Price Stickiness Heterogeneity and Equilibrium Determinacy*

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Abstract

This paper shows that the requirement for monetary policy to achieve equilibrium determinacy is substantially loosened when price change frequencies are heterogeneous. The result holds both in a simple sticky price model with the constant elasticity of substitution aggregator and no trend inflation and in an extended model with a variable elasticity of substitution aggregator that permits trend inflation at the historical level. With a realistic cross-sectional distribution of the price change frequency, monetary policy can achieve equilibrium determinacy with much weaker responses to inflation. We then revisit the debate on the role of monetary policy in the transition from the Great Inflation to the Great Moderation in the postwar US economy. The evidence that the US economy was subject to self-fulfilling expectations-driven fluctuations in the pre-Volcker period and that the systematic shift in the monetary policy rule has played a decisive role in stabilizing inflation is found to be much weaker than previously concluded in the literature.

JEL codes: E12, E31, E43, E52, N12

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1 Introduction

Inflation in the US has risen since the mid 1960s through the late 1970s and then fell sharply with Volcker disinflation in the early 1980s. It exhibited wild swings in the 1970s but remained low and stable since the early 1980s. This rise and fall of inflation in the US, often referred to as the Great Inflation and the Great Moderation with other macro variables also stabilized, is one of the most striking features of the post-war US economy. It has been an important research subject among academic researchers as well as policy makers as a proper understanding of the nature of inflation dynamics is a key to successful monetary policy.

A leading hypothesis on the source of the Great Inflation is a bad policy hypothesis proposed by Taylor (1999) and Clarida, Galí, and Gertler (2000). They found that monetary policy failed to respond sufficiently aggressively to rising inflation during the episode in contrast to the Volcker-Greenspan era and thereby violated the generalized Taylor principle that nominal interest rates should be adjusted more than one-for-one in response to changes in inflation to achieve inflation stability. Clarida, Galí, and Gertler (2000) argue that such weak responses to inflation in the 1960s and 1970s left the US economy subject to equilibrium indeterminacy or sunspot fluctuations, which could have generated much larger fluctuations in the US economy than what could be attributed to changes in the economic fundamentals. The empirical finding that monetary policy violated the generalized Taylor principle was tested and confirmed by many follow-up studies using different approaches, such as Lubik and Schorfheide (2004), Boivin and Giannoni (2006), and Bhattarai, Lee, and Park (2012, 2016).

Other hypotheses on the source of the Great Inflation were also raised in the literature. Orphanides (2004) argues that estimated monetary policy rules are not statistically different between the pre- and post-Volcker period once one accounts for the Federal Reserve's real-time forecasts. Sims and Zha (2006) and Justiniano and Primiceri (2008) find that a change in the volatility of structural shocks is primarily responsible for the shift in macroeconomic stability. Coibion and Gorodnichenko (2011) emphasize the importance of a decline in trend inflation brought about by Volcker disinflation for inflation stabilization afterwards.

Though these studies employ different empirical strategies, they examine whether an estimated monetary policy rule is associated with equilibrium determinacy or not in a sticky price model with a common assumption that the price change frequency is homogeneous across firms.[1] This homogeneous-frequency assumption is however at odds with empirical evidence on price changes found by, for example, Blinder, Canetti, and Rudd (1998), Bils and Klenow (2004), Dhyne et al. (2006), and Nakamura and Steinsson (2008): the price change frequency is very heterogeneous across firms or across goods and services in the data.

[1] An exception is Sims and Zha (2006) who compute the long-run response of the policy rate to nominal variables in an estimated vector autoregressive model with regime changes.

This paper introduces heterogeneity of price stickiness into an otherwise standard New Keynesian model and investigates how it alters the condition for equilibrium determinacy in the model. If doing so does not substantially change the determinacy property of the model and does not overturn the findings in the literature, the homogeneous-frequency assumption would be a useful simplification device for parsimoniously modeling complicated reality. We however find that it is not the case: if heterogeneity of the price change frequency is taken into account, the condition for equilibrium determinacy substantially changes so that the requirement for monetary policy to achieve equilibrium determinacy is greatly weakened. Our finding sheds new light on the debate about the source of the Great Inflation and the subsequent transition to the Great Moderation.

Specifically, we extend a standard small-scale New Keynesian model to allow for firms to change their prices at different frequencies. Our model nests the homogeneous-frequency economy as a special case so it is possible to compare the two economies, the homogeneous-frequency economy and the heterogeneous-frequency economy, within a unified framework. Three specifications of the model that are different in terms of the aggregator of differentiated goods and the level of trend inflation are considered: Specification I features the constant-elasticity of substitution (CES) aggregator and zero trend inflation; Specification II features the CES aggregator and positive trend inflation; and Specification III features a variable elasticity of substitution aggregator and zero or positive trend inflation.[2] Since we approximate equilibrium dynamics by log-linearizing the equilibrium conditions around the steady state, it is local equilibrium determinacy that we study.

We first illustrate the condition for equilibrium determinacy in each specification in the space of the coefficients in the monetary policy rule in a simple equally-sized two-sector model. It is found that the determinacy region expands in Specifications I and III in the heterogeneous-frequency economy compared to the homogeneous-frequency economy with the same mean price change frequency. On the contrary, in Specification II, the result is reversed: the determinacy region shrinks in the heterogeneous-frequency economy compared to the homogeneous-frequency economy.

Specification I extends a standard textbook New Keynesian model by, for example, Woodford (2003) and Galí (2008), to introduce heterogeneity in the price change frequency across firms as in Carvalho (2006). In particular, it features the CES aggregator and assumes that trend inflation or steady state inflation is zero. In this specification, as the price change frequency becomes more heterogeneous in a sense that its standard deviation becomes larger while its mean is preserved, the boundary between the determinacy and indeterminacy region tilts to the left in the space of the coefficients on inflation and output in the monetary policy rule. In other words, the determinacy region expands so the central bank can achieve equilibrium determinacy with weaker responses to inflation in the heterogeneous-frequency economy than in the homogeneous-frequency economy.

