cdots, n$*) from 0 to 1 is nondecreasing in* e_j *(*$j \neq i$*) if* $1 < \theta \leq 2$*.*

Proof. Let $\mathbf{E_{11}}$ and $\mathbf{E_{01}}$ be $\mathbf{E_1}$ and $\mathbf{E_0}$ when $y \in S$, $y \neq z$, respectively. Likewise, let $\mathbf{E_{10}}$ and $\mathbf{E_{00}}$ be $\mathbf{E_1}$ and $\mathbf{E_0}$ when $y \in S^c$, $y \neq z$, respectively. We denote $\mathbf{e_{11}} = \mathbf{E_{11}1}, \mathbf{e_{01}} = \mathbf{E_{01}1}, \mathbf{e_{10}} = \mathbf{E_{10}1}$, and $\mathbf{e_{00}} = \mathbf{E_{00}1}$. We denote the price matrix $\mathbf{P}_\theta$ and the consumer price index p_c corresponding to each of the four cases by the subscript 11 (firms z and y operate), 01 (firm z does not operate but firm y operates), 10 (firm z operates but firm y does not) and 00 (both do not operate), respectively. It is sufficient to show that the following difference is nonnegative.

$$
\begin{aligned}
&\Pi(\mathbf{e_{11}}) - \Pi(\mathbf{e_{01}}) - \{\Pi(\mathbf{e_{10}}) - \Pi(\mathbf{e_{00}})\} \\
&= \frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}\mathbf{A_{11}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}}) - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}\mathbf{A_{10}^{-1}}(\mathbf{e_{10}} - \mathbf{e_{00}}), && \text{(A.22)} \\
&+ \frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}(\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}})\mathbf{e_{01}} - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}(\mathbf{A_{10}^{-1}} - \mathbf{A_{00}^{-1}})\mathbf{e_{00}}, && \text{(A.23)} \\
&+ \frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{01}})\mathbf{A_{01}^{-1}}\mathbf{e_{01}} - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{10}} - \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{00}^{-1}}\mathbf{e_{00}}, && \text{(A.24)} \\
&+ \frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c01}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{01}}\mathbf{A_{01}^{-1}}\mathbf{e_{01}} - \frac{R}{\theta}(p_{c10}^{\theta-2} - p_{c00}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{00}}\mathbf{A_{00}^{-1}}\mathbf{e_{00}}. && \text{(A.25)}
\end{aligned}
$$

The term (A.22) can be expanded as follows:

$$
\begin{aligned}
&\frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}\mathbf{A_{11}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}}) - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}\mathbf{A_{10}^{-1}}(\mathbf{e_{10}} - \mathbf{e_{00}}) \\
&= \frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c10}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}\mathbf{A_{11}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}}) + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{10}})\mathbf{A_{11}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}}) \\
&+ \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}\mathbf{A_{11}^{-1}}(\mathbf{A_{10}} - \mathbf{A_{11}})\mathbf{A_{10}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}}) + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}\mathbf{A_{10}^{-1}}(\mathbf{e_{11}} - \mathbf{e_{01}} - \mathbf{e_{10}} + \mathbf{e_{00}}).
\end{aligned}
$$

The first term is nonnegative if $1 < \theta \leq 2$ by Lemma 2. The second term is also nonnegative by Lemma 1. The third term is also nonnegative by (A.15). The fourth term is zero. Thus, the term (A.22) is nonnegative.

The term (A.23) can be expanded as follows.

$$
\begin{aligned}
&\frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}(\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}})\mathbf{e_{01}} - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}(\mathbf{A_{10}^{-1}} - \mathbf{A_{00}^{-1}})\mathbf{e_{00}} \\
&= \frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c10}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{11}}(\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}})\mathbf{e_{01}} + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{10}})(\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}})\mathbf{e_{01}} \\
&+ \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}(\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}} - \mathbf{A_{10}^{-1}} + \mathbf{A_{00}^{-1}})\mathbf{e_{01}} + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta\mathbf{10}}(\mathbf{A_{10}^{-1}} - \mathbf{A_{00}^{-1}})(\mathbf{e_{01}} - \mathbf{e_{00}}).
\end{aligned}
$$

Each element of $\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}}$ is nonnegative by (A.15) because $\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}} = \mathbf{A_{11}^{-1}}(\mathbf{A_{01}} - \mathbf{A_{11}})\mathbf{A_{01}^{-1}}$. Likewise, every element of $\mathbf{A_{10}^{-1}} - \mathbf{A_{00}^{-1}}$ is nonnegative. Therefore, the last term is nonnegative. The first term is nonnegative by Lemma 2 if $1 < \theta \leq 2$. The second term is nonnegative by Lemma 1. The contents of the parentheses of the third term is:

$$
\mathbf{A_{11}^{-1}} - \mathbf{A_{01}^{-1}} - \mathbf{A_{10}^{-1}} + \mathbf{A_{00}^{-1}} = \mathbf{A_{11}^{-1}}(\mathbf{A_{01}} - \mathbf{A_{11}})\mathbf{A_{01}^{-1}} - \mathbf{A_{10}^{-1}}(\mathbf{A_{00}} - \mathbf{A_{10}})\mathbf{A_{00}^{-1}}.
$$

As

$$
\mathbf{A_{01}} - \mathbf{A_{11}} = \left(\frac{\theta-1}{\theta}\right)^{\theta}(\mathbf{E_{11}} - \mathbf{E_{10}})\mathbf{W}' = \left(\frac{\theta-1}{\theta}\right)^{\theta}(\mathbf{E_{10}} - \mathbf{E_{00}})\mathbf{W}' = \mathbf{A_{00}} - \mathbf{A_{10}} \equiv \mathbf{C},
$$

the previous expression is equal to:

$$\begin{aligned}&(\mathbf{A_{11}^{-1}} - \mathbf{A_{10}^{-1}})\mathbf{C}\mathbf{A_{01}^{-1}} + \mathbf{A_{01}^{-1}}\mathbf{C}(\mathbf{A_{01}^{-1}} - \mathbf{A_{00}^{-1}})\\&= \mathbf{A_{11}^{-1}}(\mathbf{A_{10}} - \mathbf{A_{11}})\mathbf{A_{10}^{-1}}\mathbf{C}\mathbf{A_{01}^{-1}} + \mathbf{A_{01}^{-1}}\mathbf{C}\mathbf{A_{01}^{-1}}(\mathbf{A_{00}} - \mathbf{A_{01}})\mathbf{A_{00}^{-1}}.\end{aligned}$$

Each element of this expression is nonnegative by (A.15). Thus, the third term is nonnegative. These observations establish that the term (A.23) is nonnegative.

