Irreversibility and Monitoring in Dynamic Games: Experimental Evidence*

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Abstract

This paper provides experimental evidence on the impacts of irreversibility and imperfect monitoring on the efficiency and the equity of repeated public goods game. We find that irreversibility and imperfect monitoring both lead to inefficient and unequal outcomes through different channels. Irreversibility lowers public goods contribution in earlier periods and makes the initial-period contribution gap between two players long-lasting. Imperfect monitoring hampers conditional cooperation and reduces group contribution persistently. A finite mixture estimation with conditional cooperator provides a coherent account of the treatment effects.

Keywords: repeated games; dynamic games; imperfect monitoring; irreversibility; cooperation

JEL classification: C73, C91, C92

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1 Introduction

In many economic and social settings, agents have incentives to free ride and benefit from others' contributions, often leading to sub-optimal outcomes. Repeated interactions can help resolve this problem when long term benefits from mutual cooperation outweigh a short term individual gain of free-riding. When a deviation from a cooperative path is observed, players punish the defector by choosing a non-cooperative behavior. If they are sufficiently patient, such a punishment threat can deter players from deviating for short term gains (e.g., Abreu, 1988 and Maskin and Fudenberg, 1986). The power of dynamic incentives to sustain cooperation can, however, be limited if the actions of others are imperfectly observed and the level of individual contribution is constrained to be non-decreasing. This paper examines experimentally the roles of irreversibility of actions and imperfect monitoring in cooperation in dynamic interaction.

When actions are constrained to be irreversible, cooperation must occur gradually over time (e.g., Lockwood and Thomas, 2002 and Marx and Matthews, 2000). This is because the threat to return to a non-cooperative outcome is no longer available and the only threat available is the withdrawal of future contributions. Players must then build up cooperation gradually. Such gradualism will delay players' reaching contribution levels close to the efficient level, reducing efficiency. When imperfect monitoring is added, however, Guéron (2015) shows that, under some continuity assumptions, there cannot be any contributions in equilibrium. The reason is that as increments in cooperation become arbitrarily small, any given monitoring technology will no longer be sufficiently precise to detect deviations, leading to the unravelling of any potential cooperative equilibrium.

The stage game of our experiment is a continuous action prisoner's dilemma with a linear kinked payoff structure in which free riding is strictly dominant for each player and there exists an upper bound on the limits of cooperation as socially optimal. The structure is similar to that of a linear public goods game until the kink, after which contributions no longer generate any benefits. The introduction of this upper bound reflects the fact that returns to contribution to the public good eventually decrease with the level of contributions.[1] Moreover, it allows to study efficiency and equality independently of one another. Indeed, in a standard public good game, the sum of payoffs is maximal when all contribute the full amount of their endowment. In contrast, in our setting, the sum of payoffs is maximal when the sum of contributions reaches a certain threshold, irrespective of how this is shared between players. This allows us to investigate inequality between players irrespective of efficiency.

[1] See, for the application of the diminishing returns of the public good, Demsetz (1970), Laury et al. (1999) and Battaglini et al. (2016).

We study four different types of dynamic games with reversible or irreversible actions, and perfect or imperfect public monitoring. With reversible actions, theory predicts that all feasible and individually rational payoffs can be obtained in equilibrium, both under perfect or imperfect monitoring. With irreversibility and perfect monitoring, the Pareto frontier of the equilibrium payoff set is now bounded away from the Pareto frontier of the feasible payoff set because of the inefficiency induced by gradualism (Lockwood and Thomas, 2002). Finally, with both irreversibility and imperfect monitoring, no cooperation is feasible in equilibrium (Guéron, 2015).

By using the comparative static comparison of equilibrium sets across treatments, we make two hypotheses that can be tested with our experimental data. First, irreversibility of actions lowers subjects' payoffs both in the games with perfect monitoring and in the games with imperfect monitoring. Second, imperfect monitoring lower subjects' payoffs in the games with irreversible actions but does not affect their payoffs in the games with reversible actions.

Both irreversibility and imperfect monitoring cause significant efficiency losses in our experiment. Defining the *efficiency ratio* as the ratio of the sum of the supergame payoffs of players and the maximal possible sum of the supergame payoffs in excess over the stage-game Nash equilibrium payoff, we observe a 36 percentage-point decline in efficiency due to irreversibility and a 15 percentage-point drop due to imperfect monitoring. Under irreversibility, inefficiency is caused by a delay in cooperation, as evidenced by the fact that cooperation is reduced mostly in the early periods of the game. In contrast, imperfect monitoring reduces group contributions persistently over all periods.

Our study can also shed some light on the sources of inequality, as we find that both irreversibility and imperfect monitoring tend to increase payoff inequality between players. Introducing the *inequality ratio* as the absolute value of the difference between the two players' supergame payoffs, normalized by their sum, we find that irreversibility increases inequality by a six percentage point while imperfect monitoring does so by a four percentage point. Under irreversibility, a gap in contributions at the initial period is not easily corrected and persists throughout the game.

To make a behavioral account of the treatment effects on efficiency and equity, we implement a finite mixture estimation and infer the proportion of conditional cooperator across the treatments. We find that irreversibility increases the proportion of conditional cooperator, whereas imperfect monitoring lowers it. Using our estimation of conditional cooperation, along with the behavior of subjects at the initial period, we then simulate the contribution paths of players across all treatments. The resulting efficiency and equality ratios are closely in line with our data, suggesting that this mixture model can provide a coherent account of the experimental evidence.

Our paper contributes to the experimental literature on dynamic games with irreversible actions. Studies of sequential provision of public goods (Dorsey, 1992; Duffy et al., 2007; Choi et al., 2008; Diev and Hichri, 2008; Choi et al., 2011) consider how a gradual decision process can affect the provision in a static public goods game. Though the decision is made over multiple periods, players' utilities are determined only by the final level of the public good in those studies. In our setup, each period is a distinct stage game with a flow payoff for each player.

The most closely related paper is Battaglini et al. (2016). In their irreversibility treatment, players can not decrease the amount of public goods contribution over periods which is a similar design to us. However, their reversibility treatment is also a dynamic game, but in which the public contributions can be scaled back in the future. They find that irreversibility leads to higher public good production than reversible investment. In contrast, we consider the irreversibility of actions in the standard setting of a repeated prisoner's dilemma game and find that it leads to higher inefficiency and more unequal outcomes than reversible actions.[2]

We also contribute to the experimental literature on repeated games. Many studies on infinitely repeated prisoner's dilemma games have focused on identifying complicated history-dependent strategies and understanding the determinants of cooperation (Dal Bó, 2005; Blonski et al., 2011; Fudenberg et al., 2012; Sherstyuk et al., 2013).[3] They find that going from a one-shot prisoner's dilemma games to an infinitely repeated game increases the rate of cooperative behavior significantly. Our study is the first experimental paper that examines the joint effects of irreversibility and imperfect monitoring on both efficiency and equality, in a prisoner's dilemma setting.

Another strand of the literature considers the role of imperfect monitoring in repeated games. Aoyagi and Frechette (2009) are the first to introduce imperfect monitoring in an experimental setting, and find some efficiency loss when noise is large. This is in contrast with most of the follow-up literature, such as Fudenberg et al. (2012), Embrey et al. (2013), or Aoyagi et al. (2015), who do not see an impact of imperfect public monitoring on efficiency. We use the multiplicative noise structure with a continuum of actions and find that imperfect monitoring reduces efficiency, in line with the findings of Aoyagi and Frechette (2009), and equality of payoffs between players.

The rest of this paper is organized as follows: Section 2 introduces our model and theoretical results; Section 3 describes our experimental design; Section 4 presents the results of the experiment; finally, Section 5 concludes.

[2] Kurzban et al. (2008) introduces irreversibility to the trust game and does not find any effect on the first mover's investment level.

[3] See also a recent survey is Dal Bo and Frechette (2016).

2 Theory

2.1 Setup

The stage game is a continuous action prisoner's dilemma. There are two players, $i = 1, 2$, and each player chooses a contribution level $c_i \in [0, \infty)$, $i = 1, 2$. Payoffs are given by

$$u_i(c_1, c_2) = \pi(c_i, c_j) = \begin{cases} \pi_1 c_i + \pi_2 c_j & \text{if } c_i + c_j \leq 2c^*, \\ \pi_1 c_i + \pi_2 (2c^* - c_i) & \text{if } c_i + c_j > 2c^*, \end{cases}$$

where $\pi_1 < 0$, $\pi_2 > 0$, and $\pi_1 + \pi_2 > 0$.

A player's payoff is strictly decreasing in their own contribution, weakly increasing in the other player's contribution, and total payoffs are increasing in the sum of both players' contributions until the sum reaches $2c^*$. Hence, it is socially optimal for the two players to cooperate and reach $2c^*$ although it is not in any player's interest to cooperate. In this sense, the stage game we consider has a prisoner's dilemma structure.

Time is infinite, $t = 0, 1, \ldots$. In each period there is a probability $(1 - \delta) \in (0, 1)$ that the interaction stops. Payoffs are realized once the interaction stops. In each period t, players choose contribution levels $(c_{1,t}, c_{2,t})$.

Monitoring. We consider two cases of monitoring: perfect monitoring vs. imperfect (public) monitoring. In games of perfect monitoring, the period-τ expected payoff of player i is given by the following normalized discounted sum:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t \pi(c_{i,t+\tau}, c_{j,t+\tau}).$$

In games of imperfect public monitoring, players observe a public signal y_t, which is the only information they have about their partner's play. The distribution of y_t conditional on contribution levels $(c_{1,t}, c_{2,t})$ is common knowledge and satisfy a continuity requirement: for small changes in actions, the change in the distribution of the signal is also small (see Guéron, 2015). We use a noise structure that is multiplicative in the sum of contributions made by two players:

$$y_t = (c_{1,t} + c_{2,t})\, \epsilon_t,$$

where the ϵ_t are *iid* and $\epsilon_t \sim \mathcal{U}[1 - r, 1 + r]$ for $r \in (0, 1)$, for all t. The parameter r represents the size of noise, relative to the sum of contributions. For example, if $r = 0.1$, players will observe a signal that is within plus or minus ten percent of the sum of contributions.

There are two noteworthy implications of using the multiplicative noise structure. First, if both players do not contribute at all, this will be perfectly known. Moreover, as the sum of contributions increases, the range of the public signal increases. It is simple and easy to explain this noise structure to participants of the experiment. Its main advantage over an additive noise structure in our setup is that it avoids having the possibility of subjects observing negative signals, which is not natural when interpreting actions as contributions to a public good.