[2] Trend inflation means net trend inflation here and henceforth. We explicitly refer to gross trend inflation when necessary.

We numerically verify that the generalized Taylor principle holds in Specification I. Especially, we present an exact expression of the generalized Taylor principle and establish a proposition that a mean-preserving increase in the degree of heterogeneity of the price change frequency expands the determinacy region.

The mechanism through which heterogeneity in the price change frequency shifts the boundary is the dispersion of the sectoral relative prices coupled with a larger weight on the marginal cost in the aggregate Phillips curve, which leads to a larger long-run trade-off between inflation and output or a flatter long-run slope of the aggregate Phillips curve. A strategic interaction between the strategic complementarity in price setting due to firm-specific factor market and heterogeneity in the price change frequency, identified by Carvalho (2006), amplifies the effect of the price stickiness heterogeneity on the equilibrium determinacy condition.

Then, we consider Specification II where it features the CES aggregator but trend inflation is assumed to be positive (and greater than a close-to-zero bound). As the price change frequency becomes more heterogeneous, the boundary between the determinacy and indeterminacy region now tilts to the right. In other words, the determinacy region shrinks so that the central bank has to respond more strongly to inflation to achieve equilibrium determinacy in the heterogeneous-frequency economy than in the homogeneous-frequency economy. The reason for this reversal is that the long-run slope of the aggregate Phillips curve turns negative with positive trend inflation as found by Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), and Ascari and Sbordone (2014) for the homogeneous-frequency economy. Price-updating firms raise their prices more in Specification II than in Specification I since they are worried that they may not be able to update prices again for some time in the future while their relative prices are eroded by trend inflation. It follows that their output declines substantially, which leads to a decline in aggregate output. The sectoral relative price dispersion due to the heterogeneous price change frequency makes the decline in aggregate output worse and the long-run slope of the Phillips curve drops even more. As in the homogeneous-frequency economy with positive trend inflation, the generalized Taylor principle breaks down in the heterogeneous-frequency economy.

In Specification II, however, the upper bound for trend inflation could be very tight. As acknowledged in the literature, trend inflation in a New Keynesian model with the CES aggregator is subject to an upper bound in order for the profit maximization problem of firms to be well defined. This is because the demand faced by the non-price-updating firms continually expands as their relative prices keep being eroded by trend inflation. The upper bound is not likely to bind in the homogeneous-frequency economy with a moderate price change probability, say, 20% per month. However, since the profit maximization problem should be well defined for not just an average firm but each and every firm in the economy, the upper bound for trend inflation could be very low in the heterogeneous-frequency economy where some firms change their prices rarely. For example, with the monthly price change frequency 0.016 for legal services estimated by Nakamura and Steinsson (2008), the upper bound for annual net trend inflation would be only 1.0% so does not permit 2% currently targeted by the Federal Reserve. Therefore, we do not deem Specification II suitable for modeling the US economy when heterogeneity of the price change frequency is taken into account.

To address this upper bound problem, in Specification III, we replace the CES aggregator with an aggregator proposed by Kimball (1995) that features a variable elasticity of substitution across differentiated goods. It leads to a kinked demand curve with a variable elasticity of demand which is increasing in the relative price of a good. With a reasonable rate of changes in the elasticity of demand, an increase in the demand faced by the non-price-updating firms due to trend inflation decelerates and the upper bound for trend inflation disappears. In Specification III, we obtain qualitatively same results as those in Specification I: as the price change frequency becomes more heterogeneous, the boundary between the determinacy and indeterminacy region tilts to the left so the determinacy region expands. As price-updating firms raise their prices, the demand they face becomes more elastic and their profits deteriorate more quickly. So, they do not raise prices as much as in Specification II with the CES aggregator and their output does not decline as much. Now the long-run trade-off between inflation and output stays positive. It becomes larger as in Specification I as the price change frequency becomes more heterogeneous. The generalized Taylor principle is again numerically verified to hold in Specification III.

We then investigate how introducing heterogeneity in the price change frequency affects the condition for equilibrium determinacy in a model with more disaggregated sectors. Specifically, we parameterize our model with the price change frequency estimates by Nakamura and Steinsson (2008) based on the data of price changes in 272 categories of goods and services collected by the Bureau of Labor Statistics (BLS). In this bigger model with 272 sectors, monetary policy can respond much less strongly to inflation to achieve equilibrium determinacy than in the homogeneous-frequency economy.

Lastly, we revisit the debate on the source of the Great Inflation and the subsequent stabilization of inflation by considering the monetary policy rule estimates by Clarida, Galí, and Gertler (2000), Coibion and Gorodnichenko (2011), and Carvalho, Nechio and Tristao (2019) in the heterogeneous-frequency economy parameterized with the price change frequency estimates by Nakamura and Steinsson (2008). Clarida, Galí and Gertler (2000) and Coibion and Gorodnichenko (2011) estimate a forward-looking monetary policy rule where the central bank reacts to expected inflation and the contemporaneous output gap while Carvalho, Nechio and Tristao (2019) estimate a contemporaneous monetary policy rule where the central bank reacts to contemporaneous inflation and output gap. As opposed to the previous finding in the literature, evidence in favor of the possibility that the US economy was subject to equilibrium indeterminacy and suffered from self-fulfilling expectations-driven fluctuations in the 1960s and 1970s is found to be not strong at all once the empirical cross-sectional distribution of price stickiness is accounted for. In other words, we do not find strong empirical evidence that the change in the monetary policy stance around the tenure of Paul Volcker as the chairperson of the Federal Reserve got the US economy out of equilibrium indeterminacy and led it to have a unique stable equilibrium.