The term (A.24) can be expanded as follows.

$$\begin{aligned}&\frac{Rp_{c11}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{01}})\mathbf{A_{01}^{-1}}\mathbf{e_{01}} - \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{10}} - \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{00}^{-1}}\mathbf{e_{00}}\\&= \frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c10}^{\theta-2})\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{01}})\mathbf{A_{01}^{-1}}\mathbf{e_{01}} + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{01}} - \mathbf{P}_{\theta\mathbf{10}} + \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{01}^{-1}}\mathbf{e_{01}}\\&+ \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{10}} - \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{01}^{-1}}(\mathbf{A_{00}} - \mathbf{A_{01}})\mathbf{A_{00}^{-1}}\mathbf{e_{01}} + \frac{Rp_{c10}^{\theta-2}}{\theta}\mathbf{1}'(\mathbf{P}_{\theta\mathbf{10}} - \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{00}^{-1}}(\mathbf{e_{01}} - \mathbf{e_{00}}).\end{aligned}$$

The first term is nonnegative if $1 < \theta \leq 2$ by Lemmas 1 and 2. The third term is also nonnegative by Lemmas 1 and (A.15). The fourth term is also positive by Lemma 1. The contents of the parentheses of the second term after substituting the definition (A.5) is:

$$\begin{aligned}&(\mathbf{P}_{\theta\mathbf{11}} - \mathbf{P}_{\theta\mathbf{01}} - \mathbf{P}_{\theta\mathbf{10}} + \mathbf{P}_{\theta\mathbf{00}})\mathbf{1}\\&\propto \Bigg[\mathbf{B_{11}^{-1}}(\mathbf{E_{11}} - \mathbf{E_{01}}) - \mathbf{B_{10}^{-1}}(\mathbf{E_{10}} - \mathbf{E_{00}}) + \left(\frac{\theta-1}{\theta}\right)^{\theta-1}\mathbf{B_{11}^{-1}}(\mathbf{E_{11}}\mathbf{W}\mathbf{E_{11}} - \mathbf{E_{01}}\mathbf{W}\mathbf{E_{01}})\mathbf{B_{01}^{-1}}\\&\quad + \left(\frac{\theta-1}{\theta}\right)^{\theta-1}\mathbf{B_{10}^{-1}}(\mathbf{E_{10}}\mathbf{W}\mathbf{E_{10}} - \mathbf{E_{00}}\mathbf{W}\mathbf{E_{00}})\mathbf{B_{00}^{-1}}\Bigg]\mathbf{w_0}\\&= \Big[(\mathbf{B_{11}^{-1}} - \mathbf{B_{10}^{-1}})(\mathbf{E_{11}} - \mathbf{E_{01}})\\&\quad + \left(\frac{\theta-1}{\theta}\right)^{\theta-1}(\mathbf{B_{11}^{-1}} - \mathbf{B_{10}^{-1}})(\mathbf{E_{11}}\mathbf{W}\mathbf{E_{11}} - \mathbf{E_{01}}\mathbf{W}\mathbf{E_{01}})\mathbf{B_{01}^{-1}}\\&\quad + \left(\frac{\theta-1}{\theta}\right)^{\theta-1}\mathbf{B_{10}^{-1}}(\mathbf{E_{11}}\mathbf{W}\mathbf{E_{11}} - \mathbf{E_{01}}\mathbf{W}\mathbf{E_{01}} - \mathbf{E_{10}}\mathbf{W}\mathbf{E_{10}} + \mathbf{E_{00}}\mathbf{W}\mathbf{E_{00}})\mathbf{B_{01}^{-1}}\\&\quad + \left(\frac{\theta-1}{\theta}\right)^{\theta-1}\mathbf{B_{10}^{-1}}(\mathbf{E_{10}}\mathbf{W}\mathbf{E_{10}} - \mathbf{E_{00}}\mathbf{W}\mathbf{E_{00}})(\mathbf{B_{01}^{-1}} - \mathbf{B_{00}^{-1}})\Big]\mathbf{w_0}.\end{aligned}$$

We make use of the fact that $\mathbf{E_{11}} - \mathbf{E_{01}} = \mathbf{E_{10}} - \mathbf{E_{00}}$ in the last equality. Every element of $\mathbf{B_{11}^{-1}} - \mathbf{B_{10}^{-1}} = \mathbf{B_{11}^{-1}}(\mathbf{B_{10}} - \mathbf{B_{11}})\mathbf{B_{10}^{-1}}$ in the first and second terms in the last expression is nonnegative by (A.11). Likewise, every element of $\mathbf{B_{01}^{-1}} - \mathbf{B_{00}^{-1}}$ in the last term is nonnegative. Every element of $\mathbf{E_{11}}\mathbf{W}\mathbf{E_{11}} - \mathbf{E_{01}}\mathbf{W}\mathbf{E_{01}}$ in the second term and $\mathbf{E_{10}}\mathbf{W}\mathbf{E_{10}} - \mathbf{E_{00}}\mathbf{W}\mathbf{E_{00}}$ in the last term is nonnegative by (A.11). As $\mathbf{E_{11}} - \mathbf{E_{01}} = \mathbf{E_{10}} - \mathbf{E_{00}}$, the contents of the parentheses in the third term can be rearranged as follows:

$$\mathbf{E_{11}WE_{11}} - \mathbf{E_{01}WE_{01}} - \mathbf{E_{10}WE_{10}} + \mathbf{E_{00}WE_{00}}$$
$$= (\mathbf{E_{11}} - \mathbf{E_{01}})\mathbf{W}(\mathbf{E_{11}} - \mathbf{E_{10}}) + (\mathbf{E_{01}} - \mathbf{E_{00}})\mathbf{W}(\mathbf{E_{11}} - \mathbf{E_{01}}).$$

Every element of the last expression is nonnegative. Thus, the second term of the expanded (A.24) is nonnegative. These observations establish that the expression (A.24) is nonnegative if $1 < \theta \leq 2$.

Lastly, the term (A.25) is expanded as follows.

$$\frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c01}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{01}}\mathbf{A_{01}^{-1}e_{01}} - \frac{R}{\theta}(p_{c10}^{\theta-2} - p_{c00}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{00}}\mathbf{A_{00}^{-1}e_{00}}$$
$$= \frac{R}{\theta}(p_{c11}^{\theta-2} - p_{c01}^{\theta-2} - p_{c10}^{\theta-2} + p_{c00}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{01}}\mathbf{A_{01}^{-1}e_{01}} \quad \text{(A.26)}$$
$$+ \frac{R}{\theta}(p_{c10}^{\theta-2} - p_{c00}^{\theta-2})\mathbf{1}'(\mathbf{P}_{\theta\mathbf{01}} - \mathbf{P}_{\theta\mathbf{00}})\mathbf{A_{01}^{-1}e_{01}} \quad \text{(A.27)}$$
$$+ \frac{R}{\theta}(p_{c10}^{\theta-2} - p_{c00}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{00}}(\mathbf{A_{01}^{-1}} - \mathbf{A_{00}^{-1}})\mathbf{e_{01}} \quad \text{(A.28)}$$
$$+ \frac{R}{\theta}(p_{c10}^{\theta-2} - p_{c00}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta\mathbf{00}}\mathbf{A_{00}^{-1}}(\mathbf{e_{01}} - \mathbf{e_{00}}). \quad \text{(A.29)}$$

Every element of the second (A.27), third (A.28) and fourth (A.29) terms is nonnegative if $1 < \theta \leq 2$ by Lemmas 1, 2, and (A.15). (Note $\mathbf{A_{01}^{-1}} - \mathbf{A_{00}^{-1}} = \mathbf{A_{01}^{-1}}(\mathbf{A_{00}} - \mathbf{A_{01}})\mathbf{A_{00}^{-1}}$).

