

# Labor-Market Wedge under Engel Curve Utility: Cyclical Substitution between Necessities and Luxuries\*

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## **Abstract**

In booms, households substitute luxuries for necessities, e.g., food away from home for food at home. This cyclical pattern of composition changes in the consumption basket has the potential to reduce the volatility of measures of the labor-market wedge—the gap between the marginal rate of substitution and the real wage. Based on household expenditure patterns from the Consumer Expenditure Survey, we show that this composition bias has only a limited impact on the measured labor-market wedge, accounting for 6-16% of its cyclical volatility.

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# 1 Introduction

One of the leading research questions in macroeconomics is the identification of the sources of economic fluctuations.<sup>1</sup> Economists often identify these sources through accounting procedures that are based on “wedges,” that is, violations of a model economy’s equilibrium conditions conditional on data.<sup>2</sup> For example, representative agent models impose tight restrictions on the co-movement of consumption, hours, and real wages. For an optimal allocation of consumption and hours worked, the marginal rate of substitution (*MRS*) between leisure and consumption has to equal the real wage. Conditional on consumption, hours worked should increase with the real wage, but for reasonable parameterizations of the representative household’s preferences, this prediction is inconsistent with observed movements in aggregate consumption, hours worked, and real wages over the business cycle. On the one hand, the *MRS* increases rapidly during expansions, as the marginal utility of consumption relative to leisure quickly decreases, but on the other hand, there is no corresponding strongly pro-cyclical movement in real wages. This gap between the *MRS* and the real wage, the so-called labor-market wedge, when treated as an exogenous distortion is an important source of economic fluctuations in this class of models.<sup>3</sup> Of course one would prefer to explain the wedge rather than treat it as an exogenous shock.<sup>4</sup>

Recently, Jaimovich, Rebelo, and Wong (2018) have documented that during the Great Recession, consumers reduced the quality of the goods and services they consumed. Since part of the labor wedge is due to the counter-cyclical marginal utility of consumption, pro-cyclical variation of quality can reduce the volatility of the labor wedge. While Jaimovich

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<sup>1</sup>See, for example, Christiano, Eichenbaum, and Evans (2005) or Smets and Wouters (2007).

<sup>2</sup>See Hall (1997) and Chari, Kehoe, and McGrattan (2007) for expositions of wedge accounting.

<sup>3</sup>Note that our (narrow) definition of the labor wedge represents only a part of the broader definition of the labor wedge as the gap between the *MRS* and the marginal product of labor, see, e.g., Bils, Klenow, and Marlin (2018). Nevertheless, as Karabarbounis (2014) argues, our narrow wedge accounts for most of the volatility in the overall wedge.

<sup>4</sup>The existing literature offers various interpretations for this wedge, including changes in home production technology, Benhabib, Rogerson, and Wright (1991), government spending being a part of private consumption, Christiano and Eichenbaum (1992), various frictions in the labor market, such as wage rigidity, Gali, Gertler, and Lopez-Salido (2007), or search frictions, Shimer (2010), and aggregation errors, Chang and Kim (2007).

et al. (2018) provide a general framework that includes quantity-quality substitution, the measurement of quality is very challenging. Instead, in this paper we study the “average quality ” effects stemming from composition changes in the household’s consumption basket and non-homothetic income-expenditure paths, that is, Engel curves. It is straightforward to obtain information on the shape of Engel curves from cross-sectional data such as the Consumer Expenditure Survey (CEX).

We show that accounting for the substitution between necessities and luxuries dampens the cyclical movement of the labor-market wedge, but only by a small amount. In booms, households’ consumption of luxuries (e.g., food away from home) tends to increase relatively more than the consumption of necessities (e.g., food at home). This substitution along the Engel curve slows down the increase in the MRS because the marginal utility of consumption falls more slowly as consumers switch toward luxuries. For a parameterization of non-homothetic Engel curves consistent with the cross-sectional household expenditure pattern across income quintiles in the CEX, we show that cyclical composition changes in the consumption basket can account for at most 16% of the volatility in the labor wedge measured in the aggregate time series data.

This note is organized as follows. Section 2 briefly discusses the measurement of the labor-market wedge and lays out a simple model where the household’s preferences exhibit an Engel curve. In Section 3, we compute the labor wedge corrected for the Engel curve, using data on cross-sectional household expenditure patterns across income quintiles in the CEX. Section 4 provides a concluding remark.