In addition to the literature on the source of the Great Inflation and the transition to the Great Moderation reviewed above, this paper is related to several strands of the literature. First of all, it is related to the theoretical studies about how the structure of the economy affects the condition for equilibrium determinacy. Some examples include Bullard and Mitra (2002), Lubik and Marzo (2007), Galí, López-Salido, and Vallés (2004), Carlstrom and Fuerst (2004), Carlstrom et al. (2006), Benhabib and Eusepi (2005), Sveen and Weinke (2005, 2007), Hornstein and Wolman (2005), and Bhattarai, Lee, and Park (2014). Especially, Kiley (2007), Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), Kurozumi (2014), and Ascari and Sbordone (2014) found that higher inflation induces a change in the long-run inflation-output trade-off and thus affects the condition for equilibrium determinacy. Kara and Yates (2021) study how higher trend inflation leads to a greater long-run output loss and thus shrinks the determinacy region faster in a multisector New Keynesian model than in a single-sector New Keynesian model. The model they consider assumes the standard CES aggregator so it corresponds to Specification II of our paper. We go a step further by acknowledging a possibility that high trend inflation could violate its upper bound in a multisector New Keynesian model with the CES aggregator and proposing a model specification where the histocial level of trend inflation is accommodated.

Though we present the baseline model in a general fashion that the distribution of the price change frequency across firms is continuous, we discretize the distribution when taking the model to the sectoral data. Therefore, our model is a multisector New Keynesian model. Aoki (2001) and Benigno (2004) made early contributions in this literature by studying normative implications of heterogeneity in price stickiness. We in particular build on work by Carvalho (2006). Bouakez, Cardia and Ruge-Murcia (2014) estimate a multisector sticky price model to study the implications of sectoral heterogeneity in price rigidity for the business cycle and monetary policy tranmission. Carvalho, Lee and Park (2021) show that it is very useful in matching differential responses of prices to aggregate and sectoral shocks observed in the data to introduce input-output production linkages and sectoral labor markets in a multisector sticky price model. Pasten, Schoenle and Weber (2020) find that heterogeneity in price stickiness is important for the propagation of monetary policy shocks in a similar multisector model with input-output production linkages.

Since Kimball (1995) introduced an aggregator that leads to a variable elasticity of demand, many papers adopted it to generate real rigidity. For example, see Dotsey and King (2005), Eichenbaum and Fisher (2007), Smets and Wouters (2007), Levin, Lopez-Salido, Nelson, Yun (2008), Kurozumi and van Zandweghe (2016) in monetary economics and also Gopinath and Itskhoki (2010, 2011) in international economics. Kurozumi and van Zandweghe (2016) introduce a Kimball-type aggregator into a homogeneous-frequency New Keynesian model and show that the resulting kinked demand curve mitigates the negative influence of high trend inflation on output so makes equilibrium determinacy much more likely. Our paper extends their analysis to a multisector model with heterogeneous frequency and also shows that a Kimball-type aggregator is useful in dealing with the upper bound problem for trend inflation.

Bils, Klenow and Malin (2012) raise an important criticism on a sticky price model with real rigidity, especially on those that use a Kimball-type aggregator. They criticize that New Keynesian models such as the one by Smets and Wouters (2007) imply an unrealistic behavior of reset price inflation because of strong real rigidity or strategic complementarities in price setting. Without volatile price markup shocks, these models generate too much persistence in actual inflation compared to what is observed in the data. To address this failure, these models usually introduce large transitory price markup shocks, which then result in much more volatile reset price inflation than in the data. Kara (2015) however shows that introducing heterogeneity in price stickiness helps to address this criticism. In an estimated mutlisector New Keynesian model, the volatility of price markup shocks does not have to be very large and the estimated model is able to match low variability of reset price inflation as well as low persistence in actual inflation.

2 Model

Our baseline model extends a standard New Keynesian model with homogeneous price stickiness to allow firms to change their prices at different frequencies. Another important departure from the standard New Keynesian model is that final good firms use a Kimball-type aggregator that features a variable elasticity of substitution across differentiated intermediate goods, which leads to a kinked demand curve for the intermediate goods. The kinked demand curve exhibits a variable price elasticity of demand. The Kimball-type aggregator nests the standard CES aggregator, in which case the demand function exhibits a constant price elasticity.[3] For simplicity, we do not consider idiosyncratic shocks to households and firms.

The details of the model derivation and proof are presented in the online appendix.

2.1 Households, firms and the government

The economy is populated by a large number of identical households, a large number of identical final good firms, a continuum of intermediate good firms who are monopolistically competitive, and the government. Since households and final good firms are identical with their peers, we consider only the problem of their representatives. The intermediate good firms are indexed by $i \in [0, 1]$, which is also used for the product each firm produces and the labor type it hires.

[3] Therefore, the model with the CES aggregator can be easily derived from the description of the baseline model in this section. We though present the description of the model with the CES aggregator in the online appendix as quick reference.

2.1.1 Representative household

The representative household consumes the final good and supplies firm-specific labor to intermediate good firms in order to maximize expected lifetime utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{H_t(i)^{1+\varphi}}{1+\varphi} di \right] \right\},$$

subject to the flow budget constraint

$$P_t C_t + B_t = I_{t-1} B_{t-1} + P_t \tau_t + \int_0^1 W_t(i) H_t(i) \, di + \int_0^1 \Omega_t(i) \, di,$$

where C_t is consumption of the final good, $H_t(i)$ is the supply of type-i labor, B_t is the holding of the nominal one-period riskless bonds, τ_t is the lump-sum government transfer net of taxes, and $\Omega_t(i)$ is the nominal profit distributed by the producer of intermediate good i as dividends. The unit price of the final good is denoted by P_t, the nominal wage rate of type-i labor by $W_t(i)$, and the gross interest rate for the bond between periods $t-1$ and t by I_{t-1}. The representative household takes the prices and the interest rate as given. Parameter $0 < \beta < 1$ is the discount factor, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\chi > 0$ is to adjust the level of labor disutility. A condition for no-Ponzi scheme is assumed.

The optimality conditions for the representative household are derived as

$$1 = \beta I_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right], \tag{1}$$

$$\frac{W_t(i)}{P_t} = \chi H_t(i)^{\varphi} C_t^{\sigma}, \tag{2}$$

for all $t \geq 0$ and $i \in [0, 1]$.