In games of imperfect public monitoring, a strategy depends on the public signal rather than on the other player's contribution, as well as one's own past contribution. Players will then form expectations as to which histories may have occurred and will evaluate payoffs according to those expectations.

Irreversibility. In the repeated game, actions are *reversible* in the sense that they are unconstrained and players face the same strategic interaction in each period. In the dynamic game, actions are *irreversible* in the sense that they are constrained to be weakly increasing over time:

$$c_{i,t+1} \geq c_{i,t}$$

for $i = 1, 2$ and $t = 0, 1, 2, \ldots$. With irreversibility, the game becomes a dynamic contribution game in which past contributions are not refundable, as opposed to a repeated public goods game.

2.2 Equilibrium characterization

Let us first define the set of feasible and individually rational payoffs. Consider the sum of the stage game payoffs, which is given by $u_1(c_1, c_2) + u_2(c_1, c_2) = (\pi_1 + \pi_2)(c_1 + c_2)$ if $c_1 + c_2 \leq 2c^*$ and by $u_1(c_1, c_2) + u_2(c_1, c_2) = (\pi_1 - \pi_2)(c_1 + c_2) + 4\pi_2 c^*$ if $c_1 + c_2 > 2c^*$. It is maximized when when $c_1 + c_2 = 2c^*$, and the maximum payoff a player can get (when only the other is contributing) is $2\pi_2 c^*$. Also, note that a player can always guarantee at least a payoff of zero by not contributing. Hence, the set of feasible and individually rational payoffs is given by

$$\mathcal{F}^* = \{(u_1, u_2) \mid u_1 + u_2 \leq 2(\pi_1 + \pi_2)c^*; u_1 \leq 2\pi_2 c^*; u_2 \leq 2\pi_2 c^*; u_1 \geq 0; u_2 \geq 0\}.$$

We now characterize the equilibrium payoff set for each of the four games that are used in the experiment. All proofs are deferred to Online Appendix A.

In the repeated game with perfect monitoring, we give a necessary and sufficient condition for any feasible and individually rational payoff to be an equilibrium payoff.

Proposition 1. *Consider the repeated game with perfect monitoring. When* $\delta \geq -\pi_1/\pi_2$*, any feasible and individually rational payoff is an equilibrium payoff of the repeated game. When* $\delta < -\pi_1/\pi_2$*, the only equilibrium is when players do not contribute after any history.*

To characterize the equilibrium payoff set in the repeated game with perfect monitoring, we first consider a set of simple stationary grim-trigger strategies: players play a given action profile (c_1, c_2) in each period, unless someone deviates in which case they play the unique stage-game Nash equilibrium $(0, 0)$ forever. This allows us to recover an important part of the Pareto frontier of $\mathcal{F}^*$, except for the most asymmetric payoffs.

We then consider the following non-stationary grim-trigger strategy: (i) from period 1 onward, players play an efficient stationary grim-trigger equilibrium; and (ii) in period 0, one player contributes nothing while the other contributes such that his repeated game payoff is 0. Incentive compatibility is straightforward to verify as the player contributing in period 1 gets a payoff of zero whether or not he contributes. It allows us to recover the whole Pareto frontier. Given that $(0, 0)$ is also an equilibrium payoff, convexity then ensures that any feasible and individually rational payoff is an equilibrium payoff.

Next, for the repeated game with imperfect monitoring, we provide a sufficient condition for any feasible and individually rational payoff to be an equilibrium payoffs in Proposition 2. The parameters in our experiment are chosen so as to satisfy this condition.

Proposition 2. *Consider the repeated game with imperfect monitoring and noise level* r*. When* $\delta \geq -\pi_1/\pi_2$ *and* $\frac{\pi_2\delta(1-r)+2r\pi_1(1-\delta)}{-\pi_1[\delta(1-r)+2r(1-\delta)]} \geq -\pi_2\delta/\pi_1$*, then any feasible and individually rational payoff is an equilibrium payoff of the repeated game.*

The proof of this result is similar to the case of perfect monitoring and reversible actions, first considering simple grim trigger strategies and then non-stationary ones. The proof also relies on the fact that our monitoring technology is such that the public signal has an interval support, and maximal punishments for realizations outside of this interval serve as sufficient deterrents against deviations.

In the dynamic game with irreversible actions and perfect monitoring, we show that the equilibrium payoff set is bounded away from the Pareto frontier of the feasible and individually rational payoff set.

Proposition 3. *Consider the dynamic game with irreversibility and perfect monitoring. When* $\delta \geq -\pi_1/\pi_2$*, then the equilibrium payoff set is bounded away from the Pareto frontier of the feasible payoff set by a factor* $1 - \delta$*. When* $\delta < -\pi_1/\pi_2$ *then the only equilibrium is when players do not contribute.*

Cooperation becomes more difficult with the irreversibility constraint. Under perfect monitoring, players must gradually increase their contributions (see Lockwood and Thomas, 2002). Intuitively, since players cannot lower contribution from the past level, the only way to punish deviations is to always hold back a fraction of contribution from the efficient level. Hence, players follow an increasing contribution path and threaten to stop the increase should a deviation occur. This gradualism means that the efficient frontier shifts inward.

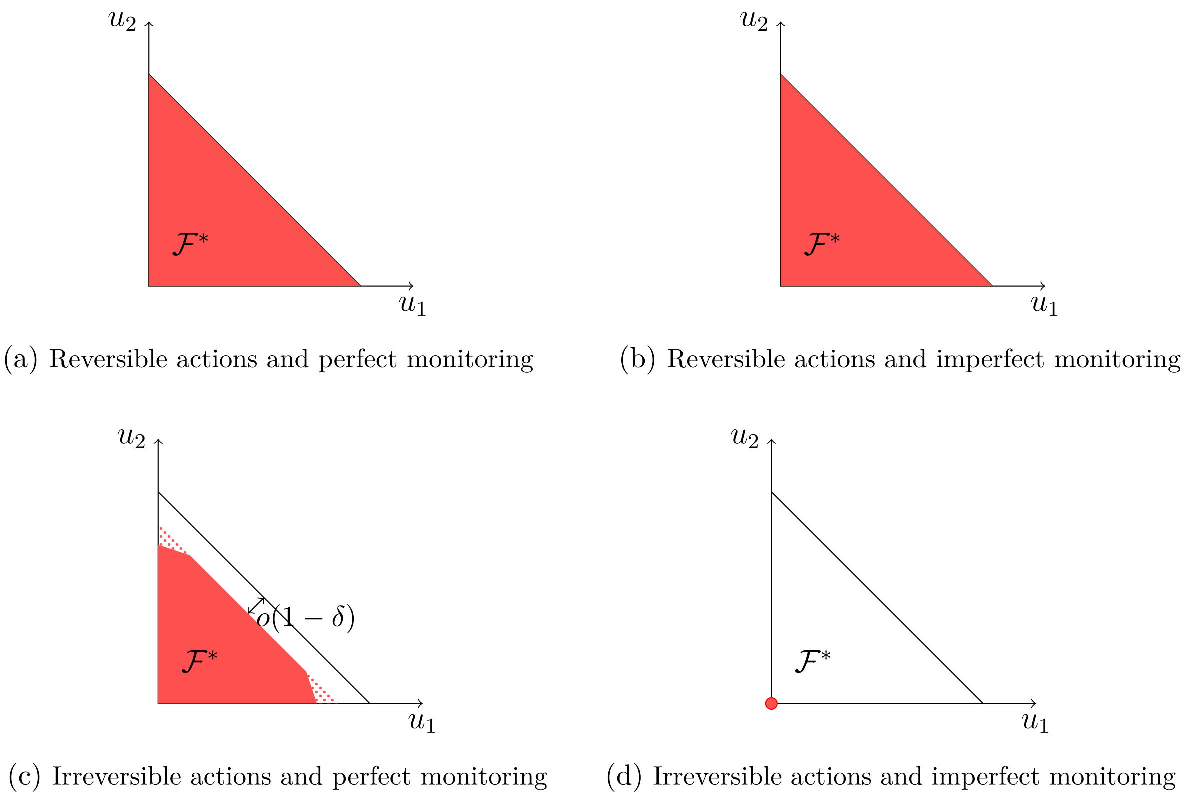
To characterize part of the efficient frontier, we consider an equilibrium path for which all the no-deviation constraints hold with equality. This gives us a system of second-order difference equations, whose solution allow us to recover part of the frontier of the equilibrium payoff set. It is bounded away from the efficient frontier (although this inefficiency disappears as the discount factor converges to one). It is however not possible to provide a full characterization of the payoff frontier in that case, but we do provide upper and lower bounds.

Finally, in the game with irreversible actions and imperfect monitoring, Guéron (2015) shows that under a continuity condition of the monitoring technology, players do not contribute in equilibrium. The continuity condition is met in our experiment.

Proposition 4. *Consider the dynamic game with imperfect perfect monitoring. Then the only equilibrium consist in players not contributing, after any history.*

To see why cooperation breaks down with imperfect monitoring, first note that in any equilibrium contributions will be bounded from above by their efficient level, and consider a potential equilibrium in which players contribute. As contributions must increase, and are bounded from above, they eventually converge. Close to the upper bound, a player can profitably deviate by slightly reducing their contribution level and then returning to the prescribed equilibrium. Such a deviation would lead to a punishment by the other player, if detected. However, close to the upper bound of contributions, such a punishment would be arbitrarily small. Moreover, when the monitoring technology is continuous, the probability of detection is small if the deviation is also small. A small punishment, coupled with a small probability of detection, eventually renders the deviation profitable, leading to an unravelling of cooperation.

The equilibrium payoff sets of the four games are visually summarized in Figure 1. The straight line in each of the four panels represent the Pareto frontier of the feasible and individually rational payoff set, and the area shaded in red represents the equilibrium payoff set. Figures 1a) and 1b) show that in the repeated games, all feasible payoffs are equilibrium payoffs. Figure 1c) shows that with perfect monitoring and irreversibility, the payoff set is bounded away from the Pareto frontier of the feasible set. Finally Figure 1d)



(a) Reversible actions and perfect monitoring
(b) Reversible actions and imperfect monitoring
(c) Irreversible actions and perfect monitoring
(d) Irreversible actions and imperfect monitoring

Figure 1: Equilibrium payoff sets

shows that the unique equilibrium outcome with irreversibility and imperfect monitoring is $(0, 0)$, which corresponds to no contribution.