To see the sign of the first term (A.26), we start by comparing $p_c^{1-\theta}$ in each case. From the definition of p_c (A.9) we have:

$$p_{c11}^{1-\theta} - p_{c01}^{1-\theta} = [\mathbf{1}'(\mathbf{E_{11}} - \mathbf{E_{01}})\mathbf{B_{11}^{-1}E_{11}} + \mathbf{1}'\mathbf{E_{01}B_{11}^{-1}}(\mathbf{B_{01}} - \mathbf{B_{11}})\mathbf{B_{01}^{-1}E_{11}} + \mathbf{1}'\mathbf{E_{01}B_{01}^{-1}}(\mathbf{E_{11}} - \mathbf{E_{01}})]\mathbf{w_0}.$$
$$p_{c10}^{1-\theta} - p_{c00}^{1-\theta} = [\mathbf{1}'(\mathbf{E_{10}} - \mathbf{E_{00}})\mathbf{B_{10}^{-1}E_{10}} + \mathbf{1}'\mathbf{E_{00}B_{10}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{10}})\mathbf{B_{00}^{-1}E_{10}} + \mathbf{1}'\mathbf{E_{00}B_{00}^{-1}}(\mathbf{E_{10}} - \mathbf{E_{00}})]\mathbf{w_0}.$$

By using the fact that $\mathbf{E_{11}} - \mathbf{E_{01}} = \mathbf{E_{10}} - \mathbf{E_{00}}$,

$$p_{c11}^{1-\theta} - p_{c01}^{1-\theta} - p_{c10}^{1-\theta} + p_{c00}^{1-\theta}$$
$$= \mathbf{1}'(\mathbf{E_{11}} - \mathbf{E_{01}})(\mathbf{B_{11}^{-1}E_{11}} - \mathbf{B_{10}^{-1}E_{10}})\mathbf{w_0}, \quad \text{(A.30)}$$
$$+ \{\mathbf{1}'\mathbf{E_{01}B_{11}^{-1}}(\mathbf{B_{01}} - \mathbf{B_{11}})\mathbf{B_{01}^{-1}E_{11}} - \mathbf{1}'\mathbf{E_{00}B_{10}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{10}})\mathbf{B_{00}^{-1}E_{10}}\}\mathbf{w_0}, \quad \text{(A.31)}$$
$$+ \mathbf{1}'(\mathbf{E_{01}B_{01}^{-1}} - \mathbf{E_{00}B_{00}^{-1}})(\mathbf{E_{11}} - \mathbf{E_{01}})\mathbf{w_0}. \quad \text{(A.32)}$$

The first term (A.30) is:

$$\mathbf{1}'(\mathbf{E_{11}} - \mathbf{E_{01}})\{\mathbf{B_{11}^{-1}}(\mathbf{B_{10}} - \mathbf{B_{11}})\mathbf{B_{10}^{-1}E_{11}} + \mathbf{B_{10}^{-1}}(\mathbf{E_{11}} - \mathbf{E_{10}})\}\mathbf{w_0} \geq 0.$$

The last inequality is from (A.11).

The contents of the parentheses of the second term (A.31) can be expanded into the following expression.

$$
\begin{aligned}
&\mathbf{1}'(\mathbf{E_{01}} - \mathbf{E_{00}})\mathbf{B_{11}^{-1}}(\mathbf{B_{01}} - \mathbf{B_{11}})\mathbf{B_{01}^{-1}}\mathbf{E_{11}} + \mathbf{1}'\mathbf{E_{00}}\mathbf{B_{11}^{-1}}(\mathbf{B_{10}} - \mathbf{B_{11}})\mathbf{B_{10}^{-1}}(\mathbf{B_{01}} - \mathbf{B_{11}})\mathbf{B_{01}^{-1}}\mathbf{E_{11}} \\
&+ \mathbf{1}'\mathbf{E_{00}}\mathbf{B_{10}^{-1}}(\mathbf{B_{01}} - \mathbf{B_{11}} - \mathbf{B_{00}} + \mathbf{B_{10}})\mathbf{B_{00}^{-1}}\mathbf{E_{10}} \\
&+ \mathbf{E_{00}}\mathbf{B_{10}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{10}})\mathbf{B_{01}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{01}})\mathbf{B_{00}^{-1}}\mathbf{E_{10}} + \mathbf{1}'\mathbf{E_{00}}\mathbf{B_{10}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{10}})\mathbf{B_{00}^{-1}}(\mathbf{E_{11}} - \mathbf{E_{10}}).
\end{aligned}
$$

Every element in the first and third lines in the above expression is nonnegative by (A.11). The contents of the parentheses in the second line are transformed into the following expression by using the fact that $\mathbf{E_{11}} - \mathbf{E_{01}} = \mathbf{E_{10}} - \mathbf{E_{00}}$,

$$
\left(\frac{\theta - 1}{\theta}\right)^{\theta - 1} (\mathbf{E_{11}} - \mathbf{E_{01}} + \mathbf{E_{01}} - \mathbf{E_{00}})\mathbf{W}(\mathbf{E_{11}} - \mathbf{E_{10}}).
$$

All elements of this expression are nonnegative. Thus, the second term (A.31) is nonnegative.

The third term (A.32) is expanded into the following expression.

$$
\mathbf{1}'\{(\mathbf{E_{01}} - \mathbf{E_{00}})\mathbf{B_{01}^{-1}} + \mathbf{E_{00}}\mathbf{B_{01}^{-1}}(\mathbf{B_{00}} - \mathbf{B_{01}})\mathbf{B_{00}^{-1}})\}(\mathbf{E_{11}} - \mathbf{E_{01}})\mathbf{w_0} \geq \mathbf{0}.
$$

The last inequality comes from (A.11).

As all the three lines (A.30), (A.31), and (A.32) are positive or nonnegative we have:

$$
p_{c11}^{1-\theta} - p_{c01}^{1-\theta} > p_{c10}^{1-\theta} - p_{c00}^{1-\theta}.
$$

We obtain the following expression by multiplying both sides by: $(p_{c10} + p_{c00})(p_{c11} + p_{c01})$.

$$
(p_{c11}^{2-\theta} - p_{c01}^{2-\theta})(p_{c10} + p_{c00}) > (p_{c10}^{2-\theta} - p_{c00}^{2-\theta})(p_{c11} + p_{c01}).
$$

We can rearrange this inequality into the following form by using Lemma 2.

$$
\frac{p_{c11}^{2-\theta} - p_{c01}^{2-\theta}}{p_{c10}^{2-\theta} - p_{c00}^{2-\theta}} < \frac{p_{c11} + p_{c01}}{p_{c10} + p_{c00}} < 1.
$$

Thus,

$$
p_{c11}^{2-\theta} - p_{c01}^{2-\theta} < p_{c10}^{2-\theta} - p_{c00}^{2-\theta}.
$$

Multiplying both sides by $(p_{c11}p_{c01}p_{c10}p_{c00})^{\theta-2}$ and rearranging gives:

$$p_{c11}^{\theta-2} - p_{c01}^{\theta-2} > \left(\frac{p_{c01}p_{c11}}{p_{c00}p_{c10}}\right)^{\theta-2} (p_{c10}^{\theta-2} - p_{c00}^{\theta-2}).$$

$\left(\frac{p_{c01}p_{c11}}{p_{c00}p_{c10}}\right)^{\theta-2}$ is larger than 1 by Lemma 2 if $1 < \theta \leq 2$. Therefore,

$$p_{c11}^{\theta-2} - p_{c01}^{\theta-2} - p_{c10}^{\theta-2} + p_{c00}^{\theta-2} > 0.$$

Thus, the first term (A.26) is positive if $1 < \theta \leq 2$. These observations establish that the term (A.25) is positive. □.