2.1.2 Representative final good firm

Final good firms are identical to each other and perfectly competitive. The representative final good firm combines differentiated intermediate goods using a Kimball-type aggregator $G(\cdot)$ to produce the final good Y_t. Taking the prices of the intermediate goods $\{P_t(i)\}_{i=0}^{1}$ as given, it chooses how much of each of the intermediate goods $\{Y_t(i)\}_{i=0}^{1}$ to purchase to maximize profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di, \tag{3}$$

subject to

$$\int_0^1 G\left(\frac{Y_t(i)}{Y_t}\right) = 1. \tag{4}$$

We follow Dotsey and King (2005) and use the following functional form for $G(\cdot)$

$$G\left(\frac{Y_t(i)}{Y_t}\right) = \frac{\theta}{\theta(1-\psi)-1}\left[(1-\psi)\left(\frac{Y_t(i)}{Y_t}\right)+\psi\right]^{\frac{\theta(1-\psi)-1}{\theta(1-\psi)}} - \left(\frac{\theta}{\theta(1-\psi)-1}-1\right),$$

with $\theta > 1$ and $\psi \geq 0$ but $\psi \neq 1$. Note that $G(\cdot)$ collapse to the standard CES aggregator if $\psi = 0$. Levin, Lopez-Salido, Nelson, Yun (2008) and Kurozumi and van Zandweghe (2016) also use the same functional form for the Kimball-type aggregator as Dotsey and King (2005).

Maximizing (3) subject to (4), the relative demand for intermediate good i is determined as

$$\frac{Y_t(i)}{Y_t} = \frac{1}{1-\psi}\left[\left(\frac{P_t(i)}{P_t}\right)^{-\theta(1-\psi)}\Lambda_t^{\theta(1-\psi)} - \psi\right], \tag{5}$$

where $\Lambda_t = \tilde{\Lambda}_t / P_t Y_t$ with $\tilde{\Lambda}_t$ being the Lagrange multiplier associated with constraint (4). Note that the demand curve (5) is downward-sloping in the relative price. The price elasticity of demand is derived as

$$\theta + \theta\psi\left(\frac{1}{Y_t(i)/Y_t} - 1\right). \tag{6}$$

If $\psi > 0$, it is decreasing in the relative demand for intermediate good i so increasing in the relative price of intermediate good i. The demand curve has a smoothed kink around a symmetric equilibrium where $Y_t(i) = Y_t$.

Parameter ψ determines the degree of the curvature of the demand function (5) so it governs how much the price elasticity of demand varies as the relative price changes. Especially, Klenow and Willis (2016) calls the following rate of changes in the elasticity of demand with respect to the relative price,

$$\frac{\theta\psi}{Y_t(i)/Y_t}, \tag{7}$$

the superelasticity. A positive superelasticity generates real rigidities since resulting variation of the price elasticity gives an incentive for intermediate good firms to keep their prices around the aggregate price. As will be discussed in Section 2.3, it plays an important role of constraining explosive effects of trend inflation on demand for intermediate goods. We will restrict $\psi > 1$ so that trend inflation is not subject to an upper bound.

Plugging (5) back into (4) leads to

$$\Lambda_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\theta(1-\psi)} di\right]^{\frac{1}{1-\theta(1-\psi)}}. \tag{8}$$

The zero profit condition for the final good producer implies that

$$\Lambda_t = (1 - \psi) + \psi S_t, \tag{9}$$

where S_t is a measure of the relative price dispersion defined as

$$S_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right) di.$$

Note that if $\psi = 0$, that is if $G(\cdot)$ collapse to the CES aggregator, $\Lambda_t = S_t = 1$ and the demand function (5) is reduced to the one with a constant price elasticity of demand. It follows that the superelasticity is zero.

2.1.3 Intermediate good firms

Intermediate good firms are monopolistically competitive and produce differentiated intermediate goods. Intermediate good firm i uses the following production technology to produce intermediate good i

$$Y_t(i) = A_t H_t(i),$$

where the total factor productivity A_t is stationary.[4] It is assumed as in Calvo (1983) and Yun (1996) that firms can reset prices only with some probability, independently of the time elapsed since they last reset their prices. Let us denote the probability of a price change by firm i in a given period by $\alpha(i) \in [0, 1]$. Firms that do not optimally adjust their prices simply keep their prices unchanged.

Suppose that firm i can optimally reset its price in period t and denote its optimal reset price by $P_t^*(i)$. If firm i does not reset its price prior to and in period $t + k$, its total revenue and cost in period $t + k$ is given as

$$TR_{t+k|t}(i) = \frac{P_t^*(i)}{P_{t+k}} Y_{t+k|t}(i),$$
$$TC_{t+k|t}(i) = \frac{W_{t+k|t}(i)}{P_{t+k}} H_{t+k|t}(i),$$

respectively, in real terms, where the demand faced by firm i is

$$Y_{t+k|t}(i) = \frac{1}{1-\psi} \left[\left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta(1-\psi)} \Lambda_{t+k}^{\theta(1-\psi)} - \psi \right] Y_{t+k},$$

[4] All the results hold even with growth in productivity. To permit balanced growth in the model, we need to set $\sigma = 1$ in the period utility function for the representative household.

and the labor input is

$$H_{t+k|t}(i) = \frac{Y_{t+k|t}(i)}{A_{t+k}},$$

with nominal wage $W_{t+k|t}(i)$ given. The profit maximization problem for firm i in period t can be written as

$$\max_{P_t^*(i)} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (1-\alpha(i))^k Q_{t,t+k} \left[TR_{t+k|t}(i) - TC_{t+k|t}(i) \right] \right\}, \tag{10}$$

where $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}$ is the real stochastic discount factor between periods t and $t+k$. The first order condition with respect to $P_t^*(i)$ is