By comparing the equilibrim payoff sets, we hypothesize in regard to the impacts of irreversibility and imperfect monitoring as follows.

Hypothesis 1 *Irreversibility lowers payoffs both in the games with perfect monitoring and in the games with imperfect monitoring.*

Hypothesis 2 *Imperfect monitoring lowers payoffs in the games with irreversible actions but does not affect payoffs in the games with reversible actions.*

3 Experimental Design and Procedures

In the experiment we use the following parameter values of the linear kinked specification: $\pi_1 = -1$, $\pi_2 = 3$, and $c^* = 50$. Payoffs are calculated in terms of South Korean Won (KRW).[4] To preclude negative payoffs, 100 is added to each subject's stage game payoff. Therefore, the stage game payoff function is given by

[4]In 2016, the average exchange rate is around 1 USD = 1,050 KRW.

$$u_i(c_{i,t}, c_{j,t}) = \begin{cases} 100 - c_{i,t} + 3c_{j,t} & \text{if } c_{i,t} + c_{j,t} \leq 100, \\ 400 - 4c_{i,t} & \text{if } c_{i,t} + c_{j,t} > 100. \end{cases}$$

Subjects choose a contribution level between 0 and 100 using a slide bar, with a precision of two decimal places. Each player has incentives to free ride and receives benefit from the other's contribution as long as the sum of contributions is below 100.

The experiment consists of the four dynamic game treatments by varying the structures of reversibility and monitoring: the game with reversible actions and perfect monitoring (RA-PM), the game with reversible actions and imperfect monitoring (RA-IPM), the game with irreversible actions and perfect monitoring (IRA-PM), and the game with irreversible actions and imperfect monitoring (IRA-IPM). With regard to the noise structure in the experiment, it follows a uniform distribution and can be as large as 10% of the sum of contributions. For example, if the total contribution in the previous period is 50, the noise is randomly drawn from the distribution $\mathcal{U}[-5, 5]$. In each treatment, after each stage game is played, supergames continue to a next stage game with probability 90% ($\delta = 0.9$) and terminates with probability 10%.

There are 16 sessions in total, with four for each treatment. The experiment was computerized with z-Tree (Fischbacher, 2007) and run at the Center for Experimental and Behavioral Social Science Lab at Seoul National University during the period between September and November 2016. The subjects in the experiment were recruited from the pool of Seoul National University undergraduate students from all disciplines through official website of the university. Each subject participated in only one of the experimental sessions. After subjects had read the instructions, the experimenter read aloud the instructions.[5] In each session subjects played a series of supergames, at each of which they were randomly and anonymously paired with their partners. Subjects played supergames for 75 minutes in a session. To control supergame lengths across treatments, we use the random termination rule ($\delta = 0.9$) for the four sessions with the IRA-IPM treatment. Then we take the realized series of supergame lengths from each session and use these series for the sessions of the other treatments.[6] After each supergame ends, subjects observed the results of that supergame, including their own actions, their partners' actions, public signals, and their payoffs across periods. To make a parallel structure between IPM and PM, we also give public signal as the sum of contribution in the previous round in the PM treatments.

In the experiment the average length of the supergames is 9.7 periods, close to the

[5]Sample instructions are available in Online Appendix B.

[6]This method is commonly used in the experimental literature of repeated games (e.g., Aoyagi et al., 2015) to control the effect of length in supergames across treatments.

Table 1: Summary information on the treatments

Treatment	Subjects	Session	Subjects per session	Supergames	Rounds per supergame			Average earnings
					Average	max	min	(unit:KRW)
RA-PM	56	4	12,14,14,16	13,14,16,16	9.8	28	1	24,100
RA-IPM	58	4	12,14,16,16	10,14,18,21	9.6	28	1	22,100
IRA-PM	58	4	14,14,14,16	13,17,18,19	9.6	28	1	21,200
IRA-IPM	54	4	12,14,14,14	14,16,17,23	9.9	34	1	19,400

expected length of 10 given the value of the continuation probability, with the maximum length being 34 periods. Participants accumulated their earnings across supergames and on averaged earned KRW 25,800 including the show-up fee of KRW 5,000. Table 1 provides the description of the experimental sessions of the four treatments. In total, 226 subjects participated in the experiment.

4 Results

We begin our analysis of the experimental data by plotting the two players' supergame-level earnings for each treatment. In order to facilitate the comparison between observed earnings and equilibrium payoff sets presented in Figure 1, we compute the normalized discounted sum of a subject's payoffs across periods at the supergame level, conditional on the realization of random termination. Suppose that a supergame lasted for K periods and a subject received a stream of payoffs $P_1, ..., P_K$. The normalized sum of payoffs for this subject is defined as $\frac{P_1+\delta P_2+...+\delta^{K-1}P_K}{1+\delta+\delta^2+...+\delta^{K-1}}$. For the sake of brevity, we refer to the normalized discounted sum of payoffs as the supergame payoff.

Figure 2 shows the scatter plots of two players' supergame payoffs for each of the four treatments. Because player labeling is arbitrary, each figure contains two players' supergame payoffs by permuting their labels. The set of feasible and individually rational payoffs is represented by the triangle made by orange (horizontal and vertical) lines.[7] Note that it is equivalent to the equilibrium payoff sets for the RA-PM and RA-IPM treatments.

The top left panel of Figure 2 shows that a large proportion of the data in the RA-PM treatment are clustered around the Pareto optimal and symmetric outcome (200, 200). This outcome can be achieved only if both players contributed 50 from the first period onward. The introduction of imperfect monitoring (the top right panel) or irreversibility (the bottom left panel) appears to make subjects' supergame payoffs further away from

[7]Note that $(100, 100)$ is the supergame payoff obtained when both players make zero contributions throughout the game. Thus, $\mathcal{F}^* = \{(u_1, u_2) \mid u_1 + u_2 \leq 400; u_1 \geq 100; u_2 \geq 100\}$.

the Pareto optimal outcome. The treatment with both irreversible actions and imperfect monitoring (the bottom right panel) shows most clearly that a substantial proportion of the data are clustered around the stage-game Nash equilibrium payoffs (100, 100). We also note that dispersion of the data from the symmetric outcomes (45-degree line) appears to increase with the introduction of imperfect monitoring and irreversibility from the RA-PM treatment. [8]

We also note that the proportion of observed supergame payoffs that fall below the individually rational payoff of a player appears to small but increase with both imperfect monitoring and irreversibility of actions. The theory predicts that people should never get worse than the stage-game Nash equilibrium payoff of 100. Nonetheless, we observe that some subjects occasionally got a supergame payoff lower than the stage-game Nash equilibrium. Specifically, these frequencies are 2.4% in the RA-PM treatment, 6.1% in the RA-IPM, 8.5% in the IRA-PM, and 19.9% in the IRA-IPM treatment.

4.1 Efficiency and Equity

We examine the effects of irreversibility and imperfect monitoring on the efficiency and equity of supergame outcomes by introducing two ratios, the *efficiency ratio* and the *inequality ratio*.

The efficiency ratio is defined as the ratio of the sum of the players' supergame payoffs and their maximum possible sum, where both sums are taken in excess of the stage-game Nash equilibrium payoffs. This allows for the ratio to range between 0 and 1, being zero when both player earn the stage-game Nash equilibrium payoff and 1 when the two players coordinate to achieve the maximal sum of payoffs. The inequality ratio is defined as the absolute value of the difference between two players' supergame payoffs, normalized by their sum. It also ranges between 0 and 1, 0 meaning that both players have the same supergame payoff and 1 meaning a maximal payoff difference.

Table 2 reports the overall summary of how efficient and unequal supergame outcomes emerge across treatments. First, the efficiency level substantially declines with the introduction of irreversibility and imperfect monitoring. Irreversibility reduces efficiency by 37 percentage point in the games with perfect monitoring and by 35 percentage point in the games with imperfect monitoring. Imperfect monitoring lowers efficiency by 16 percentage point in the games with reversible actions and by 14 percentage point in the games with irreversible actions.

Second, payoff inequality increases substantially with irreversibility and imperfect

[8]In Online Appendix C.1, in order to control potential learning effects across supergames, we plotted the same figures by focusing on last 5 supergames in each session. The data patterns remain to be robust.