Proposition 1 *If $1 < \theta \leq 2$, there exists a fixed point for the recursive map (A.20), i.e., a maximum of the monopolistic-bank profit.*

Proof. The domain and region of the map (A.20) is defined on the complete lattice by Lemma 6. The map is monotonically increasing (order-preserving) by Lemma 7. Therefore, there exists a fixed point $\mathbf{e}^* = \mathcal{H}(\mathbf{e}^*)$ by Tarski's fixed point theorem (Theorem 1 in Tarski, 1955). □

A1.3 Comparative statics with respect to the demand influence coefficient

Proposition 3 *For a given set of operating firms $S = \{i | e_i = 1, i \neq j\}$, the monopolistic bank is more likely to provide a loan to firm z with a larger demand influence coefficient v_z if the bank assumes that its decision does not change product prices p_i $(i = 1, 2, \cdots, n)$.*

Proof.

$$\begin{aligned}
\Pi(\mathbf{e}_1) - \Pi(\mathbf{e}_0) =& \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 1}\mathbf{A}_1^{-1}\mathbf{e_1} - \frac{Rp_{c0}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 0}\mathbf{A}_0^{-1}\mathbf{e_0} \\
=& \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 1}\mathbf{A}_1^{-1}\left(\mathbf{e}_1 - \mathbf{e}_0\right) && \text{(A.33)} \\
&+ \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 1}\left(\mathbf{A}_1^{-1} - \mathbf{A}_0^{-1}\right)\mathbf{e}_0 && \text{(A.34)} \\
&+ \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\left(\mathbf{P}_{\theta 1} - \mathbf{P}_{\theta 0}\right)\mathbf{A}_0^{-1}\mathbf{e}_0 && \text{(A.35)} \\
&+ \frac{R}{\theta}(p_{c1}^{\theta-2} - p_{c0}^{\theta-2})\mathbf{1}'\mathbf{P}_{\theta 0}.\mathbf{A}_0^{-1}\mathbf{e}_0. && \text{(A.36)}
\end{aligned}$$

Under the assumptions about $p_{c1} = p_{c0}$, (A.36) is zero. (A.35) is also zero under the assumption that $p_{j1} = p_{j0}$ $(j \neq z)$, because the z th column of $\mathbf{A_0^{-1}e_0}$ is zero.

The second term (A.34) can be transformed as follows.

$$\begin{aligned}
&\frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 1}\left(\mathbf{A}_1^{-1}-\mathbf{A}_0^{-1}\right)\mathbf{e}_0 \\
&= \frac{Rp_{c1}^{\theta-2}}{\theta}\mathbf{1}'\mathbf{P}_{\theta 1}\left(\mathbf{A}_1^{-1}\mathbf{E}_1-\mathbf{A}_0^{-1}\mathbf{E}_0\right)\mathbf{e}_0 \\
&= \frac{Rp_{c1}^{\theta-2}}{\theta}\left(\mathbf{1}'\mathbf{P}_{\theta 1}\mathbf{A}_1^{-1}\mathbf{E}_1-\mathbf{1}'\mathbf{P}_{\theta 0}\mathbf{A}_0^{-1}\mathbf{E}_0\right)\mathbf{e}_0 \\
&= \frac{Rp_{c1}^{\theta-2}}{\theta}\left(\mathbf{v}_\mathbf{1}'\mathbf{P}_{\theta 1}-\mathbf{v}_\mathbf{0}'\mathbf{P}_{\theta 0}\right)\mathbf{e}_0 \\
&= \frac{Rp_{c1}^{\theta-2}}{\theta}\left(\mathbf{v}_\mathbf{1}'-\mathbf{v}_\mathbf{0}'\right)\mathbf{P}_{\theta 0}\mathbf{e}_0 \\
&= \frac{Rp_{c1}^{\theta-2}}{\theta}\sum_{j\neq z}(v_{j1}-v_{j0})p_{j0}^{1-\theta}.
\end{aligned}$$

We use the property that $\mathbf{E_1 e_0} = \mathbf{E_0 e_0} = \mathbf{e_0}$ in deriving the second line, and the property $\mathbf{P_{\theta 1} e_0} = \mathbf{P_{\theta 0} e_0}$ for the fifth line. The first term (A.33) can be transformed into the following expression.

$$\frac{Rp_{c1}^{\theta-2}v_{z1}p_{z1}^{1-\theta}}{\theta}.$$

Thus, under the assumptions about prices,

$$\Pi(\mathbf{e_1})-\Pi(\mathbf{e_0}) = \frac{Rp_{c1}^{\theta-2}}{\theta}\{v_{z1}p_{z1}^{1-\theta}+\sum_{j\neq z}(v_{j1}-v_{j0})p_{j0}^{1-\theta}\}.$$

For a given v_{j1}, v_{j0} for $j \neq z$, and p_{j1}, p_{j0} for all j, the marginal increase of the monopolistic-bank profit is increasing in v_z. Thus, the condition for the forbearance lending (A.19) is more likely to hold when v_z is higher for a given $(1+\rho)F_z$. $\square$

The first term (A.33) in the profit difference represents the propagation effect, or the positive externality of firm z's operation. The second term (A.34) is the effect of the change in $v_i p_i^{1-\theta}$ $(i \neq z)$. This part is nonnegative as shown in Lemma 3. The third term (A.35) captures the effect of the change in $p_i^{1-\theta}$. As the variety of inputs increases, the product price decreases because of the love-of-variety assumption in the production function. This has a positive effect on firm profits. The fourth term (A.36) captures the effect of the change in CPI. This increases the relative price of each product, while it also increases the real income of households. The former decreases the demand, while the latter increases it. If θ is sufficiently small, the latter effect exceeds the former, and so it increases firm profits.