$$\begin{aligned} 0 = \mathbb{E}_t \Bigg\{ \sum_{k=0}^{\infty} (1-\alpha(i))^k Q_{t,t+k} \Bigg[& \left(\frac{P_t^*(i)}{P_t}\right) \left(\frac{P_t}{P_{t+k}}\right)^{1-\theta(1-\psi)} \Lambda_{t+k}^{\theta(1-\psi)} Y_{t+k} \\ & + \frac{\psi}{\theta(1-\psi)-1} \left(\frac{P_t^*(i)}{P_t}\right)^{1+\theta(1-\psi)} \left(\frac{P_t}{P_{t+k}}\right) Y_{t+k} \\ & - \frac{\theta(1-\psi)}{\theta(1-\psi)-1} MC_{t+k|t}(i) \left(\frac{P_t}{P_{t+k}}\right)^{-\theta(1-\psi)} \Lambda_{t+k}^{\theta(1-\psi)} Y_{t+k} \Bigg] \Bigg\}, \end{aligned} \tag{11}$$

where the real marginal cost is given as

$$MC_{t+k|t}(i) = \frac{W_{t+k|t}(i)}{P_{t+k}} \frac{1}{A_{t+k}} = \chi \left\{ \frac{1}{1-\psi} \left[\left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta(1-\psi)} \Lambda_{t+k}^{\theta(1-\psi)} - \psi \right] \right\}^{\varphi} \frac{Y_{t+k}^{\sigma+\varphi}}{A_{t+k}^{1+\varphi}}.$$

Without any firm-specific shocks in the economy, firms who have an identical price change frequency would choose the same optimal price in any given period if they get a chance to adjust their prices. So we can describe the dynamics of the optimal reset price for each group of the firms who share an identical price change frequency. Let us denote the set of the firms with price change frequency α by $\mathcal{I}_\alpha = \{i; \alpha(i) = \alpha\} \subset [0,1]$ and call it sector α. Its size is $f(\alpha) = \int_{\mathcal{I}_\alpha} di$. Also, let us assume that the infinite sum of each term in the first order condition (11) is well defined. Then, its infinite sum can be written recursively if φ is zero or a positive integer. We assume that $\varphi = 1$ for simplicity,[5] and rewrite (11) recursively as

$$\begin{aligned} 0 = {} & Z_{1,t}(\alpha) \left(\frac{P_{\alpha,t}^*}{P_t}\right) + \frac{\psi}{\theta(1-\psi)-1} Z_{2,t}(\alpha) \left(\frac{P_{\alpha,t}^*}{P_t}\right)^{1+\theta(1-\psi)} - \frac{\theta\chi}{\theta(1-\psi)-1} Z_{3,t}(\alpha) \left(\frac{P_{\alpha,t}^*}{P_t}\right)^{-\theta(1-\psi)} \\ & + \frac{\theta\chi\psi}{\theta(1-\psi)-1} Z_{4,t}(\alpha), \end{aligned} \tag{12}$$

[5]We consider a case with $\varphi = 0$ and compare it with the baseline model in sensitivity analysis to investigate the role of the real rigidity generated by the firm-specific labor market assumption. The model specification and solution with $\varphi = 0$ is provided in the online appendix. Note that with the CES aggregator ($\psi = 0$), (11) can be written recursively with any positive, either non-integer or integer, values of φ.

where $P^*_{\alpha,t}$ is the optimal reset price of the firms in sector α, and

$$Z_{1,t}(\alpha) = \Lambda_t^{\theta(1-\psi)} Y_t^{1-\sigma} + \beta(1-\alpha)\mathbb{E}_t Z_{1,t+1}(\alpha)\left(\frac{P_t}{P_{t+1}}\right)^{1-\theta(1-\psi)}, \tag{13}$$

$$Z_{2,t}(\alpha) = Y_t^{1-\sigma} + \beta(1-\alpha)\mathbb{E}_t Z_{2,t+1}(\alpha)\left(\frac{P_t}{P_{t+1}}\right), \tag{14}$$

$$Z_{3,t}(\alpha) = \Lambda_t^{2\theta(1-\psi)}\left(\frac{Y_t}{A_t}\right)^2 + \beta(1-\alpha)\mathbb{E}_t Z_{3,t+1}(\alpha)\left(\frac{P_t}{P_{t+1}}\right)^{-2\theta(1-\psi)}, \tag{15}$$

$$Z_{4,t}(\alpha) = \Lambda_t^{\theta(1-\psi)}\left(\frac{Y_t}{A_t}\right)^2 + \beta(1-\alpha)\mathbb{E}_t Z_{4,t+1}(\alpha)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta(1-\psi)}. \tag{16}$$

To derive inflation dynamics for sector α, it is useful to define

$$\Lambda_{\alpha,t} = \left[\frac{1}{f(\alpha)}\int_{\mathcal{I}_\alpha}\left(\frac{P_t(i)}{P_t}\right)^{1-\theta(1-\psi)} di\right]^{\frac{1}{1-\theta(1-\psi)}}, \tag{17}$$

which is aggregated as

$$\Lambda_t = \left[\int_0^1 f(\alpha)\,\Lambda_{\alpha,t}^{1-\theta(1-\psi)} d\alpha\right]^{\frac{1}{1-\theta(1-\psi)}}, \tag{18}$$

and

$$S_{\alpha,t} = \frac{1}{f(\alpha)}\int_{\mathcal{I}_\alpha}\left(\frac{P_t(i)}{P_t}\right) di, \tag{19}$$

which implies

$$S_t = \int_0^1 f(\alpha)\,S_{\alpha,t}\,d\alpha. \tag{20}$$

2.1.4 Government

The central bank adjusts nominal interest rates using a Taylor-type monetary policy rule

$$\frac{I_t}{\bar{I}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}, \tag{21}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation and $\bar{I}$, $\bar{\Pi}$ and $\bar{Y}$ are the steady state values of I_t, Π_t, and Y_t, respectively. In this model, gross inflation in the steady state represents gross trend inflation.

Fiscal policy is Ricardian and trivial. We do not consider government spending.

2.2 Equilibrium

The market clearing condition is $Y_t(i) = C_t(i)$ for intermediate good i and

$$Y_t = C_t, \tag{22}$$

for the final good, and $B_t = 0$ for the nominal bond. We implicitly assumed that the labor market clears all the time.