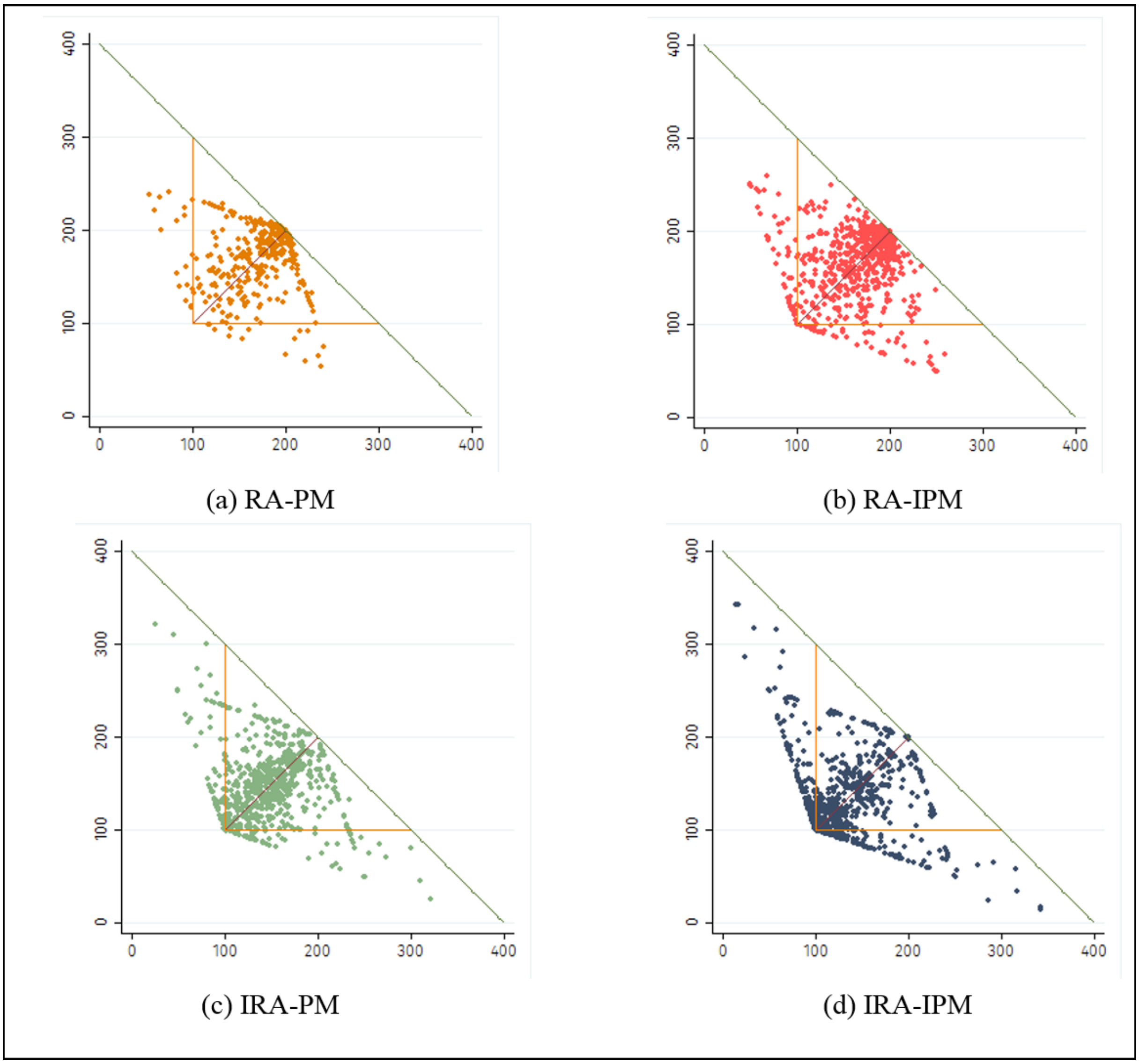


Figure 2: Scatter plots of supergame payoffs across treatments

Table 2: Summary statistics of supergame payoffs

Treatments	RA-PM	RA-IPM	IRA-PM	IRA-IPM
Variable: sum of supergame payoffs				
Mean	369.1	338.2	295.5	268.3
S.D.	45.3	53.1	51.2	52.8
Efficiency ratio	0.85	0.69	0.48	0.34
Variable: difference between supergame payoffs				
Mean	18.1	31.7	38.4	50.2
S.D.	31.3	39.1	49.7	61.5
Inequality ratio	0.06	0.10	0.13	0.18
Observations	351	383	377	354

monitoring, too. With regard to inequality ratio, irreversibility increases inequality by 7 percentage point in the games with perfect monitoring and by 8 percentage point in the games with imperfect monitoring. Imperfect monitoring makes inequality go up by 4 percentage point in the games with reversible actions and by 5 percentage point in the games with irreversible actions.

We next perform regression analysis on the effects of irreversibility and monitoring. We start with efficiency. Table 3 shows the linear regression results for efficiency ratio. The first column shows results with the pooled data across all treatments. Other columns reports results with the subsample of the data corresponding to a treatment. For example, column (2) includes the sample of the games of RA-PM and IRA-PM. Standard errors are clustered at the session level. All regression specifications include fixed effects by controlling for sequence of supergames in a session and their lengths.

In column (1), both the IRA and IPM dummy variables are significantly negative at 1% level. The efficiency loss is large: 36 percentage-point decline due to irreversibility of actions and 15 percentage-point drop due to imperfect monitoring. This effect of irreversibility is significantly greater than that of imperfect monitoring in our experimental setup (p-value < 0.01). Columns (2)$-$(5) in Table 3 show the regression analysis for each sub-sample. In all specifications the coefficients of IRA and IPM remain significant and quantitatively robust.

We connect our findings on efficiency to those in the literature. Firstly, in regard to the role of irreversibilty, Battaglini et al. (2016) studied experimentally a dynamic public

Table 3: Treatment effects on efficiency

	Dependent variable: efficiency ratio				
	(1)	(2)	(3)	(4)	(5)
Sample	All	PM	IPM	RA	IRA
IRA	-0.359***	-0.367***	-0.350***		
	(0.032)	(0.046)	(0.044)		
IPM	-0.146***			-0.155***	-0.137**
	(0.032)			(0.027)	(0.057)
Constant	0.713***	0.689***	0.592***	0.760***	0.309***
	(0.039)	(0.043)	(0.046)	(0.041)	(0.055)
Fixed effect	Y	Y	Y	Y	Y
Observation	1,465	728	737	734	731
R^2	0.412	0.444	0.354	0.177	0.282

Notes: Session level clustered standard error in parenthesis. */**/*** represent 10/5/1% significance level respectively. Fixed effects include sequence of supergame in a session and length of supergames.

goods provision game with perfect monitoring and compared between cases of reversible and irreversible actions. They found that the case of irreversible actions leads to a higher level of efficiency than that of reversion actions, which is in line with the theoretical prediction based on Markov Perfect equilibrium. Their theoretical and experimental findings are opposite to ours. Our theory predicts, based on the comparative static exercise of equilibrium sets, and the experimental data support that irreversibility leads to a lower level of efficiency. A main difference between their setup and ours is that subjects in Battaglini et al. (2016) accumulate the pubic goods provision over periods even in the game with reversible actions, whereas our game with reversible actions is the pure repetition of stage games of public goods provision.

Secondly, the experimental literature has found mixed evidence on the role of imperfect monitoring in efficiency. Aoyagi and Frechette (2009) found that imperfect monitoring leads to an increase in efficiency loss, while Aoyagi et al. (2015) found no impact of imperfect monitoring on efficiency. We believe that the detail of repeated games, including noise structure and action space, plays a role in such contrasting findings. Our experiment contributes to this literature by providing evidence that imperfect monitoring lowers efficiency in both conventional repeated games and dynamic games with irreversible actions, with the use of the multiplicative noise structure and continuous action space.

Result 1. *Each of irreversibility and imperfect monitoring causes inefficiency. Irreversibility results in greater efficiency loss than imperfect monitoring.*

Table 4: Treatment effects on equality

	Dependent variable: inequality ratio				
	(1)	(2)	(3)	(4)	(5)
Sample	All	PM	IPM	RA	IRA
IRA	0.064^{***}	0.059^{**}	0.066^{***}		
	(0.015)	(0.024)	(0.016)		
IPM	0.041^{***}			0.028^{***}	0.055^{**}
	(0.014)			(0.007)	(0.023)
Constant	0.193^{***}	0.193^{***}	0.236^{***}	0.224^{***}	0.229^{***}
	(0.021)	(0.021)	(0.030)	(0.021)	(0.030)
Fixed effect	Y	Y	Y	Y	Y
Observation	1,465	728	737	734	731
R^2	0.141	0.171	0.115	0.240	0.090

Notes: Session level clustered standard error in parenthesis. */**/*** represent significance level 10/5/1%. Fixed effects include sequence of supergame in a session, length of supergames, and sum of supergame payoff.

We move to the regression analysis regarding the effects of irreversibility and imperfect monitoring on payoff inequality between two players. The dependent variable is the inequality ratio. Table 4 reports the linear regression results in the same structure as Table 3, controlling for fixed effects on the sequence of supergames and their lengths and the sum of supergame payoffs as well. Standard errors are clustered at the session level.

The regression results confirm that both irreversibility and imperfect monitoring increase significantly payoff inequality between two subjects. Column (1), based on all data, shows that irreversibility increases inequality by 6 percentage point and imperfect monitoring does so by 4 percentage point. Both coefficients are significant at 1% level. The effect of irreversibility on unequal outcomes is significantly larger than that of imperfect monitoring (F statistics: 12.01 p-value<0.01). Columns (2)–(5) confirm the robust effects of irreversibility and imperfect monitoring on equity. We note that the coefficient of imperfect monitoring dummy is larger in the games with irreversible actions than in the games with reversible actions.

Result 2. *Each of irreversibility and imperfect monitoring causes inequality. Irreversibility results greater inequaility than imperfect monitoring.*

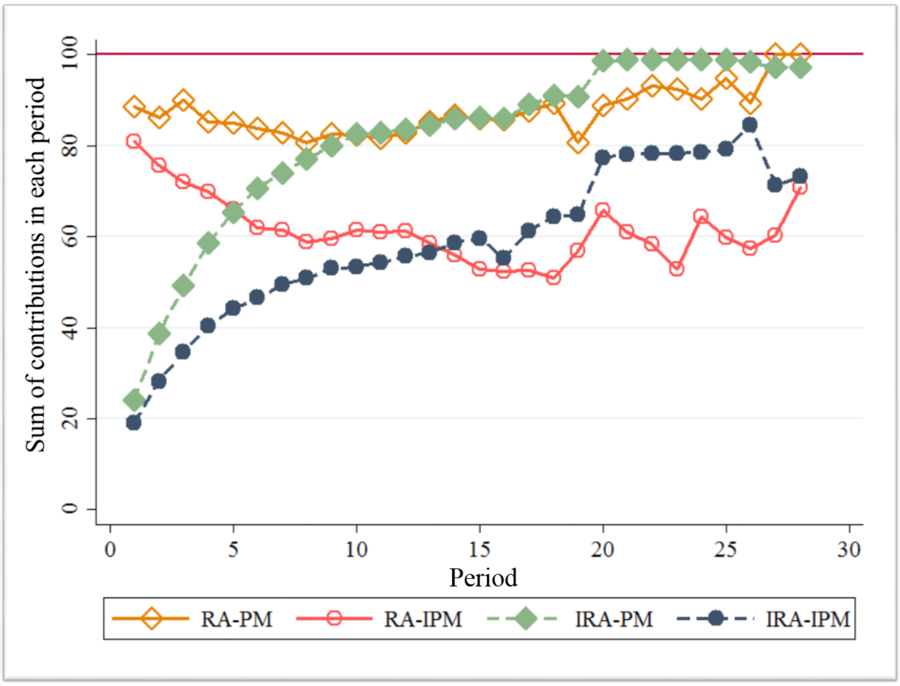


Figure 3: Sum of contributions by period

4.2 Contribution Dynamics

We have shown that both irreversibility and imperfect monitoring lead to inefficient and unequal supergame payoffs. In this section we investigate subjects' behavior and group dynamics to understand the differential treatment effects on efficiency and equality.

We begin by presenting the sum of two players' contributions per period across treatments in Figure 3. We take the average of the sum of contributions in each period over groups and sessions. The red horizontal line represents the efficient level of group contributions.