A1.4 Equilibrium in the decentralized financial market of many infinitesimal investors

We assume that each infinitesimal investor in the decentralized financial market invests in firm i if and only if:

$$\pi_i(\mathbf{e}) - (1+\rho)F_i \geq 0 \tag{A.37}$$

for a given $\mathbf{e}$. Let us consider the following recursive map, such that:

$$\bar{\mathbf{e}} = \mathcal{G}(\mathbf{e}), \tag{A.38}$$

where the ith element of $\bar{\mathbf{e}}$ $(n \times 1)$ is given by

$$\bar{e}_i = \mathbb{1}\left[\pi_i(\mathbf{e}) - (1+\rho)F_i \geq 0\right]. \tag{A.39}$$

Proposition 4 *If $1 < \theta \leq 2$, there exists a fixed point $\mathbf{e}^\dagger = \mathcal{G}(\mathbf{e}^\dagger)$ for the recursive map (A.38), i.e., a competitive equilibrium in the decentralized financial market.*

Proof. The domain and region of the map (A.38) is defined on the complete lattice by Lemma 6. The map is monotonically increasing (order-preserving) by Lemma 4. Therefore, there exists a fixed point $\mathbf{e}^\dagger = \mathcal{G}(\mathbf{e}^\dagger)$ by Tarski's fixed point theorem (Theorem 1 in Tarski, 1955). □

Online Appendix 2: Numerical example

We present a numerical example to illustrate Proposition 2. In the example, there are four firms that supply inputs to each other in a network similar to Figure 1.

The first example illustrates network-motivated forbearance lending. We set

$$\mathbf{W} = \begin{pmatrix} 0 & .3 & .3 & .3 \\ .1 & 0 & .1 & 0 \\ .1 & 0 & 0 & .1 \\ .1 & .1 & 0 & 0 \end{pmatrix}, \quad \mathbf{w}_0 = \begin{pmatrix} .1 \\ .8 \\ .8 \\ .8 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 14 \\ 10 \\ 10 \\ 10 \end{pmatrix},$$

where $\mathbf{F}$ is the vector of the required amount of refinancing, $\theta = 2$, $\rho = 0.1$, $p_0 = 1$, and $R = 100$. Firm 1 depends on the inputs from the other firms more than the others, and its required refinancing is larger than that of the others. The other firms are symmetric. When all firms are refinanced, the demand influence coefficient of Firm 1 is 1.417 and those of the others are 1.046. The monopolistic bank can achieve the profit of 37.07 by refinancing all these firms and making them operate despite the loan to Firm 1 incurring a loss of 2.12.

In the case of infinitesimal investors in the decentralized financial market, the loss-making Firm 1 cannot be refinanced, whereas the others make profits and are refinanced. In this case, the demand influence coefficient of each operating firm is 1.0256 and the profit of each firm is 10.59. Thus, the total profit of the network is 31.78. This is smaller than the monopolistic-bank case in the previous paragraph, which is 37.07.

The marginal profit from forbearing loans to firm 1 is $37.07 - 31.78 = 5.29$. The marginal cost is 2.12. The bank can benefit by forbearing the loss-making firm 1.

Online Appendix 3: The bias of the OLS estimator

We provide a justification for the use of the OLS estimator to estimate the spatial autoregressive models. We derive the formula for the bias of the OLS estimator and evaluate the bias in our sample. As a result, we find that the bias is likely to be small.[1]

Recall that the model is:

$$\Delta \mathbf{s} = \beta \mathbf{GM}\boldsymbol{\Delta}\mathbf{s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon,$$

where $\beta \equiv \gamma_0 \mathbf{I} + \gamma_1 \mathbf{I_{mfg}} + \gamma_2 \mathbf{I_{ws}}$. We assume that **Ind** and **Pre** are exogenously given variables, so that they are uncorrelated with ϵ. In this section, we assume that ϵ is homoskedastic. However, note that the standard errors reported in the main text are robust to heteroskedasticity.

The bias of the OLS estimator of $\gamma = (\gamma_0, \gamma_1, \gamma_1)'$ can be derived in the following way. Let $\tilde{\mathbf{X}}$ be the matrix of the residuals from the regression of $\mathbf{GM}\Delta\mathbf{s}$, $\mathbf{I_{mfg}GM}\Delta\mathbf{s}$ and $\mathbf{I_{ws}GM}\Delta\mathbf{s}$ on **Ind** and **Pre**. Thus, $\tilde{\mathbf{X}}$ is an $n \times 3$ matrix. The OLS estimator of γ is:

$$\hat{\gamma} = \left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1} \tilde{\mathbf{X}}'\Delta\mathbf{s} = \gamma + \left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1} \tilde{\mathbf{X}}'\Delta\epsilon.$$

As **Ind** and **Pre** are assumed to be exogenous, $E(\tilde{\mathbf{X}}'\epsilon) \approx E((\mathbf{GM}\Delta s, \mathbf{I_{mfg}GM}\Delta\mathbf{s}, \mathbf{I_{ws}GM}\Delta\mathbf{s})'\epsilon)$. The reduced-form equation for $\Delta\mathbf{s}$ is:

$$\Delta\mathbf{s} = (\mathbf{I}_n - \beta\mathbf{GM})^{-1}(\gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon).$$

As all terms appearing on the right-hand side except ϵ are assumed to be exogenous, we have:

$$E((\mathbf{GM}\Delta s)'\epsilon) = E\left(\epsilon'(\mathbf{GM})'(\mathbf{I}_n - \beta\mathbf{GM})^{-1}\epsilon\right) = \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{GM})'(\beta\mathbf{GM})^k \epsilon\right).$$

Similarly, we have:

$$E((\mathbf{I_{mfg}GM}\Delta s)'\epsilon) = \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{mfg}GM})'(\beta\mathbf{GM})^k \epsilon\right),$$
$$E((\mathbf{I_{ws}GM}\Delta s)'\epsilon) = \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{ws}GM})'(\beta\mathbf{GM})^k \epsilon\right).$$

[1] Note that Lee (2002) also examines the conditions under which the OLS estimator is consistent. However, their argument is not applicable to our model.

Let

$$\mathbf{B} = \begin{pmatrix} \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{GM})'(\beta\mathbf{GM})^k\epsilon\right) \\ \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{mfg}GM})'(\beta\mathbf{GM})^k\epsilon\right) \\ \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{ws}GM})'(\beta\mathbf{GM})^k\epsilon\right). \end{pmatrix}$$

The asymptotic bias of the OLS estimator is:

$$\left(\operatorname*{plim}_{N\to\infty} \frac{1}{N}\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1} \lim_{N\to\infty} \frac{1}{N}\mathbf{B}.$$

The bias of the OLS estimator may be numerically evaluated. The value of $\tilde{\mathbf{X}}'\tilde{\mathbf{X}}$ can be computed from the data. Let $g_{0ii}(k)$ be the i-th element on the main diagonal of $(\mathbf{GM})'(\beta\mathbf{GM})^k$. Similarly, let $g_{1ii}(k)$ and $g_{2ii}(k)$ be the i-th elements on the main diagonals of $(\mathbf{I_{mfg}GM})'(\beta\mathbf{GM})^k$ and $(\mathbf{I_{ws}GM})'(\beta\mathbf{GM})^k$, respectively. Assume that ϵ is homoskedastic with variance σ^2. Noting also that the elements of ϵ are uncorrelated across i, we have:

$$\begin{aligned} \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{GM})'(\beta\mathbf{GM})^k\epsilon\right) =& \sigma^2 \sum_{k=0}^{\infty}\sum_{i=1}^{n} g_{0ii}(k), \\ \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{mfg}GM})'(\beta\mathbf{GM})^k\epsilon\right) =& \sigma^2 \sum_{k=0}^{\infty}\sum_{i=1}^{n} g_{1ii}(k), \\ \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{I_{ws}GM})'(\beta\mathbf{GM})^k\epsilon\right) =& \sigma^2 \sum_{k=0}^{\infty}\sum_{i=1}^{n} g_{2ii}(k). \end{aligned}$$