## 2 Labor-Market Wedge

To understand the role of the Engel curve in the measurement of the labor-market wedge, we first present the standard labor wedge for household preferences expressed with respect

to an aggregate consumption good,  $C$ , and hours worked,  $H$ :

$$\begin{aligned} U(C, H) &= \frac{C^{1-1/\sigma}}{1-1/\sigma} - \psi \frac{H^{1+1/\gamma}}{1+1/\gamma} \\ P \cdot C &= W \cdot H \end{aligned}$$

where  $\sigma$  is the inter-temporal elasticity of substitution (IES) for consumption and  $\gamma$  is the Frisch elasticity of labor supply.<sup>5</sup> The labor wedge  $\tau$  is defined as the ratio between the marginal rate of substitution (between leisure and consumption) and the real wage ( $W/P$ ):

$$\frac{\psi H^{1/\gamma}}{C^{-1/\sigma}} = \frac{MU_L}{MU_C} = MRS = \tau \frac{W}{P}. \quad (1)$$

When we denote  $\hat{x}$  for the cyclical component of  $x$  (de-measured growth rate or percentage deviation from the trend), the cyclical component of the labor wedge can be expressed as:

$$\hat{\tau} = \frac{1}{\gamma} \hat{H} + \frac{1}{\sigma} \hat{C} - \widehat{W/P} \quad (2)$$

Figure 1 shows the cyclical component of aggregate GDP and the labor wedge for a baseline parameterization of preferences using aggregate times series data. The measured wedge is highly volatile and pro-cyclical because: (i) hours worked and consumption are both pro-cyclical, with hours being very volatile, and (ii) the real wage is neither highly pro-cyclical nor volatile. As shown in the table of Figure 1, (i) hours are slightly more volatile than GDP and highly pro-cyclical with a 0.95 elasticity with respect to GDP growth, while (ii) consumption and the real wage exhibit similar volatility, and the real wage is only mildly pro-cyclical with a mere 0.19 elasticity with respect to GDP growth. As a result, the labor wedge is tightly correlated with GDP and more than twice as volatile: a 1% increase in GDP is associated with a nearly 2% increase in the labor wedge for our baseline parameterization,  $\sigma = 0.5$  and  $\gamma = 1$ .

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<sup>5</sup>Since the labor-market wedge is entirely based on the intra-temporal optimality condition, we abstract from the dynamic decisions of households, e.g., savings, etc.

We believe our baseline parameterization is plausible since (i) there is ample evidence for an IES in consumption that is much smaller than one, and (ii) variations in aggregate hours reflect the extensive margin as well as the intensive margins of labor supply decisions.<sup>6</sup> In addition, we obtain similar results for the labor wedge volatility for a range of empirically plausible values of  $\sigma$  and  $\gamma$ , see columns (2) through (5) in Table 2.

Now, suppose that the household purchases  $N$  types of consumption goods,  $\{c_1, \dots, c_N\}$ , at prices  $\{p_1, \dots, p_N\}$ . The household maximizes a utility function with inter-temporal elasticities of substitution that differ across goods

$$\begin{aligned}
 U(c_1, \dots, c_N, H) &= \sum_{i=1}^N \phi_i \frac{c_i^{1-1/\sigma_i}}{1-1/\sigma_i} - \psi \frac{H^{1+1/\gamma}}{1+1/\gamma} \\
 P^m \cdot C^m &= \sum_{i=1}^N p_i c_i = W \cdot H
 \end{aligned}$$

where  $P^m$  and  $C^m$  represent the measured aggregate price and consumption index. The FOCs are

$$\begin{aligned}
 \phi_i c_i^{-1/\sigma_i} &= \lambda p_i, \text{ for } i = 1, \dots, N \\
 \psi H^{1/\gamma} &= \lambda W
 \end{aligned} \tag{3}$$

where  $\lambda$  is the marginal utility of nominal expenditures. This specification yields non-homothetic Engel curves across goods. A good with a small  $\sigma_i$  is a necessity (e.g., food) whose marginal utility decreases rapidly with increased consumption. A good with a large  $\sigma_i$  is a luxury whose marginal utility decreases slowly. **Consequently, as total expenditures increase for fixed prices and the marginal utility of expenditures decline, consumption of luxury goods increases faster than does consumption of necessities.**

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<sup>6</sup>For example, Havranek (2015) in a meta analysis of 169 published articles finds a mean estimate of 0.5 for the IES, and Keane and Rogerson (2012) discuss the relevance of intensive and extensive margins for estimates of the aggregate labor supply elasticity.