The competitive equilibrium of this economy consists of the sequence of prices and allocations such that given the sequence of prices, the sequence of allocations solve the utility maximization problem of the representative household, the profit maximization problem of the representative final good firm and all the intermediate good firms, the government sticks to the description of monetary policy and fiscal policy, and all the markets clear. Specifically, the equilibrium dynamics of Y_t, C_t, Π_t, $\left\{P^*_{\alpha,t}/P_t\right\}$, Λ_t, $\{\Lambda_{\alpha,t}\}$, $\{Z_{1,t}(\alpha)\}$, $\{Z_{2,t}(\alpha)\}$, $\{Z_{3,t}(\alpha)\}$, $\{Z_{4,t}(\alpha)\}$, $\{S_{\alpha,t}\}$, S_t, and I_t are determined by (1), (9), (12) and (13)-(22).

To study the equilibrium dynamics of the model, we log-linearize the equilibrium equations around the deterministic steady state. Then the set of the log-linearized equilibrium equations constitute a linear rational expectations model and can be solved using a solution method such as Sims (2002). Depending on the magnitude of the monetary policy parameters ϕ_π and ϕ_y, the solution of the model may be unique or not.

2.3 Upper bound for trend inflation

As noted in the literature, trend inflation is subject to an upper bound in a New Keynesian model that features the CES aggregator and no or less than full indexation of prices. The upper bound is usually not binding in a New Keynesian model with homogeneous price stickiness, which explains why it was not considered seriously in the literature. It can be however severely binding in the presence of heterogeneity in price stickiness, in particular when there are some firms with a very low price change frequency.

To explain the upper bound problem, let us first consider the CES-aggregator case of the baseline model by letting $\psi = 0$. Take, for illustration, an intermediate good firm i who can adjust its price with probability $0 \le \alpha(i) < 1$. Since its profit maximization problem is an infinite-horizon problem, a necessary condition for the problem to be well defined is that the infinite sums in (10) and (11) converge in the steady state. That is, in the steady state, gross trend inflation is required to be smaller than the following upper bound

$$\left[\frac{1}{\beta\left(1-\alpha(i)\right)}\right]^{\frac{1}{\theta(1+\varphi)}}.$$

Suppose that the frequency of the model is a quarter and set $\beta = 0.99$, $\theta = 10$ and $\varphi = 1$, which are common in the literature.[6] A typical value for $\alpha(i)$ in a New Keynesian model with homogeneous price stickiness, usually set at the average price change frequency in the economy, is greater than 0.5. It implies that the upper bound for quarterly gross trend inflation is greater than 1.0358. So, the upper bound for annual net trend inflation is greater than 15.1%, which is sufficiently high to be not binding.

However, the objective function of (10) and the first order condition (11) should be well defined not just for a representative firm with mean or median frequency but for each and every firm in the economy. Therefore, in an economy with different frequencies of price changes across firms, the effective upper bound for the economy is

$$\min\left\{\left[\frac{1}{\beta\left(1-\alpha\left(i\right)\right)}\right]^{\frac{1}{\theta(1+\varphi)}}\right\} = \left[\frac{1}{\beta\left(1-\min\left\{\alpha\left(i\right)\right\}\right)}\right]^{\frac{1}{\theta(1+\varphi)}},$$

where $\min\{\alpha(i)\}$ is the lowest frequency of price changes among all the firms. For example, the lowest price change frequency estimated by Nakamura and Steinsson (2008) is 0.016 per month for legal services.[7] The upper bound of trend inflation for a firm with monthly frequency 0.016 is 1.00086 with $\beta = 0.999$, which is converted to 1.0% in terms of annual net inflation. It is so low that it does not permit even 2% currently targeted by the Federal Reserve.[8] There could be a firm that rarely updates its price, which pushes the upper bound further down so that positive trend inflation is practically not allowed.

The reason why trend inflation cannot be too high with the CES aggregator is that the demand quickly expands as the relative price is eroded by trend inflation. With positive trend inflation, the price of intermediate goods, if not updated again after adjustment, is continually eroded relative to the aggregate price level as time passes by. The demand continues to expand since the demand curve exhibits a constant price elasticity. Because of this perpetual expansion of the demand, the infinite sums in the objective function of (10) and the first order condition (11) become likely to explode.

The Kimball-type aggregator leads to a variable price elasticity of demand and prevents such effects of trend inflation on the demand: the price elasticity of demand declines as the relative price decreases so the demand expansion decelerates. Especially, if $\psi > 1$, it can be easily shown

[6]See Section 3.1 for a discussion on parameterization.

[7]It is based on the regular price change frequencies. Legal services category has the lowest except girls' outerwear whose price change frequency estimate is 0 during the sample period possibly due to a small sample.

[8]It could be the case that sectoral inflation is lower than the upper bound in very sticky sectors. Indeed, inflation in each category, or sectoral inflation, is positively correlated with the frequency estimate of the same category during the sample period of Nakamura and Steinsson (2008). Nevertheless, the inflation rate in those categories with very low price change frequencies is higher than the implied upper bound. For example, the average inflation rate in legal services, computed using the BLS CPI data, is 4.87% per annum during the period of 1998-2005.

that the upper bound disappears. If $0 < \psi < 1$, the upper bound is given as

$$\left[\frac{1}{\beta\left(1-\min\left\{\alpha\left(i\right)\right\}\right)}\right]^{\frac{1}{\theta(1-\psi)(1+\varphi)}},$$

which is greater than the one with the CES aggregator.

3 Condition for equilibrium determinacy

In this section, we study how introducing heterogeneity in price stickiness changes the condition for equilibrium determinacy in the baseline model. We log-linearize the equilibrium equations so, to be precise, it is local equilibrium determinacy around the steady state that we study. Before presenting the main result, parameterization of the baseline model is discussed.

3.1 Parameterization

Parameters other than those associated with the demand curve for intermediate goods are set to the typical values in the literature. The frequency of the model is a quarter so the discount factor β is set at 0.99. The inverse of the Frisch elasticity of labor supply φ is set at 1. The inverse of the intertemporal elasticity of substitution σ is set at 1, which leads to a log function for period utility. We set $\chi = 8$ so that labor supply is around 0.33 in the steady state.