Figure 3 reveals two prominent features about the dynamics of group contributions. First, as the theoretical literature predicts (e.g., Lockwood and Thomas, 2002), cooperation is built gradually over time in both IRA-PM and IRA-IPM treatments when actions are irreversible. Comparing between RA and IRA treatments under each of monitoring technologies, we observe that contributions in early periods are much lower in an IRA treatment than in an RA treatment but they become similar or even higher in later periods. On the other hand, the first-period group contribution is similar in the IRA-PM and IRA-IPM treatments. But the gradient of gradual cooperation is much lower in the IRA-IPM treatment than in the IRA-PM treatment. As a result, we observe larger departure from the efficient level of cooperation when irreversibility is combined with imperfect monitoring. Given that the experimental literature has documented that human subjects often cooperate even when free riding is the only Nash equilibrium (Cooper et al., 1996; Ledyard, 1995), the theoretical prediction of zero contribution in the IRA-IPM treatment may be too stringent to hold in the experiment. Having pointed out this, a lower gradient of gradual cooperation and as a result greater inefficiency in the IRA-IPM treatment can be interpreted as qualitatively consistent with the theory. Overall, we conclude that irreversibility reduces players' supergame payoffs because of low contributions in early periods which are resulted from gradual cooperation.[9]

Second, imperfect monitoring lowers group contributions throughout the supergame. Comparing the RA-PM and RA-IPM treatments, the gap of group contributions in the first period is small but becomes wider and persists over later periods. This pattern is also similar when comparing the IRA-PM and IRA-IPM treatments.

Result 3. *(i) Irreversibility induces gradual cooperation and makes group contributions low in early periods, which results in greater efficiency loss. (ii) Imperfect monitoring reduces group contributions persistently over periods, leading to larger inefficiency.*

We next turn to check if the first-period contribution difference between two players has a long lasting effect over dynamics and results in payoff inequality at the supergame level. If one player contributes excessively but the other player does not in the first period, such initial discrepancy of contribution may be rectifiable by the former player in the games with reversible actions but may not be easily fixable in the games with irreversible actions. If it is the case, we can take it as one factor driving the effect of irreversibility on payoff inequality.

Table 5 shows linear regression analyses showing the impact of the first-period contribution difference on the average contribution difference in subsequent periods (that is, from period 2 to the end) of the supergame. Specifically, we identify who contributes more in the first period in each supergame and take the contribution difference between this subject and the other subject. For the dependent variable, we define the average contribution difference in subsequent periods between this subject and the other subject.

As we expected, in the games with irreversible actions (columns (3)–(4)), the first-period contribution difference has a positive effect on contribution gap in the following periods. On the other hand, it has no significant effect and the magnitude of the coefficient is close to zero in each game with reversible actions (columns (1)–(2)). It suggests that when we consider a behavioral model to explain the effect of irreversibility on equality, as presented in the next section, the initial contribution gap in the experimental data is one factor we need to take into account.

[9] To control the effect of learning, we report the comparable figure using the last 5 supergames in each session in Online Appendix C.2.

Table 5: Regression for persistent effect of period 1 difference

	Dependent variable: Overall contribution difference			
	(1)	(2)	(3)	(4)
Sample	RA-PM	RA-IPM	IRA-PM	IRA-IPM
Period 1 difference	-0.07	0.02	0.54^{***}	0.68^{**}
	(0.05)	(0.06)	(0.06)	(0.21)
Fixed effect	Y	Y	Y	Y
Constant	-18.19	0.32	0.70	1.85
	(8.11)	(3.26)	(1.33)	(6.16)
Observation	324	351	348	327
R^2	0.173	0.068	0.312	0.348

Notes: Session level clustered standard error in parenthesis. */**/*** represent significance level 10/5/1%. Fixed effects include sequence of supergame in a session and length of supergames.

4.3 Econometric Analysis of Individual Behavior

In order to make a parsimonious account of the treatment effects on efficiency and equality we found in the data, we consider a simple model that incorporates conditional cooperation or reciprocal behavior. It is well-known that human subjects behave reciprocally in a wide range of strategic situations including public goods provision and social dilemmas (e.g., Sugden, 1984; Fischbacher et al., 2001; Fischbacher and Gachter, 2010). On the other hand, diverse motives ranging from selfish preferences to altruism also drive heterogeneous human behaviors.

We adopt a finite mixture approach (McLachlan and Peel, 2004) in which each subject in the experiment can belong to one of two behavioral types τ: conditional cooperator ($\tau = cc$); or others ($\tau = others$). Because the types are not directly observable, from our perspective, each subject is assigned a probability of being a conditional cooperator. To classify, let the incremental contribution of individual i in period t be denoted by INC_t^i which is the difference of contribution between period t and $t-1$. We focus on increments rather than absolute levels of contributions because of the dynamic structure of the games. A conditional cooperator is assumed to positively adjust her increment in period t to their opponent j's increment of period $t-1$.[10] Suppose an individual i's increment at t is a function of j's increment at $t-1$ and the error term $\epsilon_{\tau t}^i$ with a normal

[10] We assume that subject i views the contribution of opponent j at period t as the difference between public signal and contribution of i at period t, containing some noise in IPM treatments.

distribution with zero-mean:

$$INC_t^i = \gamma_\tau + \beta_\tau INC_{t-1}^j + \epsilon_{\tau t}^i.$$

Our mixture model is characterized by eight parameters: $\beta = (\beta_{cc}, \beta_{others})$, $\gamma = (\gamma_{cc}, \gamma_{others})$, the fraction of conditional cooperator $\pi = (\pi_{cc}, \pi_{others})$[11], and the variance of error term $\sigma^2 = (\sigma_{cc}^2, \sigma_{others}^2)$. Then given the normal distribution error term assumption, we first construct the density of type τ for the ith individual as follows.

$$f(INC_t^i, INC_{t-1}^j; \beta_\tau, \gamma_\tau, \sigma_\tau) = \prod_{t=3}^{T} \phi(\frac{INC_t^i - (\gamma_\tau + \beta_\tau INC_{t-1}^j)}{\sigma_\tau})$$

where T and ϕ denote the number of rounds in the session and the standard normal distribution. We then construct a log likelihood function as follows:

$$\ln L(\beta, \gamma, \pi, \sigma^2 \mid INC_t^i, INC_{t-1}^j) = \sum_{i=1}^{N} \ln \sum_{\tau \in \{cc, others\}} \pi_\tau f\left(INC_t^i, INC_{t-1}^j \mid \beta_\tau, \gamma_\tau, \sigma_\tau\right)$$

Table 6 reports separately the results of the maximum likelihood estimation for each sample of the games with reversible actions and those with irreversible actions because of their different dynamic structure of the games. Our econometric method produces parameter estimates of the two types with one of them having a significantly positive value of β. We refer to that type as a conditional cooperator (cc) and the other type as others. After the model is estimated, we compute a posterior probability of an individual being a conditional cooperator, using the data of her choices. The rightmost two columns report the average of individual posterior probabilities of two types.

The estimation results reveal notable differences of subjects' behavior across treatments. First, the irreversibility structure of actions promotes conditional cooperation. It corroborate with the main feature of gradual cooperation predicted by theory and borne out by our data. The proportion of a conditional cooperator is higher in the IRA treatments than in the RA treatments: 69% for the IRA treatments and 36% for the RA treatments. The responsiveness of conditional cooperation appears to be higher in the IRA treatments than in the RA treatments: 0.41 in the IRA and 0.33 in the RA treatments. Second, perfect monitoring facilitates conditional cooperation. The posterior proportion of a conditional cooperator is higher in each game with perfect monitoring than that with imperfect monitoring: 51% vs. 19% in the RA games and 76% vs. 61% in the IRA games.

[11] π_{others} is equal to $1 - \pi_{cc}$ in 2 types classification case.

Table 6: Finite Mixture Estimation Results

Treatment	Type 1 (cc)			Type 2 (others)			Average posterior	
	β	γ	π	β	γ	π	Type 1	Type 2
RA-PM	0.33***	−0.45**	0.36***	0.01	0.02	0.64	0.51	0.49
	(0.01)	(0.19)	(0.03)	(0.13)	(0.23)			
RA-IPM							0.19	0.81
IRA-PM	0.41***	1.56***	0.69***	0.00	0.07	0.31	0.76	0.24
	(0.01)	(0.06)	(0.03)	(0.43)	(1.08)			
IRA-IPM							0.61	0.39

Notes: */**/*** represent significance level 10/5/1%. We note that in RA sample type 1 (2) σ is 22.56 (0.23). In IRA sample type 1 (2) σ is 4.61 (0.20). In both samples, type 1 sigma is significant at 5% level. The log-likelihoods of RA (IRA) is -21,188 (-18,723). The number of observations in RA (IRA) is 9,054 (8,906).

In order to evaluate whether the variations of conditional cooperation can explain the treatment effects on efficiency and equality, we conduct a simulation exercise using subjects' initial contributions, initial increments, realized length of supergame, and their posterior probabilities of the two behavioral types at each supergame in the experimental data. Specifically, each pair of subjects who played a supergame in the experiment is assigned to a joint posterior distribution of four possible matchings of behavioral types estimated in Table 6. This allows us to compute four possible contribution paths of a super game. By using the joint posterior distribution of the four type matchings, we compute the expected supergame payoffs of two players and derive the simulated efficiency and equality outcomes.

Table 7 shows the simulation results of efficiency and equality based on the finite mixture estimation results, and compare them with the observational patterns of the data. The simulated outcomes match quantitatively closely the corresponding observational outcomes across the treatments. In addition, the observational rankings of efficiency and equity over the treatments hold in the simulation exercise. Hence, we conclude that the variations of conditional cooperation across the treatments explain well the effects of irreversibility and imperfect monitoring on efficiency and equality.

Result 4. *(i) Irreversibility promotes conditional cooperation, while imperfect monitoring hampers it. (ii) These estimated variations of conditional cooperation explains well the treatment effects on efficiency and equality.*

Table 7: Comparing between observed and simulated outcomes

Treatments	RA-PM	RA-IPM	IRA-PM	IRA-IPM
Variable: sum of supergame payoff				
Observed mean	369.1	338.2	295.5	268.3
Simulated mean	363.6	344.5	305.4	278.3
Variable: efficiency ratio				
Observed mean	84.5%	69.1%	47.8%	34.2%
Simulated mean	81.8%	72.2%	52.7%	39.2%
Variable: difference between supergame payoff				
Observed mean	18.1	31.7	38.4	50.2
Simulated mean	24.5	38.8	45.7	55.2
Variable: inequality ratio				
Observed mean	5.6%	10.2%	12.9%	17.6%
Simulated mean	6.7%	11.3%	14.9%	19.8%

5 Concluding Remarks

We consider four dynamic public goods games, which vary along two dimensions. First, actions can either be reversible (i.e., unconstrained) or irreversible (i.e., constrained to be non-decreasing over time). Second, actions can either be perfectly monitored or imperfectly monitored. In the latter case, the distribution of public signal satisfies a mild continuity requirement.