Summing over the FOCs for the consumption goods, we get the marginal utility of expenditures

$$\lambda = \frac{\sum_i \phi_i c_i^{1-1/\sigma_i}}{\sum_i p_i c_i} = \frac{\tilde{c}}{P^m \cdot C^m} \text{ with } \tilde{c} \equiv \sum_i \phi_i c_i^{1-1/\sigma_i}. \quad (4)$$

Allowing for a labor wedge in equation (3) and using the marginal utility of expenditures, the *true* labor wedge,  $\tau^*$ , is then defined by the expression

$$\psi H^{1/\gamma} \frac{C^m}{\tilde{c}} = \frac{MU_L C^m}{\tilde{c}} = \tau^* \frac{W}{P^m}. \quad (5)$$

Compared to the standard measure of the labor wedge in (1) with aggregate consumption, this wedge with multiple goods is likely to be less cyclical because in economic booms households' consumption moves toward luxuries whose marginal utility decreases more slowly. The cyclical component (growth rate) of the labor wedge is<sup>7</sup>

$$\hat{\tau}^* = \frac{1}{\gamma} \hat{H} + \hat{C}^m - \sum_i \left(1 - \frac{1}{\sigma_i}\right) \omega_i \hat{c}_i - \widehat{W/P^m}. \quad (6)$$

where  $\hat{C}^m = \sum_i \omega_i \hat{c}_i$  and  $\hat{P}^m = \sum_i \omega_i \hat{p}_i$  are Divisia quantity and price indices of aggregate consumption. Measured quantity and price indices of aggregate consumption are essentially constructed as Divisia indices. Using these quantity and price measures of aggregate consumption in expression (2) we obtain the difference between the measured wedge and true wedge

$$\hat{\tau}^m - \hat{\tau}^* = \sum_i \left(1 - \frac{1}{\sigma_i}\right) \omega_i \hat{c}_i - \left(1 - \frac{1}{\sigma}\right) \sum_{i=1}^N \omega_i \hat{c}_i = \sum_i \left(\frac{1}{\sigma} - \frac{1}{\sigma_i}\right) \omega_i \hat{c}_i.$$

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<sup>7</sup>From the definition of  $\tilde{c}$  in equation (4) we get  $\hat{\tilde{c}} = \sum_i \left(1 - \frac{1}{\sigma_i}\right) \omega_i \hat{c}_i$  where  $\omega_i$  is the expenditure share of the  $i$ -th good.

## 3 Empirical Analysis

### 3.1 Engel Curves from the CEX

We use eight categories of household expenditures in the CEX: food at home, food away from home, transportation (excluding vehicle purchases), housing, health care, apparel, entertainment, and cash contribution. In Table 1, first and second column, we report their expenditure shares in years 2005 and 2015. The expenditure shares of the eight categories are quite stable over the decade, and in total (CEX8) they make up about 75% of total expenditures—which is close to 89% of the consumption-related expenditure (total expenditure net of those on personal insurance and pensions, CEX NET). We exclude vehicle purchases because vehicles are durable goods, and we exclude “insurance and pensions” because they may reflect the household’s savings rather than consumption.

For each consumption category  $i$ , the Engel curve parameter,  $\sigma_i$ , can be estimated as follows. The FOCs of the household’s utility maximization for consumption goods (3) imply that for any two goods

$$\ln c_i = \frac{\sigma_i}{\sigma_j} \ln c_j - \sigma_i \ln(p_i/p_j). \quad (7)$$

Let  $c_i^{Q^k}$  denote the quantity of consumption for category  $i$  by the household in the  $k$ -th quintile of the income distribution. [BEGIN EDIT] Assuming that households face the same prices we get

$$\ln \left( \frac{p_i c_i^{Q^5}}{p_i c_i^{Q^1}} \right) = \frac{\sigma_i}{\sigma_j} \ln \left( \frac{p_j c_j^{Q^5}}{p_j c_j^{Q^1}} \right) \quad (8)$$

and we can infer the *relative* Engel curves between categories  $i$  and  $j$ ,  $\sigma_i/\sigma_j$ , from the cross-sectional nominal consumption ratios of the respective categories for households in the 5th and 1st income quintile.