Parameter θ denotes the elasticity of substitution across intermediate goods and also the price elasticity of demand in any symmetric equilibrium where all the intermediate good firms produce the same amount. Its product with ψ, $\theta\psi$, denotes the superelasticity in a symmetric equilibrium.[9] We set θ at 10, which implies the steady state markup of 11% in a symmetric equilibrium, following the macroeconomics literature. We set ψ at 1.1, which implies the superelasticity of 11. It is at the low end of the range considered in the macroeconomics literature and close to the estimates based on micro data. Note that our result holds robustly with smaller values for θ as favored by some studies based on the micro data, as long as $\psi > 1$ or ψ is smaller than 1 but very close to 1.

We briefly review the literature in terms of the superelasticity before moving on to present the main result. The value of the superelasticity chosen in the macroeconomics literature varies widely.

[9] With the CES aggregator ($\psi = 0$), the elasticity of substitution across intermediate goods is θ and the superelasticity is zero. With the Kimball-type aggregator ($\psi > 0$ but $\psi \neq 1$), in asymmetric equilibrium where intermediate good firms produce different amounts, the price elasticity of demand and the superelasticity faced by the intermediate good firm can deviate from θ and $\theta\psi$ even in the steady state. See Equations (6) and (7). Asymmetric equilibrium arises if inflation is not zero in the steady state. However, unless the prices are extremely sticky for the majority of the firms in the economy, relative output of the firms is close to 1 and the distribution of the exact price elasticity and the exact superelasticity is highly concentrated around θ and $\theta\psi$. Their median is close to θ and $\theta\psi$, respectively. Therefore, we consider the two parameters as the price elasticity and the superelasticity when choosing their values.

Typically, however, this literature chooses a fairly large value for the superelasticity, which implies that ψ is greater than 1 when translated in our specification of the Kimball-type aggregator. For example, Kimball (1995) calibrates the elasticity of the marginal cost relative to the marginal revenue with respect to firm output, holding aggregate output fixed, and parameterizes his model so that the implied superelasticity is 470.8. With the price elasticity at 11 as chosen by Kimball (1995), this implies that $\psi = 42.8$. Calibration by Dotsey and King (2005) implies $\psi = 6$ while Eichenbaum and Fisher (2007) consider 10 and 33 for the superelasticity, which is equivalent to $\psi = 0.909$ or 3 with their choice of the price elasticity 11. In Smets and Wouters (2007), the posterior mode of the price elasticity is 2.35 while the curvature of the demand curve is fixed at 10, which implies that $\psi = 4.26$. Levin, Lopez-Salido, Nelson, Yun (2008) and Kurozumi and van Zandweghe (2016) use the same functional form for the Kimball-type aggregator as ours. By targeting the slope of the NK Phillips curve, Levin, Lopez-Salido, Nelson, Yun (2008) and Kurozumi and van Zandweghe (2016) set ψ at 8 and 9, respectively. Klenow and Wilis (2016) sets the price elasticity at 5 and the superelasticity at 10, which implies that $\psi = 2$. Gopinath and Itskhoki (2010, 2011) empirically document pass-through using international price data and evaluate the role of real rigidities in pass-through using the Kimball-type aggregator. Targeting the elasticity of markup, Gopinath and Itskhoki (2010) set the price elasticity and the superelasticity at 5 and 4 while Gopinath and Itskhoki (2011) set them at 5 and 6. The implied value of ψ is 0.8 and 1.2 for the two studies, respectively.

The empirical studies that directly estimate the superelasticity using micro data find evidence supporting a kinked demand curve but their estimates are relatively small compared to the value used in the macroeconomics literature. Also, the estimate for the superelasticity exhibits a wide variation across different products and markets. Nevertheless, the assumption that $\psi > 1$, or the superelasticity is greater than the price elasticity, is not greatly at odds with the estimates based on micro data.

Nakamura and Zerom (2010) estimate the demand curve for ground coffee to account for incomplete pass-through in the US ground coffee market. Their median price elasticity is 3.46 and the estimate of the superelasticity of demand is 4.64. So their estimates imply $\psi = 1.34$ in our specification of the Kimball-type aggregator. Dossche, Heylen and Van den Poel (2010) estimate the slope and curvature of the demand curve for a wide range of retail consumer products using scanner data from a large euro area retailer. Their median estimate for the price elasticity and the curvature is 1.4 and 0.8, respectively, so the implied value for ψ is smaller than 1. However, the scatter plot of the two estimates across different products and stores in Dossche, Heylen and Van den Poel (2010) reveals that for many products, the curvature estimate is greater than the price elasticity estimate, which implies that $\psi > 1$ for them.[10] Especially, they find that, for products with price elasticity greater than 6, the superelasticity is greater than the price elasticity

[10] See Figure 3 in Dossche, Heylen and Van den Poel (2010).

on average. For products with price elasticity between 3 and 6, the superelasticity is smaller than the price elasticity but very close to. Beck and Lein (2020) use a large set of European homescan data and estimate both the price elasticity and the superelasticity. The median estimate for the price elasticity and the superelasticity is 3.2 and 1.93, respectively, so the implied value for ψ is smaller than 1. They do not provide the estimates across products and markets so it is not possible to check the relative size of the price elasticity and the superelasticity at the product level. The dataset of Dossche, Heylen and Van den Poel (2010) and Beck and Lein (2020) is very large but mainly consists of transactions of the retail consumer goods. In light of the result by Dossche, Heylen and Van den Poel (2010) that products with bigger price elasticity exhibit bigger superelasticity, we conjecture that $\psi > 1$ for goods other than retail consumer goods.

3.2 Boundary between the determinacy and indeterminacy region

The baseline model may have a unique stable equilibrium (equilibrium determinacy) or multiple stable equilibria (equilibrium indeterminacy) depending on the magnitude of the policy parameters (ϕ_π, ϕ_y) in the Taylor-type interest rate rule (21) as the standard New Keynesian model.[11] Below we graphically illustrate the condition for equilibrium determinacy of the baseline model by presenting the boundary between the determinacy and indeterminacy region on (ϕ_π, ϕ_y) space. To facilitate visualization and intuitive discussions, we consider a simple case where there are only two groups of the intermediate good firms that share an identical price change frequency within each group. This simple case can be thought of as a two-sector New Keynesian model. Since the baseline model is complicated even with two sectors only, we could not analytically find a condition for equilibrium determinacy. So, we numerically find the boundary between the determinacy and indeterminacy region.[12] A realistic case with more disaggregate sectors is considered later on.