As $g_{lii}(1) = 0$ for $l = 0, 1, 2$ by the definition of $\mathbf{G}$ and $\mathbf{M}$ being a diagonal matrix, the value at $k = 0$ does not contribute to the sum. Note that $g_{lii}(k)$ can be computed from the data once we have the values of σ^2 and γ. We then compute the bias using the formula:

$$\left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1} \sigma^2 \sum_{k=1}^{\infty}\sum_{i=1}^{n} \begin{pmatrix} g_{0ii}(k) \\ g_{1ii}(k) \\ g_{2ii}(k) \end{pmatrix}.$$

We compute an approximated value of the bias in the data and find that the bias is likely to be small. To do so, we use the OLS estimator of β and the estimate σ^2 from the OLS estimation to evaluate: $\sigma^2 \sum_{k=0}^{\infty}\sum_{i=1}^{n} g_{lii}(k)$ for $l = 0, 1, 2$. The infinite sum is truncated at $k = 2$. This truncation is justified because the value of β is small (recall that the OLS estimate is at most 0.0103 for a seller in the wholesale sector, and $\mathbf{M}$ is a diagonal matrix with elements less than one).

The bias is listed in Table A.1. The ratios of the bias over the estimated coefficients are less than 1% in for 2005 and 2013. The ratios are somewhat higher for 2010, but they are still at most 2.5%. Thus, the bias is negligible and so we can rely on the OLS estimator. Here, we use the

Table A.1: Bias of the spatial autoregressive model

	2005 series	2010 series	2013 series
γ_0	9.52E-06	6.42E-05	1.82E-05
γ_1	-6.59E-06	-5.34E-05	-1.53E-05
γ_2	4.07E-05	1.58E-04	4.19E-05

values of σ^2 and β from the OLS estimation. The computed bias would be small even if different (reasonable) values of σ^2 and β were used.

Online Appendix 4: Standard errors when regressors are generated

In this appendix, we provide the formula for the standard errors for the forbearance lending probability regression, taking into account the fact that the key regressor (demand influence coefficient) is estimated. We first briefly review the effect of a generated regressor in general linear regression frameworks. We then provide the formula for the specific case in which the generated regressor is a demand influence coefficient.

General results

We first examine the effect of a generated regressor in linear regression models. In particular, we explain how to modify the asymptotic variance estimator for the OLS estimator. Note that this is a rather standard exercise in econometrics.

We consider the following linear regression model:

$$y_i = x_i'\beta + u_i.$$

However, x_i is not directly observed. We know that x_i can be written as $x_i = x_i(\gamma_0)$, where the function $x_i(\cdot)$ is known and γ_0 is estimable. Let $\hat{\gamma}$ be an estimate of γ_0. We thus use a generated regressor $\hat{x}_i = x_i(\hat{\gamma})$ instead of x_i. The OLS estimator of β with $\hat{x}_i$ is:

$$\hat{\beta} = \left(\sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \sum_{i=1}^{N} \hat{x}_i y_i.$$

We make the following assumptions about the generated regressor. We assume that $x_i(\cdot)$ is differentiable and use the mean value theorem so that:

$$x_i - \hat{x}_i = -\frac{\partial x_i}{\partial \gamma}(\tilde{\gamma})(\hat{\gamma} - \gamma_0),$$

where $\tilde{\gamma}$ is between $\hat{\gamma}$ and γ_0. We also assume that $\hat{\gamma}$ is asymptotically linear:

$$\hat{\gamma} - \gamma = \frac{1}{N^*} \sum_{j=1}^{N^*} \phi_j,$$

where ϕ_j has mean zero and finite variance. We note that here we allow γ to be estimated from a different sample than the sample used in the estimation of β. These samples are allowed to be overlapping or disjointed. Let N^* denote the sample size of the sample used in the estimation of γ; N^* may be different from N. We assume that $\lim_{N,N^*\to\infty}(N/N^*) = \kappa < \infty$.

The asymptotic distribution of the OLS estimator $\hat{\beta}$ depends on the estimation error in $\hat{\gamma}$. We observe that:

$$y_i = \hat{x}_i'\beta + (x_i - \hat{x}_i)'\beta + u_i.$$

We therefore have the following expansion of $\hat{\beta}$:

$$\hat{\beta} = \beta + \left(\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \sum_{i=1}^{N} \hat{x}_i u_i + \left(\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \sum_{i=1}^{N} \hat{x}_i(x_i - \hat{x}_i)'\beta.$$

The second term on the right-hand side yields the usual asymptotic distribution of the OLS estimator. The third term depends on the estimation error in $\hat{\gamma}$. Using the assumption on $\hat{x}_i$, we have:

$$\hat{\beta} - \beta = \left(\frac{1}{N}\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \left(\frac{1}{N}\sum_{i=1}^{N} \hat{x}_i u_i - \frac{1}{N}\sum_{i=1}^{N} \hat{x}_i\beta'\frac{\partial x_i}{\partial\gamma}(\tilde{\gamma})\frac{1}{N^*}\sum_{j=1}^{N^*}\phi_j\right).$$

Let

$$A = E\left(x_i\beta'\frac{\partial x_i}{\partial\gamma}(\gamma_0)\right),$$

and

$$B = \text{plim}_{N\to\infty}\frac{1}{N}\sum_i x_i u_i \phi_i',$$

where $\sum_i$ in B is taken over the set of observations that appear in both the sample used for the estimation γ and that for β. From the expansion of $\hat{\beta}$, it is easy to derive the asymptotic distribution of $\hat{\beta}$, which is:

$$\sqrt{N}(\hat{\beta} - \beta) \to_d N(0, V),$$

where

$$V = \left(E(x_i x_i')\right)^{-1}\left(E(u_i^2 x_i x_i') - BA' - AB' + A\kappa E(\phi_j\phi_j')A'\right)\left(E(x_i x_i')\right)^{-1}.$$

We note that when $\kappa = 0$ (i.e., N^* is much larger than N), the estimation error in the generated regressor does not affect the asymptotic distribution of $\hat{\beta}$.

The asymptotic variance can be estimated by:

$$\hat{V} = \left(\frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{x}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2\hat{x}_i\hat{x}_i' - \hat{B}\hat{A}' - \hat{A}\hat{B}' + \hat{A}\frac{N}{N^*}\frac{1}{N^*}\sum_{j=1}^{N^*}\hat{\phi}_j\hat{\phi}_j'\hat{A}'\right)\left(\frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{x}_i\right)^{-1},$$

where

$$\hat{A} = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{\beta}'\frac{\partial x_i}{\partial\gamma}(\hat{\gamma}),$$

and

$$\hat{B} = \frac{1}{N}\sum_{i}\hat{x}_i\hat{u}_i\hat{\phi}_i'.$$

In this, $\hat{\phi}_i$ is an estimate of ϕ_i.