Based on the cross-sectional CEX of 2005 and 2015, we compute the relative (to total

expenditure) Engel curve parameters,  $s_i$ , third and fourth column of Table 1,

$$s_i = \frac{\ln \left( p_i c_i^{Q5} / p_i c_i^{Q1} \right)}{\ln (PC^{Q5} / PC^{Q1})} \quad (9)$$

The relative Engel curve parameters for the two years differ somewhat, but they do not change much over the decade, and their ranking stays roughly constant. The last column of Table 1 displays the average relative Engel curve parameters for the two years, which we use in our calculation of the composition adjusted labor wedges.

For a given aggregate intertemporal elasticity of substitution we calculate the levels of the corresponding Engel curve parameters as  $\sigma_i = s_i \sigma$ . The measured relative Engel curve parameters indicate an above (below) average response of a category's consumption to an increase of income for  $s_i > 1 (< 1)$ . The average relative parameter is about 1.1, thus the average Engel curve parameter is close to  $\sigma$ . [END EDIT]

While the CEX contains information which we can use to calculate the slope of household Engel-curves, it does not contain information on prices, and it is well known that aggregate nominal expenditures from the CEX and the more widely used NIPA Personal Consumption Expenditures (PCE) diverge over time. For the prices of CEX consumption categories, we use the corresponding price index from the CPI, except for "Entertainment" and "Cash Contribution." For the latter two categories we use the aggregate CPI, since the CPI does not have separate price indexes for them. Aggregate nominal CEX expenditures are growing at a much slower pace than aggregate PCE in the NIPA because the CEX systematically understates durable goods and luxuries in households' expenditures. Figure 2 shows that aggregate PCE increased 4.6 times from 1985 to 2015, whereas aggregate CEX expenditures (CEXNET) has increased 2.4 times. We, however, focus on the cyclical components of consumption, and the de-meaned growth rates of the two consumption aggregates comove fairly closely, Figure 3. The correlation coefficient for the two consumption growth rates is 0.45, and the projection of the growth rates of aggregate PCE on those of aggregate CEX



yields an  $R^2$  of 0.80.

### 3.2 Cyclical Behavior of Labor-Market Wedges

We first show that the cyclical behavior of the labor-market wedge constructed with our aggregate measure of consumption from the CEX is comparable with that of labor wedges constructed from more standard measures of aggregate consumption. We then show that the labor wedge constructed from the disaggregated CEX categories is less cyclical than the labor wedge from the CEX aggregate. We start with our baseline parameterization, and then show that similar results obtain for other empirically reasonable parameterizations.

The first column of Table 2 displays the cyclical behavior of the labor wedge for our baseline parameterization and different measures of consumption.<sup>8</sup> The first three rows of Table 2 display the cyclical behavior of the labor wedge based on the standard single-goods utility for three measures of aggregate consumption: all items of PCE in the NIPA, “PCE-All”, nondurable goods and services PCE, “PCE-NDS”, and a Divisia-aggregate of our 8 CEX expenditure categories, “CEX8-Aggregate.” The PCE-All is more cyclical than the PCE-NDS, but since our framework applies to nondurable goods, the PCE-NDS is the appropriate aggregate consumption measure. The labor wedge cyclical behavior from the CEX8-Aggregate and the PCE-NDS are of similar magnitude, with the CEX8-Aggregate based labor wedge slightly less cyclical.

We now use the 8 CEX consumption categories and construct a labor wedge, “CEX8-Engel”, that allows for differences in income expansion paths of consumption, fourth row of Table 2. Comparing CEX8-Engel with CEX8-Aggregate we can see that accounting for differences in income elasticities across commodities reduces the volatility of the labor wedge by 9.3%. In other words, recognizing the differences in marginal utility across commodities together with the pro/counter-cyclical nature of luxuries/necessities makes true marginal utility move less than is implied by the usual aggregate consumption measure and results in

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<sup>8</sup>Again, as in Figure 1, “cyclical behavior” is defined as the regression coefficient of the labor-market wedge growth rate on the GDP growth rate.

a less volatile labor wedge.