Theory predicts that in the repeated public good game, whether monitoring is perfect or imperfect, the set of equilibrium payoffs is the whole set of feasible and individually rational payoffs. With irreversibility and perfect monitoring, the equilibrium payoff set is strictly included in the set of feasible payoffs, with an efficiency loss on the order $(1-\delta)$, where δ is the discount factor. Finally, with irreversibility and imperfect monitoring, cooperation is no longer feasible and no contributions ever take place in equilibrium.

Our experiments show that irreversibility and imperfect monitoring cause inefficiency and inequality. Irreversibility affects efficiency through gradualism, whereas imperfect monitoring lowers group contributions persistently over periods. We also find that irreversibility makes it hard to correct the initial contribution gap between players which results in inequality in the supergame. Lastly, both irreversibility and imperfect monitoring decreases the degree of conditional cooperation. This variation of conditional cooperation explains well the experimental data.

References

Abreu, D. (1988). On the theory of infinitely repeated games with discounting. *Econometrica: Journal of the Econometric Society*, 383–396.

Aoyagi, M., V. Bhaskar, and G. R. Frechette (2015). The impact of monitoring in infinitely repeated games: Perfect, public, and private. *43*(2), 113–149.

Aoyagi, M. and G. Frechette (2009). Collusion as public monitoring becomes noisy: Experimental evidence. *Journal of Economic theory 144*(3), 1135–1165.

Battaglini, M., S. Nunnari, and T. R. Palfrey (2016). The dynamic free rider problem: A laboratory study. *American Economic Journal: Microeconomics 8*(4), 268–308.

Blonski, M., P. Ockenfels, and G. Spagnolo (2011). Equilibrium selection in the repeated prisoner's dilemma: Axiomatic approach and experimental evidence. *American Economic Journal: Microeconomics 3*(3), 164–192.

Choi, S., D. Gale, and S. Kariv (2008). Sequential equilibrium in monotone games: A theory-based analysis of experimental data. *Journal of Economic Theory 143*(1), 302–330.

Choi, S., D. Gale, S. Kariv, and T. Palfrey (2011). Network architecture, salience and coordination. *Games and Economic Behavior 73*(1), 76–90.

Cooper, R., D. V. DeJong, R. Forsythe, and T. W. Ross (1996). Cooperation without reputation: Experimental evidence from prisoner's dilemma games. *Games and Economic Behavior 12*(2), 187–218.

Dal Bó, P. (2005). Cooperation under the shadow of the future: experimental evidence from infinitely repeated games. *The American Economic Review 95*(5), 1591–1604.

Dal Bo, P. and G. R. Frechette (2016). On the determinants of cooperation in infinitely repeated games: A survey. *Journal of Economic Literature*.

Demsetz, H. (1970). The private production of public goods. *The Journal of Law and Economics 13*(2), 293–306.

Diev, P. and W. Hichri (2008). Dynamic voluntary contributions to a discrete public good: Experimental evidence.

Dorsey, R. E. (1992). The voluntary contributions mechanism with real time revisions. *Public Choice 73*(3), 261–282.

Duffy, J., J. Ochs, and L. Vesterlund (2007). Giving little by little: Dynamic voluntary contribution games. *Journal of Public Economics 91*(9), 1708–1730.

Embrey, M., G. R. Fréchette, and E. Stacchetti (2013). An experimental study of imperfect public monitoring: Efficiency versus renegotiation-proofness.

Fischbacher, U. (2007). z-tree: Zurich toolbox for ready-made economic experiments. *Experimental economics* (2), 171–178.

Fischbacher, U. and S. Gachter (2010). Social preferences, beliefs, and the dynamics of free riding in public goods experiments. *American economic review 100*(1), 541–56.

Fischbacher, U., S. Gächter, and E. Fehr (2001). Are people conditionally cooperative? evidence from a public goods experiment. *Economics letters 71*(3), 397–404.

Fudenberg, D., D. G. Rand, and A. Dreber (2012). Slow to anger and fast to forgive: Cooperation in an uncertain world. *The American Economic Review 102*(2), 720–749.

Guéron, Y. (2015). Failure of gradualism under imperfect monitoring. *Journal of Economic Theory 157*, 128–145.

Kurzban, R., M. L. Rigdon, and B. J. Wilson (2008). Incremental approaches to establishing trust. *Experimental Economics 11*(4), 370–389.

Laury, S. K., J. M. Walker, and A. W. Williams (1999). The voluntary provision of a pure public good with diminishing marginal returns. *Public Choice 99*(1-2), 139–160.

Ledyard, J. (1995). Public goods: A survey of experimental research. kagel j, roth a, editörler. handbook of experimental economics içinde.

Lockwood, B. and J. P. Thomas (2002). Gradualism and irreversibility. *The Review of Economic Studies 69*(2), 339–356.

Marx, L. and S. Matthews (2000). Dynamic voluntary contribution to a public project. *Review of Economic Studies 67*(2), 327–358.

Maskin, E. and D. Fudenberg (1986). The folk theorem in repeated games with discounting or with incomplete information. *Econometrica 53*(3).

McLachlan, G. and D. Peel (2004). *Finite mixture models.* John Wiley & Sons.

Sherstyuk, K., N. Tarui, and T. Saijo (2013). Payment schemes in infinite-horizon experimental games. *Experimental Economics 16*(1), 125–153.

Sugden, R. (1984). Reciprocity: the supply of public goods through voluntary contributions. *The Economic Journal 94*(376), 772–787.

ONLINE APPENDICES

A Equilibrium payoffs

A.1 Feasible and individually rational payoffs

The sum of stage game payoffs is given by $u_1(c_1, c_2) + u_2(c_1, c_2) = (\pi_1 + \pi_2)(c_1 + c_2)$ if $c_1 + c_2 \leq 2c^*$, which is maximized when $c_1 + c_2 = 2c^*$. In that case, $u_1 + u_2 = 2(\pi_1 + \pi_2)c^*$. Note that the maximum payoff a player can get is when the other player is the only one to contribute, up to $2c^*$. Therefore we must have $u_i \leq 2\pi_2 c^*$, and the set of feasible payoffs is given by

$$\mathcal{F} = \{(u_1, u_2) \mid u_1 + u_2 \leq 2(\pi_1 + \pi_2)c^*; u_1 \leq 2\pi_2 c^*; u_2 \leq 2\pi_2 c^*\}.$$

However, a player can always guarantee a payoff of zero by not contributing, so that the set of feasible and individually rational payoffs is given by

$$\mathcal{F}^* = \{(u_1, u_2) \mid (u_1, u_2) \in \mathcal{F}; u_1 \geq 0; u_2 \geq 0\}.$$

A.2 Proof of Proposition 1: reversibility and perfect monitoring

In this section we show that any feasible and individually rational payoff profile $u \in \mathcal{F}^*$ is the payoff of a subgame-perfect equilibrium. We do so in two steps.

First, we characterize the payoffs that can be obtained by playing a simple stationary grim-trigger strategy: players play a given action profile (c_1, c_2) in each period, unless someone deviates in which case they play the unique stage-game Nash equilibrium $(0, 0)$ forever.

We then consider a "modified" grim-trigger strategy in the following way: 1) from period 1 onwards, players play an efficient stationary grim-trigger equilibrium; and 2) in period 0, one player contributes nothing while the other contributes such that his repeated game payoff is 0.

Stationary grim-trigger

We first restrict ourselves to a stationary grim-trigger, where players play a fixed profile (c_1, c_2) forever. If there is a deviation, they play the unique Nash equilibrium $(0, 0)$ forever.

Without loss of generality, we consider $c_i \geq c_j$, and check that Pi has no incentives to deviate. If Pi follows the strategy, he gets a payoff of $\pi_1 c_i + \pi_2 c_j$ in each period. If he deviates, he gets $\pi_2 c_j$ today followed by 0 forever afterwards.[12] Therefore Pi will not deviate if

$$\pi_1 c_i + \pi_2 c_j \geq (1-\delta)\pi_2 c_j,$$

and we will have an equilibrium if

$$c_j \leq c_i \leq -\frac{\pi_2}{\pi_1}\delta c_j.$$

Taking $i = 1, 2$, we obtain the following characterization of the set of actions that can be supported by a stationary grim-trigger equilibrium using Nash reversion:

$$\left\{(c_1, c_2) \mid c_1 + c_2 \leq 2c^*; -\frac{\pi_1}{\delta\pi_2}c_1 \leq c_2 \leq -\frac{\delta\pi_2}{\pi_1}c_1\right\}.$$

Note that for this to be possible we must have $-\frac{\pi_2}{\pi_1}\delta \geq 1$, that is $\delta \geq -\frac{\pi_1}{\pi_2}$. If not, the only equilibrium of the repeated game is to play $(0, 0)$ forever.

We see in particular that we can reach points on the Pareto frontier of the feasible payoffs when $c_1 + c_2 = 2c^*$. However not all the frontier can be recovered because of the constraints $-\frac{\pi_1}{\delta\pi_2}c_1 \leq c_2 \leq -\frac{\delta\pi_2}{\pi_1}c_1$. The maximal payoff that P1 can get will be when $c_1 + c_2 = 2c^*$ and $c_1 < c_2 = -\frac{\delta\pi_2}{\pi_1}c_1$, which corresponds to $c_1 = \frac{-\pi_1}{\delta\pi_2 - \pi_1}2c^*$ and $c_2 = \frac{\delta\pi_2}{\delta\pi_2 - \pi_1}$. The payoffs are then given by

$$(u_1, u_2) = \left(\frac{\pi_2^2\delta - \pi_1^2}{\pi_2^2\delta - \pi_1^2 - \pi_1\pi_2(1-\delta)}2c^*, \frac{-\pi_1\pi_2(1-\delta)}{\pi_2^2\delta - \pi_1^2 - \pi_1\pi_2(1-\delta)}2c^*\right).$$

Symmetrically, the highest payoff that P2 can obtain from a stationary grim-trigger strategy is when $c_1 + c_2 = 2c^*$ and $c_2 < c_1 = -\frac{\delta\pi_2}{\pi_1}c_2$, which corresponds to $c_2 = \frac{-\pi_1}{\delta\pi_2 - \pi_1}2c^*$ and $c_1 = \frac{\delta\pi_2}{\delta\pi_2 - \pi_1}$. The payoffs are then given by

$$(u_1, u_2) = \left(\frac{-\pi_1\pi_2(1-\delta)}{\pi_2^2\delta - \pi_1^2 - \pi_1\pi_2(1-\delta)}2c^*, \frac{\pi_2^2\delta - \pi_1^2}{\pi_2^2\delta - \pi_1^2 - \pi_1\pi_2(1-\delta)}2c^*\right).$$

Non-stationary grim-trigger

We now consider a class of non-stationary grim-trigger strategies with the following properties: 1) from period 1 onwards, both players play an efficient stationary grim-trigger equilibrium; and 2) in period 0, player i contributes 0 and player j contributes (less than

[12] This is because the best deviation possible is not to contribute, since contributing is strictly dominated.