When the demand influence coefficient is generated

In our application, the generated regressor is the value of the demand influence coefficient. This subsection provides the formula for ϕ_j and $x_i(\cdot)$ in our case.

Recall that the demand influence coefficient is computed from the OLS estimate of the model:

$$\Delta\mathbf{s} = \beta\mathbf{GM}\Delta\mathbf{s} + \gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \epsilon,$$

where $\beta \equiv \gamma_0\mathbf{I} + \gamma_1\mathbf{I_{mfg}} + \gamma_2\mathbf{I_{ws}}$. Thus, in our setting, $\gamma = (\gamma_0, \gamma_1, \gamma_2)'$. As x_i is the logarithm of the demand influence coefficient, we have $x_i(\gamma) = \log((\sum_{k=0}^{\infty}\mathbf{1}'(\beta\mathbf{GM})^k)_i)$, where $(a)_i$ denotes the element of a corresponding to the i-th firm.

The formula for ϕ_j and $\partial x_i/(\partial\gamma)$ can be derived easily in our setting. Let $\tilde{\mathbf{X}}$ be the matrix of the residuals from the regression of $\mathbf{GM}\Delta\mathbf{s}$, $\mathbf{I_{mfg}GM}\Delta\mathbf{s}$ and $\mathbf{I_{ws}GM}\Delta\mathbf{s}$ on $\mathbf{Ind}$ and $\mathbf{Pre}$. The OLS estimator of γ is:

$$\hat{\gamma} = \left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\tilde{\mathbf{X}}'\Delta\mathbf{s} = \gamma + \left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\tilde{\mathbf{X}}'\Delta\epsilon.$$

Therefore, the formula for ϕ_j is:

$$\phi_j = \left(\frac{1}{N^*}\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\tilde{\mathbf{X}}_j'\epsilon_j,$$

where $\tilde{\mathbf{X}}_j$ is the j-th row of $\tilde{\mathbf{X}}$. The formula for $\partial x_i/(\partial\gamma)$ can be computed directly by taking the derivative. This gives:

$$\frac{\partial x_i}{\partial\gamma}(\gamma) = \frac{1}{(\sum_{k=0}^{\infty}\mathbf{1}'(\beta\mathbf{GM})^k)_i}\sum_{k=1}^{\infty}k\begin{pmatrix}\left(\mathbf{1}'(\beta\mathbf{GM})^{k-1}\mathbf{GM}\right)_i\\ \left(\mathbf{1}'(\beta\mathbf{GM})^{k-1}\mathbf{I_{mfg}GM}\right)_i\\ \left(\mathbf{1}'(\beta\mathbf{GM})^{k-1}\mathbf{I_{ws}GM}\right)_i\end{pmatrix}'.$$

Online Appendix 5: Additional empirical results

A5.1 Result from the annual cross-section data

We estimate the baseline model (28) with the annual cross-section data of distressed firms in each year as a robustness check. We assign lagged influence coefficients for each annual set of financial data, i.e., v_{2005} for the observations in 2008–10, v_{2010} for those in 2011–13, and v_{2013} for those in 2014–16. The set of control variables is almost the same as in the time-aggregated estimation except that we refrain from including the main-bank fixed effects because of the small sample size. The shortcoming of this method is that the number of observations in each cross-section is much smaller and this makes it difficult to introduce the bank fixed effects.

Table A.2 shows that the demand influence coefficient is highly positive in 2010 and 2011, right after the global financial crisis. The coefficient in 2010 is statistically significant at the 1% level. The estimated coefficient implies that a 0.4 percentage point increase from one in the demand influence coefficient increases the probability of obtaining forbearance by 2.4 percentage points in 2010 and 2.1 percentage points in 2011. This is economically significant because the average forbearance probability is 12.7%. The estimated coefficient of the demand influence coefficient in the period from 2008 to 2013 is positive and its value is relatively high. However, many of the coefficients of the demand influence coefficients are not statistically significant because of the high standard errors, probably because of the small sample size. The estimated coefficient becomes smaller and insignificant in the years after 2014. We assume that this is due to QQE and the economic recovery since April 2013 as mentioned before.

A5.2 Sample selection model

So far, we have used the dataset comprising distressed firms only to test our hypothesis that a distressed firm with a higher demand influence coefficient has a higher probability of obtaining a forbearance loan. A potential problem is sample selection bias due to dropping firms that are not in financial distress. To address this potential problem, we apply the Heckit procedure (Heckman, 1979).

In the first stage, we estimate the probability of $Distress_{it} = 1$ using a probit model. We include all the independent variables used in the baseline estimation for the forbearance loan probability

Table A.2: Result from the annual cross-section data

(Notes) Estimated by OLS. The dependent variable is forbear (0,1). Each row is the cross-section regression for a particular year. The estimated coefficients of the control variables, industry dummies, regional dummies, and the constant term are omitted from the report. The cross-section data for the accounting year ending in each period are used. The sample includes firms that have been distressed (distress = 1) in each year. v for 2005 is in rows 1–3), for 2010 is in rows 4–6 and for 2013 is in rows 7–9, respectively. The set of controls includes those in Table 9. The set of industry dummies is based on the large classification listed in Table 4 (seven dummies). The region dummies are the same as those in Table 9. Main-bank-clustered standard errors of the estimated coefficient of $ln(v)$ in parentheses. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

Year	Coef. of ln(v)	(s.e.)	N	Adj.R^2
2008	3.508	(3.562)	749	0.023
2009	2.005	(3.725)	881	0.035
2010	6.008***	(2.211)	1,115	0.046
2011	5.347	(3.937)	925	0.030
2012	3.536	(4.117)	808	0.007
2013	4.364	(3.885)	818	0.012
2014	0.215	(1.614)	705	0.008
2015	-0.111	(1.163)	572	0.044
2016	1.097	(1.840)	431	0.024

in the previous section. In the second stage, we regress the forbearance loan dummy on the set of the independent variables that we used in the baseline estimation and the inverse Mills ratio, which is calculated by the predicted value in the first stage. We can estimate the model including main-bank fixed effect in the second stage using the procedure proposed by Wooldridge (1995).

The estimated coefficients with the time-aggregated dataset are listed in Table A.3. The estimated coefficients from the first stage probit for the distress probability are listed in panel (a). The demand influence coefficients are positive in all periods. This suggests the possibility of moral hazard because of the anticipated network-motivated forbearance lending. However, they are not statistically significant. The firms with higher interest coverage ratio $sale/R^*$, higher sales growth $\Delta real\ sale$, larger tangible assets $collateral$, and larger total assets $asset$ are less likely to face financial distress. Older firms and firms with larger numbers of employees are more likely to be distressed. The negative and significant coefficient of the number of lenders $\#banks$ suggests that the variable works as a proxy for firm size or creditworthiness.