In the remaining columns of Table 2 we report the cyclical behavior of the labor wedge based on alternative values of the preference parameters  $\sigma$  and  $\gamma$ . Using a smaller inter-temporal elasticity of consumption magnifies the labor-wedge cyclical behavior—it is even harder to justify the cyclical behavior of consumption and hours as an optimal choice of the stand-in household. With  $\sigma = 0.1$ , the cyclical behavior based on the CEX8-Aggregate increases to 4.55—the wedge moves five times as much as GDP over the business cycle. The cyclical behavior of the “true” wedge (CEX8-Engel) is 3.85, roughly 16% smaller than the standard measure. Using the larger value  $\sigma = 1$ , that is, log utility in consumption, accounting for non-homothetic Engel curves reduces the wedge cyclical behavior by only 6.2%. A larger labor supply elasticity reduces the cyclical behavior of the wedge because the marginal utility of leisure increases at a slower rate in booms. The same reduction in the cyclical behavior of the marginal utility of consumption from using disaggregated Engel curves then implies a larger percentage reduction in the labor wedge cyclical behavior. Overall, correcting the movement of the marginal utility of consumption based on the differences in the Engel curve across the eight consumption categories in the CEX decreases the cyclical behavior of the wedge by 6-16%; see row (6) of Table 2.

We obtain an upper bound on how much one can reduce the labor wedge through modifications of the marginal utility of consumption by making the marginal utility of consumption a constant,  $\sigma = \infty$ ; equation (2) and row (5) of Table 2. From equation (2) it follows that this specification provides an upper bound for *any* specification of preferences for which the implied consumption index and labor supply are positively correlated and the real wage is essentially acyclical.<sup>9</sup> For example, with  $\gamma = 1$  and  $\sigma = 0.5$ , assuming a constant marginal utility of consumption reduces the estimated cyclical behavior of the wedge by half relative to the benchmark case. Our treatment based on non-homothetic Engel curves across eight categories in the CEX materialize 18.5% of this potential reduction in the cyclical behavior of wedge. Note also that the *relative* contribution of our correction of the wedge remains at

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<sup>9</sup>In particular, it includes preference specifications with a quality-quantity trade-off along the lines of Jaimovich et al (2018).

18.5% regardless of  $\gamma$ 's and  $\sigma$ 's; see row (8) of Table 2. **In the Appendix we show that this feature is a consequence of fixing the relative Engel-curve parameters, and defining their levels proportional to the aggregate intertemporal elasticity of substitution.**

## 4 Concluding Remark

Estimated DSGE models have been widely used to study economic fluctuations. One popular way to identify the sources of fluctuation in these DSGE models is to measure shocks as ‘wedges’ in model-implied relationships among key aggregate time series, e.g., an optimality condition or a resource constraint. According to this method, the labor-market wedge—the gap between the MRS between consumption and leisure and the real wage—often emerges as an important source of aggregate fluctuations.

In this note, we have studied the extent to which pro-cyclical changes in the ‘average quality’ of aggregate consumption can account for the volatility of the labor wedge when Engel curves are non-homothetic. Using information on changes in consumption patterns from the CEX we have found that the impact of these composition effects on the labor wedge is of limited quantitative importance. They can account for at most 6-16% of the labor wedge volatility. We have also derived an upper bound on how much more general approaches that allow for unobserved quantity-quality substitution in consumption, such as Jaimovich et al. (2018), can reduce volatility of the measured labor wedge. These more general specifications of preferences can reduce the cyclical volatility of the labor wedge by at most 80%. The particular preferences we consider, non-homothetic Engel-curves disciplined by the cross-sectional Engel curves over eight expenditures categories in the CEX, can account for only one-fifth of that maximal reduction.

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## Appendix

We can rewrite the equations for the growth rates in the measured, true, and limiting labor wedge with  $\sigma = \infty$ , as follows

$$\begin{aligned}\hat{\tau}^m &= \hat{\tau}^\infty + \frac{1}{\sigma}\hat{C}^m, \\ \hat{\tau}^* &= \hat{\tau}^\infty + \frac{1}{\sigma}\hat{C}^*, \\ \hat{\tau}^\infty &= \frac{1}{\gamma}\widehat{H} - \widehat{W/P^m},\end{aligned}$$

where  $\hat{C}^* = \sum s_i^{-1}\omega_i\hat{c}_i$ .