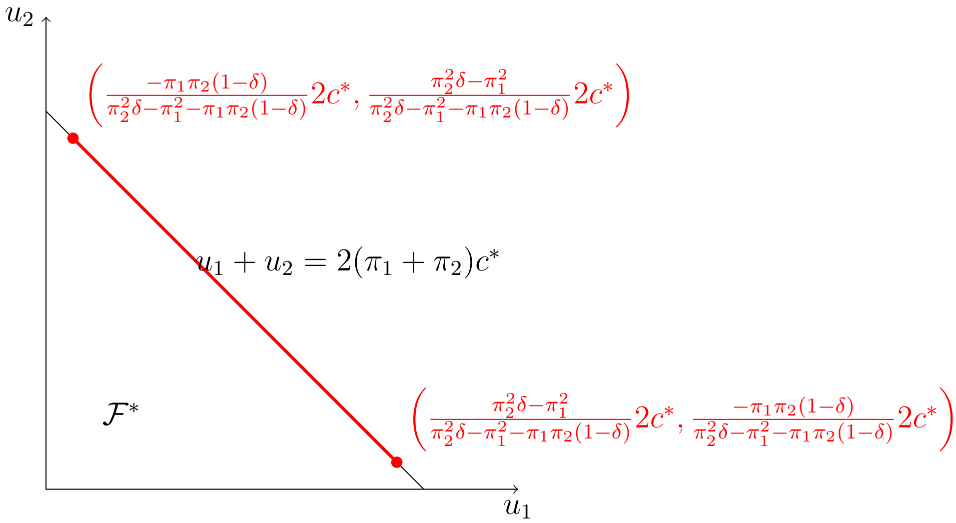


Figure 4: Pareto frontier using stationary grim-trigger strategies payoffs under perfect monitoring (in red)

$2c^*$) so that his repeated game payoff is 0, and any deviation triggers an infinite play of the stage-game Nash equilibrium.

Such a strategy is trivially a subgame-perfect equilibrium. First, from period 1 onward, players play a subgame perfect equilibrium. In period 0, Pi clearly has no incentive to deviate, as he is contributing zero. Finally, if Pj deviates and contributes zero in period 0, this is followed by an infinite repetition of $(0,0)$, and the payoff from such a deviation is 0, the same as the repeated game payoff under this modified grim-trigger strategy.

Fix a stationary grim-trigger equilibrium (c_1, c_2), which is played from period 1 onward. For it to be an equilibrium, we must have $c_1 + c_2 \leq 2c^*$ and $-\frac{\pi_1}{\delta\pi_2}c_1 \leq c_2 \leq -\frac{\delta\pi_2}{\pi_1}c_1$.

In period 0 player i contributes $c_{i,0} = 0$ and player j contributes $c_{j,0} \leq 2c^*$ so that the repeated game payoff of player j is 0, that is, such that:

$$(1-\delta)\pi_1 c_{j,0} + \delta(\pi_1 c_j + \pi_2 c_i) = 0.$$

This gives

$$c_{j,0} = \frac{\delta}{-\pi_1(1-\delta)}(\pi_1 c_j + \pi_2 c_i).$$

The equilibrium payoff of Pj from this non-stationary equilibrium is zero by construction, while the payoff of Pi is given by

$$(1-\delta)\pi_2 c_{j,0} + \delta\left(\pi_1 c_i + \pi_2 c_j\right) = (1-\delta)\pi_2 \frac{\delta}{-\pi_1(1-\delta)}\left(\pi_1 c_j + \pi_2 c_i\right) + \delta\left(\pi_1 c_i + \pi_2 c_j\right)$$
$$= \frac{(\pi_2-\pi_1)(\pi_1+\pi_2)}{-\pi_1}\delta c_i.$$

This payoff will be maximized when c_i is maximized, given the constraints $c_{j,0} \leq 2c^*$, $c_i + c_j \leq 2c^*$ and $-\frac{\pi_1}{\delta\pi_2}c_j \leq c_i \leq -\frac{\delta\pi_2}{\pi_1}c_j$. The solution involves having $c_i + c_j = 2c^*$ and $c_{j,0} = 2c^*$, in which that case we have

$$c_i = \frac{-\pi_1}{\delta(\pi_2-\pi_1)}2c^*,$$

and

$$c_j = \frac{-\delta\pi_2 + (1-\delta)\pi_1}{\delta(\pi_2-\pi_1)}2c^*,$$

and the repeated game payoff of Pi is therefore

$$\frac{(\pi_2-\pi_1)(\pi_1+\pi_2)}{-\pi_1}\delta\frac{-2c^*\pi_1}{\delta(\pi_2-\pi_1)} = (\pi_1+\pi_2)2c^*.$$

That is, the payoff from this strategy is one of the end point of the Pareto frontier of the feasible and individually rational payoff set. Given that $(0,0)$ is an equilibrium payoff and that the equilibrium payoff set is convex, any feasible and individually rational payoff must be an equilibrium payoff.

A.3 Proof of Proposition 2: Reversibility and imperfect monitoring

Again, we characterize the equilibrium payoff space in two steps. First we look at stationary grim-trigger strategies, and then consider non-stationary ones. We show that the set of equilibrium payoffs is again the full set of feasible and individually rational payoffs. This is due to our choice of parameters, and also our special monitoring structure, which does not have full support. The harshest punishment possible will therefore take place if the signal is outside of its expected support, and this is enough to prevent deviations and recover the whole payoff space in equilibrium.

Stationary grim-trigger

First, we consider the payoffs sustainable using the following stationary grim trigger strategy: play (c_1, c_2) forever, where $c_1+c_2 = 2c^*$, unless the signal falls bellow $(c_1+c_2)(1-r)$,

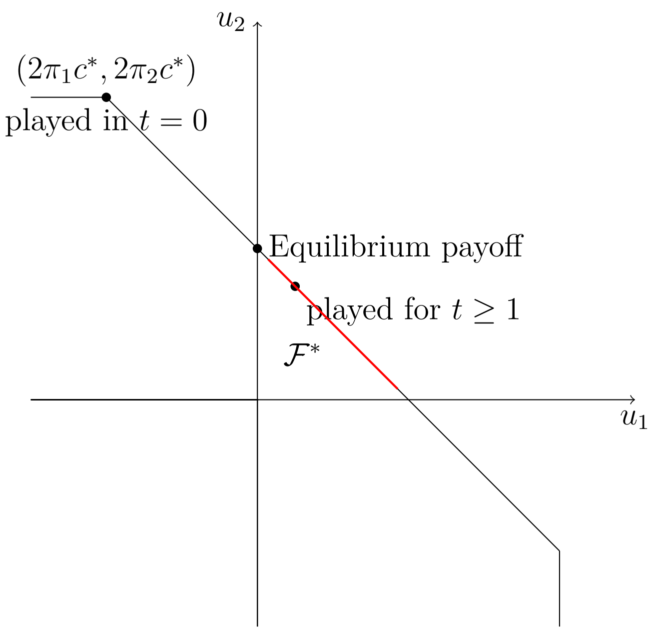


Figure 5: Equilibrium in which one player gets a payoff of zero using a non-stationary grim-trigger strategy

in which case play $(0, 0)$ forever. That is, players only punish if they know with certainty that one of them deviated.

Let us check under which condition there is no profitable one-shot deviation for P1. If P1 follows the strategy, his payoff is given by

$$\pi_1 c_1 + \pi_2 c_2.$$

Now consider a one-shot downward deviation $c_1 - x$, where $x \in [0, c_1]$. The probability that this deviation is detected is given by $\mathbb{P}\Big[(c_1 - x + c_2)\,\epsilon \leq (c_1 + c_2)\,(1 - r)\Big] = \min\Big\{1, \frac{1}{2}\frac{x}{c_1 - x + c_2}\frac{1-r}{r}\Big\}$, so that for moderate levels of deviation[13] the payoff is given by

$$(1 - \delta)\Big[\pi_1(c_1 - x) + \pi_2 c_2\Big] + \delta\frac{2(c_1 + c_2)r - x(1 + r)}{2r(c_1 + c_2 - x)}\Big[\pi_1 c_1 + \pi_2 c_2\Big].$$

Differentiating with respect to x gives

$$-(1 - \delta)\pi_1 - \delta(\pi_1 c_1 + \pi_2 c_2)\frac{(1 - r)(c_1 + c_2)}{2r(c_1 + c_2 - x)^2}. \tag{1}$$

For $r < 1$, this expression is decreasing in x. Therefore if P1 had a profitable deviation, it can occur either for either $x = c_1$ or for a very small deviation. The latter kind is not

[13] That is, as long as $\frac{1}{2}\frac{x}{c_1 - x + c_2}\frac{1-r}{r} \leq 1$.

profitable if and only if (1) evaluated at $x = 0$ is non-positive. This gives

$$c_1 \leq \frac{\pi_2 \delta(1-r) + 2r\pi_1(1-\delta)}{-\pi_1[\delta(1-r) + 2r(1-\delta)]} c_2 \equiv K c_2, \tag{2}$$

and the corresponding constraint for payoffs is given by $u_1 \geq \frac{K\pi_1+\pi_2}{\pi_1+K\pi_2} u_2$.

If the deviation is large enough - so that it is detected with probability 1, then the best deviation is not to contribute and the corresponding constraint is similar to the case with perfect monitoring.

Given our choice of parameters, $K \geq \frac{\pi_2}{-\pi_1}\delta$. Furthermore recall that in the case of perfect monitoring, a stationary grim-trigger strategy profile with $c_1 \geq c_2$ is be an equilibrium if $c_1 \leq \frac{\pi_2}{-\pi_1}\delta c_2$. Hence any stationary grim-trigger equilibrium in the case of perfect monitoring is also a stationary grim-trigger equilibrium in the case of imperfect monitoring.