Panel (b) shows the estimated coefficients in the second stage. The coefficient of the inverse

Mills ratio is positive and significant. This implies that the sample selection bias does matter in our baseline estimation. However, the estimated coefficient and its statistical significance with respect to the demand influence coefficient are not affected by it.

Table A.4 presents the estimated coefficients of important variables from the sample selection model with the year-by-year repeated cross-section data. The coefficient of the inverse Mills ratio is positive and statistically significant in all years after 2011. However, the estimated coefficients of the demand influence coefficient in the second stage regression are only marginally different. The influence coefficient in the first stage is positive in most years, but statistically significant only in 2008 and 2015.

Thus, the estimations by the sample selection model support the baseline results consistently. In addition, they provide evidence suggestive of possible moral hazard by firms who may expect network-motivated forbearance lending.

Table A.3: Sample selection model with time-aggregated data

(Notes) Estimated by the sample selection model with the main-bank fixed effect. The first stage is the probit model for the distress probability. The dependent variable in the second stage is forbear (0,1). Time-aggregated data for each period are used. The control variables are those corresponding to the first year of each period. Column 1 uses v for 2005, column 2 for 2010, and column 3 for 2013. The year dummy is a dummy indicating the first year of distress (two dummies). The industry dummy is based on the detailed classification listed in Table 4 (seven dummies) in the first stage, whereas it is based on the two-digit classification in the second stage. Region dummies are the same as in Table 9. The estimates of the constant term and the coefficients of these dummies are not reported. The contents of the parentheses are standard errors of the estimated coefficients, which are clustered in the second stage by main bank. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

(a) First stage: sample selection (distress probability)

	(1)	(2)	(3)
	$v = v_{2005}$	v_{2010}	v_{2013}
VARIABLES	Period: 2008-10	2011-13	2014-16
ln(v)	2.279	3.598	2.473
	(1.600)	(2.671)	(1.752)
ln(sale/R^*)	-0.418***	-0.473***	-0.428***
	(0.024)	(0.029)	(0.035)
ln(1+Δ real sale)	-1.317***	-0.923***	-1.214***
	(0.102)	(0.113)	(0.163)
ln(1 + collateral(t-1))	-0.525***	-0.589***	-0.269***
	(0.070)	(0.085)	(0.098)
ln(age)	0.111**	0.221***	0.254***
	(0.050)	(0.058)	(0.069)
ln(#employee(t-1))	0.074**	0.098***	0.146***
	(0.030)	(0.035)	(0.044)
ln(asset(t-1))	-0.270***	-0.373***	-0.340***
	(0.025)	(0.030)	(0.038)
leverage(t-1)	3.777***	4.280***	4.999***
	(0.145)	(0.161)	(0.204)
switch	-0.073	0.036	-0.034
	(0.118)	(0.161)	(0.203)
#bank	-0.039***	-0.053***	-0.053***
	(0.010)	(0.012)	(0.014)
share	-0.224	-0.024	-0.104
	(0.187)	(0.216)	(0.262)
hi	-0.155	-0.278	0.172
	(0.252)	(0.277)	(0.355)
ln(MB asset)	0.024*	0.009	0.004
	(0.014)	(0.015)	(0.018)
MB cap ratio	-1.193	-0.179	-1.135
	(1.183)	(1.793)	(2.206)
Industry dummy	yes	yes	yes
Year dummy	yes	yes	yes
Region dummy	yes	yes	yes

Table A.3: (cont.)

(b) Second stage: forbearance lending probability

VARIABLES	(1) $v = v_{2005}$ Period: 2008-10	(2) v_{2010} 2011-13	(3) v_{2013} 2014-16
ln(v)	3.476	8.370***	0.777
	(2.897)	(2.856)	(1.590)
ln(sale/R^*)	-0.029**	-0.061***	-0.027
	(0.014)	(0.016)	(0.021)
ln(1+Δ real sale)	-0.065	0.026	-0.056
	(0.057)	(0.055)	(0.073)
ln(1 + collateral(t-1))	-0.039	-0.093*	0.037
	(0.040)	(0.053)	(0.068)
ln(age)	-0.014	0.017	0.019
	(0.024)	(0.025)	(0.037)
ln(#employee(t-1))	-0.021	0.007	0.022
	(0.016)	(0.020)	(0.028)
ln(asset(t-1))	-0.006	-0.043***	-0.051**
	(0.012)	(0.015)	(0.023)
leverage(t-1)	0.091***	0.036*	0.030
	(0.031)	(0.019)	(0.021)
switch	-0.019	0.163*	0.081
	(0.040)	(0.093)	(0.123)
#bank	-0.009	-0.017**	-0.016**
	(0.006)	(0.007)	(0.008)
share	-0.068	-0.327*	-0.371*
	(0.133)	(0.187)	(0.213)
hi	-0.050	-0.098	0.112
	(0.173)	(0.154)	(0.184)
ln(MB asset)	0.380	0.575**	-2.284*
	(0.567)	(0.288)	(1.283)
MB cap ratio	-0.786	7.426	-1.702
	(1.247)	(5.032)	(16.355)
Inverse Mills Ratio	0.069**	0.103***	0.096***
	(0.030)	(0.022)	(0.026)
N	10,638	9,050	8,584
Adj. R^2 in 2nd stage	0.083	0.066	0.066
Bank FE	yes	yes	yes
Industry (2 digit) dummy	yes	yes	yes
Year dummy	yes	yes	yes
Region dummy	yes	yes	yes
#group(banks)	253	243	208

Table A.4: Sample selection model with year-by-year data

(Notes) Estimated by the sample selection model. The first stage is the probit model for the distress probability. The dependent variable in the second stage is forbear (0,1). Each row represents a separate year. The estimated coefficients of the control variables, industry dummies, regional dummies, and the constant term are omitted from the report. Rows 1–3 represent v for 2005, rows 4–6 for 2010, and rows 7–9 for 2013. The set of independent variables is the same as in those in Table A.3 except that the industry dummies in the second stage are based on the classification in Table 4 (seven dummies). Main-bank-clustered standard errors are reported for the second stage regression. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

Year	Coef. of ln(v) for forbearance prob. (clustered s.e.)	Coef. of inverse-Mills ratio (clustered s.e.)	Coef. of ln(v) for distress prob. (s.e.)	N
2008	3.541	0.011	5.836**	7,823
	(2.483)	(0.028)	(2.705)	
2009	2.413	0.024	-0.156	7,806
	(4.414)	(0.028)	(5.122)	
2010	5.964**	0.041	1.961	8,436
	(2.376)	(0.033)	(1.762)	
2011	5.366	0.071***	3.607	7,418
	(3.578)	(0.023)	(3.444)	
2012	3.881	0.095***	4.531	7,299
	(3.832)	(0.023)	(2.881)	
2013	4.843	0.067***	3.744	7,398
	(3.885)	(0.025)	(3.679)	
2014	0.820	0.079***	2.229	7,676
	(2.396)	(0.024)	(2.973)	
2015	-0.045	0.087***	3.683*	7,200
	(1.494)	(0.022)	(2.091)	
2016	1.155	0.051**	3.005	6,188
	(2.160)	(0.022)	(2.892)	

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