In Table 2 we list the regression coefficients of the growth rate in the three labor wedges on GDP growth in rows (3), (4), and (5). Across columns the aggregate IES and labor supply elasticity change, but the relative IES across categories,  $s_i$ , remain fixed. This means that the right hand side variables,  $\widehat{H}$ ,  $\widehat{W/P^m}$ ,  $\hat{C}^m$ , and  $\hat{C}^*$ , are all independent of  $\sigma$  and  $\gamma$ .

The regressions asymptotically reflect the linear projections of the labor wedges on output

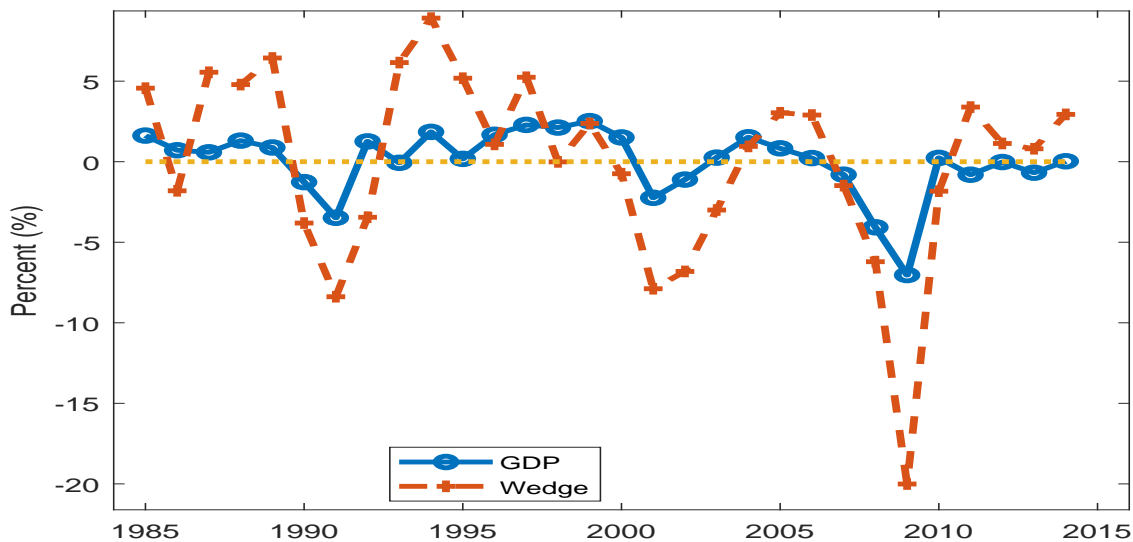
$$\begin{aligned}(3) : \quad E[\hat{\tau}^m|\hat{y}] &= E[\hat{\tau}^\infty|\hat{y}] + \frac{1}{\sigma}E[\hat{C}^m|\hat{y}] = \left[ \beta^\infty + \frac{1}{\sigma}\beta^m \right] \hat{y}, \\ (4) : \quad E[\hat{\tau}^*|\hat{y}] &= E[\hat{\tau}^\infty|\hat{y}] + \frac{1}{\sigma}E[\hat{C}^*|\hat{y}] = \left[ \beta^\infty + \frac{1}{\sigma}\beta^* \right] \hat{y}, \\ (5) : \quad E[\hat{\tau}^\infty|\hat{y}] &= \beta^\infty \hat{y}\end{aligned}$$

Therefore the ratios in rows (6), (7), and (8) are given by

$$\begin{aligned}(6) : \quad \frac{(4)}{(3)} - 1 &= \frac{(\beta^* - \beta^m)/\sigma}{\beta^\infty + \beta^m/\sigma} \\ (7) : \quad \frac{(5)}{(3)} - 1 &= \frac{-(1/\sigma)\beta^m}{\beta^\infty + \beta^m/\sigma} \\ (8) : \quad \frac{(6)}{(7)} &= -\frac{\beta^* - \beta^m}{\beta^m}\end{aligned}$$

As you can see the relative improvements are independent of  $\sigma$ .