Modified grim-trigger

We now consider the non-stationary grim-trigger strategy studied in the case of perfect monitoring. In period 0, Pi plays $c_{i,0} = 2c^*$ and Pj plays 0. From period 1 onward players plays $c_i = \frac{-\pi_1}{\delta(\pi_2-\pi_1)} 2c^*$ and $c_j = 2c^* - c_i = \frac{\delta\pi_2+\pi_1(1-\delta)}{\delta(\pi_2-\pi_1)} 2c^*$. Any identified deviation leads to a constant play of $(0, 0)$ forever.

For this to be an equilibrium from period 1 onward, it must be the case that $c_i \leq K c_j$ (see (2)), which as we have argued will hold given our choice of parameters.

We now check whether Pi wants to deviate in period 0. Again, his options are deviating all the way to 0 or deviating slightly from $2c^*$. The former was already shown not to be profitable, so we only consider the case of a small deviation.

The deviation payoff in period 0 can be written as

$$(1-\delta)\Big[\pi_1(c_{i,0} - x) + \pi_2 c_{2,0}\Big] + \delta \frac{2(c_{1,0} + c_{2,0})r - x(1+r)}{2(c_{1,0} - x + c_{2,0})r} \Big[\pi_1 c_1 + \pi_2 c_2\Big].$$

We can see that this deviation is not profitable by comparing the above value with that from stationary grim trigger. First, the benefit from deviation (the first term) is independent of contribution level $c_{1,0}$. Second, the probability of being detected only depends on the sum of contributions, which we have set to equal $2c^*$. Finally, the continuation payoff is the same. So as long as the stationary grim trigger after period 1 is sustainable, this strategy profile is an equilibrium.

A.4 Proof of Proposition 3: Irreversibility and perfect monitoring

Cooperation becomes more difficult with the irreversibility constraint. Under perfect monitoring, players must gradually increase their contributions (LT). Intuitively, since players cannot reduce contributions, the only way to be able to punish deviations is to always hold back a fraction of contribution from the efficient level. Hence, players follow an increasing contribution path and threaten to stop the increase should a deviation occur. This gradualism means that the efficient frontier shifts inward.

Consider an equilibrium path $(c_{1,t}, c_{2,t})_{t\geq 0}$, and let $V_{i,t}$ denote the period t non-normalized payoff from this path: $V_{i,t} = \pi_1 c_{i,t} + \pi_2 c_{j,t} + \delta[\pi_1 c_{i,t+1} + \pi_2 c_{j,t+1}] + \dots$

The best deviation in any period, because of irreversibility, is to not increase contributions, and the harshest punishment is that both players maintain their contributions constant in every subsequent period, which is an equilibrium given that it is strictly dominated to increase contributions. Therefore for $(c_{1,t}, c_{2,t})_{t\geq 0}$ to be an equilibrium, we must have

$$\pi_1 c_{i,t-1} + \pi_2 c_{j,t} \leq (1-\delta)\Big[\pi_1 c_{i,t} + \pi_2 c_{j,t} + \delta[\pi_1 c_{i,t+1} + \pi_2 c_{j,t+1}] + \dots\Big], \tag{3}$$

for $i = 1, 2$ and $t \geq 1$.

The left-hand side of (3) is the payoff from the optimal deviation, given that the other player then maintains contributions at a level of $c_{j,t}$ in the future. The right-hand side of (3) is the payoff from following the equilibrium path.

Binding constraints

We first consider the case in which (3) is binding for $i = 1, 2$ and any $t \geq 1$. Consider two successive constraints:

$$\begin{aligned}
\pi_1 c_{i,t-1} + \pi_2 c_{j,t} &= (1-\delta)\Big[\pi_1 c_{i,t} + \pi_2 c_{j,t} + \delta S_{i,t+1}\Big] \\
\pi_1 c_{i,t} + \pi_2 c_{j,t+1} &= (1-\delta) S_{i,t+1}.
\end{aligned}$$

We can combine those two equations to get rid of $S_{i,t+1}$, which gives us the following system of second-order difference equation:

$$\begin{cases}
c_{i,t+1} - c_{i,t} = a[c_{j,t} - c_{j,t-1}] \\
c_{j,t+1} - c_{j,t} = a[c_{i,t} - c_{i,t-1}],
\end{cases}$$

where $a = \frac{-\pi_1}{\delta\pi_2} < 1$ given our assumption about the discount factor.

Given initial contribution levels $c_{i,0}$ and $c_{j,0}$, the solution of this system is given by

$$c_{i,t} = \begin{cases} \frac{1}{1-a^2}[c_{i,0}(1-a^{t+1}) + ac_{j,0}(1-a)^{t-1})], & t \text{ even} \\ \frac{1}{1-a^2}(1-a^t)[c_{i,0} + ac_{j,0}], & t \text{ odd} \end{cases}$$

Taking the limits as $t \to \infty$, we find the asymptotic contribution levels to be

$$\begin{cases} c_{i,\infty} = \frac{1}{1-a^2}[c_{i,0} + ac_{j,0}] \\ c_{j,\infty} = \frac{1}{1-a^2}[c_{j,0} + ac_{i,0}]. \end{cases}$$

We can also express initial contribution levels as a function of the limit contribution levels:

$$\begin{cases} c_{i,0} = c_{i,\infty} - ac_{j,\infty} \\ c_{j,0} = c_{j,\infty} - ac_{i,\infty}. \end{cases}$$

We have an equilibrium if $c_{i,\infty} + c_{j,\infty} \leq 2c^*$ and $c_{i,0} \geq 0$, $c_{j,0} \geq 0$.

Substituting contribution levels back into payoffs, we find the equilibrium payoff

$$V_i = [1 - \frac{(1-\delta)(a + a^2\delta)}{1 - a^2\delta^2}](\pi_1 c_{i,\infty} + \pi_2 c_{j,\infty}),$$

so that

$$V_1 + V_2 = \phi(\delta)(\pi_1 + \pi_2)(c_{i,\infty} + c_{j,\infty}),$$

where

$$\phi(\delta) = [1 - \frac{(1-\delta)(a + a^2\delta)}{1 - a^2\delta^2}].$$

When $c_{i,\infty} + c_{j,\infty} = 2c^*$, then $V_1 + V_2 = \phi(\delta)(\pi_1 + \pi_2)2c^*$, which is constant. The payoff-frontier from such strategies is therefore a line with slope -1 and endpoints A and B defined by the restrictions $c_{i,0} \geq 0$, $i = 1, 2$.

It can then be showed (see LT) by considering the efficient symmetric path and from convexity considerations that this frontier is indeed part of the equilibrium payoff frontier. Note that it is bounded away from the payoff frontier without irreversibility, since $\phi(\delta) < 1$.

Equilibrium in which one player gets a zero payoff

Consider the point A of Figure 6, which is at the bottom-right corner of the efficient frontier with binding constraints. In the contribution path that leads to A, in period 0, Player 1 plays 0 while Player 2 plays $c_{2,0} = c_{2,\infty} - ac_{1,\infty} = 2c^*(1-a)$. This latter value acts as the upper bound for player 2's "upfront" payments.

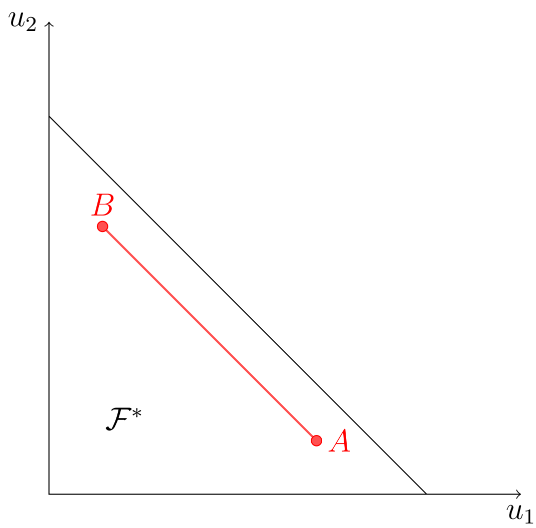


Figure 6: Equilibrium payoff frontier with irreversibility when all incentive constraints are binding

Fix an integer $k \geq 1$ and a contribution level $x \in [0, 2c^*(1-a)]$, and consider the following strategy: first, players contribute $(0, x)$ for a fixed number of periods k, and then they switch to the path that leads to A. That is, P2 makes an upfront payment of x for k periods, after which players start the path that generates point A. For $x = 0$, this is simply a delay in starting the path that leads to point A, and both players will have positive payoffs. For each k, let $x(k) \in [0, 2c^*(1-a)]$ be the highest contributions such that the previous strategy gives a non-negative payoff to P2.

It is not difficult to see that all incentive constraints will now be satisfied. First, after period $k-1$, players start playing the path that leads to A, for which we know that all incentive constraints hold (with equality). Prior to that, P1 does not contribute, so we only need to consider incentives for P2. In $t = 0$, P2 gets a non-negative payoff by following the strategy, and a payoff of zero if he deviates. Therefore there are no incentives to deviate. For $0 < t \leq k-1$, P2 does not have a possible deviation, as his contribution remains constant.

In our parametric setup, we use $k = 1$ and $k = 2$, and find $x(1) = 2c^*(1-a) = 63$ and $x(2) \sim 32.6$, which gives repeated game payoffs of $(182, 0.6)$ and $(165.4, 0)$ respectively. This allows us to give a lower bound on the set of equilibrium payoffs, which is depicted in Figure 7. The solid red set is contained in the equilibrium payoff set, and the equilibrium payoff set is contained in the union of the solid red set and the dotted red set.

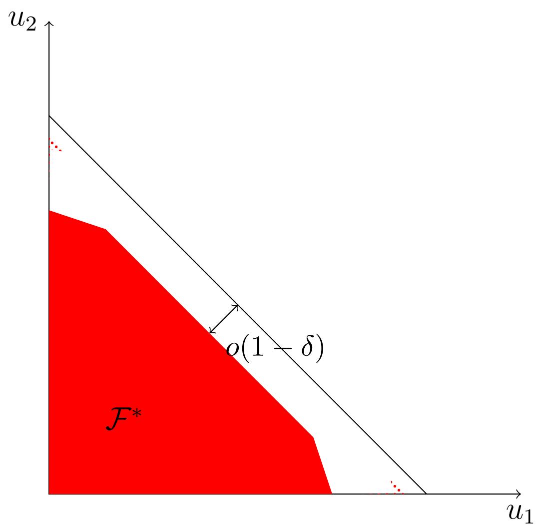


Figure 7: Bounds for the equilibrium payoff frontier with irreversibility and perfect monitoring