Figure 1: Cyclical Behavior of the Labor-Market Wedge




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	GDP	H	C	W/P	Wedge ( $\tau$ )
SD (%)	2.06	2.34	1.31	1.50	4.55
Cyclical	1	0.95	0.56	0.19	1.88

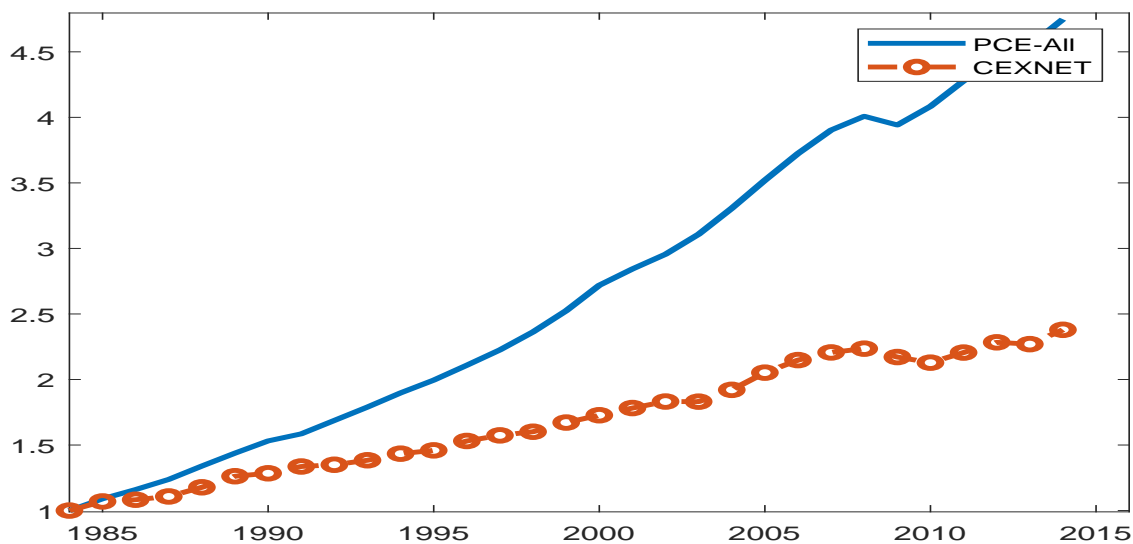
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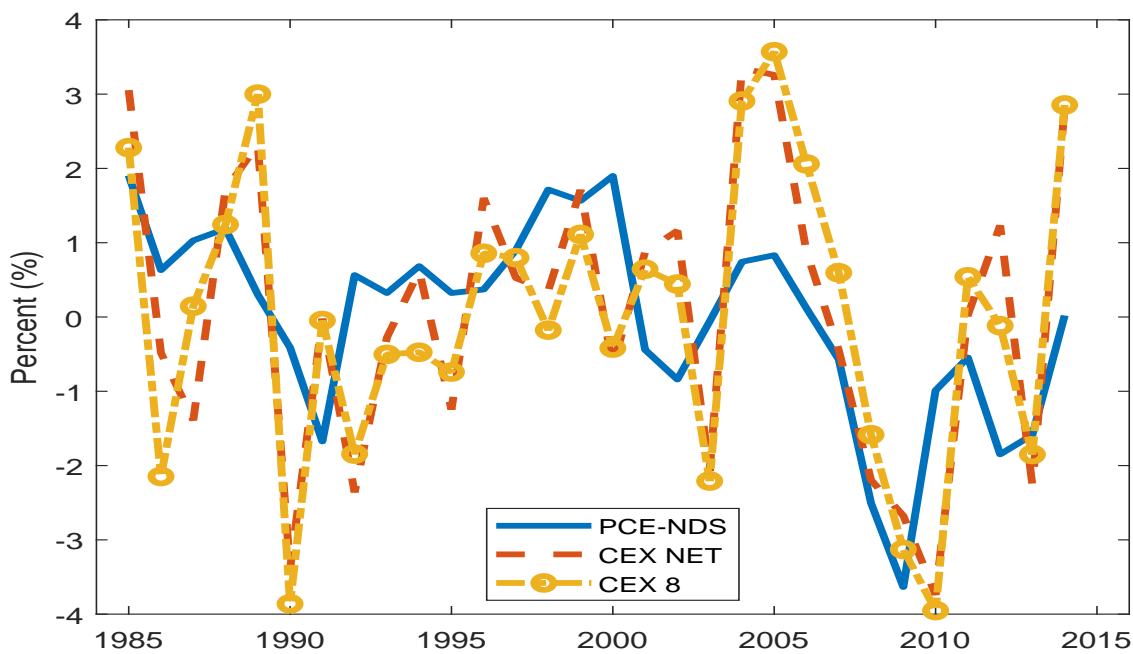
Note: Aggregate consumption ( $C$ ) and its price are based on personal consumption expenditure (PCE) data for nondurables and services from the NIPA. Aggregate hours ( $H$ ) and nominal wages ( $W$ ) are total hours and wages from the BLS’s Labor Productivity and Cost index (LPC) for nonfarm business sectors (<https://www.bls.gov/lpc/>). We use annual data and calculate their growth rates as 100 times first differences in logs. The labor-market wedge is computed for  $\sigma = 0.5$  and  $\gamma = 1$ . SD denotes the standard deviation, and “Cyclical” denotes the regression coefficient on GDP growth.

Figure 2: Nominal Expenditures on Consumption



Note: Nominal expenditures of personal consumption expenditure of all categories (PCE-All) and those of CEX net of “pension and insurance” (CEX NET).

Figure 3: Cyclical Components of Consumption



Note: Real consumption growth of PCE nondurables and services (PCE-NDS), CEX NET, and CEX 8.

Table 1: Relative Engel Curves

Category	Share (%)		Relative Engel ( $\frac{\sigma_i}{\sigma}$ )		
	2005	2015	2005	2015	Avg.
Food at Home	7.1	7.2	0.78	0.68	0.73
Food away from Home	5.4	5.7	1.30	1.15	1.23
Transportation	10.3	9.9	1.28	1.18	1.24
Housing	32.6	32.9	1.10	1.01	1.06
Health Care	5.7	7.8	0.84	0.95	0.90
Apparel	4.1	3.3	1.23	1.12	1.18
Entertainment	5.1	5.1	1.45	1.13	1.29
Cash Contribution	3.5	3.2	1.64	1.29	1.47
Sum of 8 Categories ( <b>CEX8</b> )	<b>73.8</b>	<b>75.1</b>	–	–	–
Others	15.0	13.6	–	–	–
Sum of All Above ( <b>CEX NET</b> )	<b>88.8</b>	<b>88.7</b>	–	–	–
Personal Insurance and Pension	11.2	11.3	2.82	2.5	2.66
All Items	100.0	100.0	1.00	1.00	1.00

Note: The data are based on the annual overall expenditure shares and mean expenditures of the 1st and 5th income quintile (before taxes) from the Consumer Expenditure Survey 2005 (Table 1) and 2015 (Table 1101). “Transportation” excludes vehicle purchases. “Others” are other miscellaneous categories and “Cash Contribution” is cash donation.



Table 2: Cyclicalilty of Labor Wedges

Consumption Measure for Marginal Utility	$\sigma = 0.5$ $\gamma = 1$	$\sigma = 0.1$ $\gamma = 1$	$\sigma = 1$ $\gamma = 1$	$\sigma = 0.5$ $\gamma = 2$	$\sigma = 0.5$ $\gamma = 0.5$
(1) PCE-All	2.15	7.71	1.46	1.68	3.10
(2) PCE-NDS	1.88	6.35	1.32	1.40	2.83
(3) <b>CEX8-Aggregate</b>	<b>1.52</b>	<b>4.55</b>	<b>1.14</b>	<b>1.05</b>	<b>2.47</b>
(4) <b>CEX8-Engel</b>	<b>1.38</b>	<b>3.85</b>	<b>1.07</b>	<b>0.90</b>	<b>2.33</b>
(5) Constant $MU_C$	0.76	0.76	0.76	0.29	1.71
(6) $\frac{(4)}{(3)} - 1$	-9.2%	-15.4%	-6.2%	-13.4%	-5.7%
(7) $\frac{(5)}{(3)} - 1$	-50%	-83%	-33%	-73%	-31%
(8) $\frac{(6)}{(7)}$	18.5%	18.5%	18.5%	18.5%	18.5%

Note: Rows (1) through (5) display the regression coefficient of labor-market wedge growth rates on GDP growth rates for different measures of consumption in the construction of marginal utility of consumption ( $MU_C$ ). Rows (1) and (2) use personal consumption expenditures (PCE) from the NIPA, all categories or nondurable goods and services only. Rows (3) and (4) use the 8 categories in the CEX, where CEX8-Aggregate uses the Divisia-Aggregate and CEX8-Engel uses the CEX8-components together with the relative Engel-curve parameters from the last column of Table 1. Row (5), considers the limit for  $\sigma$  large, when  $MU_C$  is a constant and independent of the measure of consumption.