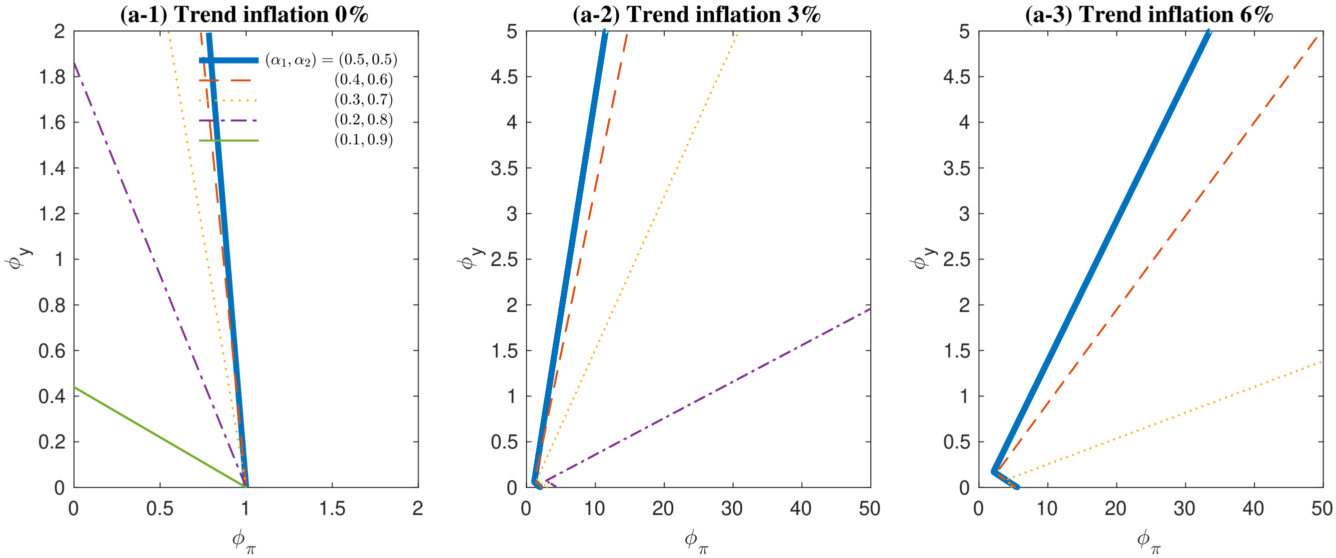
To show how a change in the degree of heterogeneity affects the boundary between the determinacy and indeterminacy region, we trace out a change in the boundary by starting from a homogeneous-frequency economy with $(\alpha_1, \alpha_2) = (0.5, 0.5)$ and increasing the distance of the two sectors in terms of the price change frequency. We keep the average price change frequency fixed at 0.5 in order to control the overall price stickiness of the economy. So, our exercise amounts to an increase in the mean-preserving spread of the price change frequency. The size of the two sectors is assumed to be identical.

Three distinct cases are identified in terms of the type of the aggregator and the level of trend inflation:

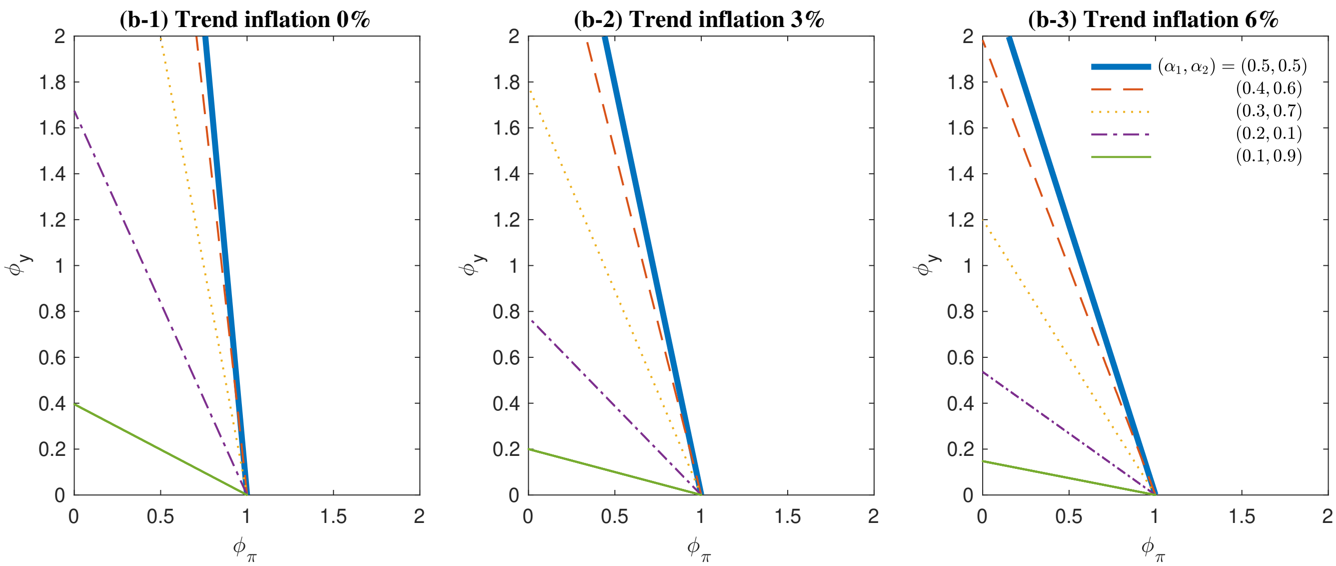
- Specification I: featuring the CES aggregator ($\psi = 0$) and zero trend inflation ($\bar{\Pi} = 1$);

[11] With non-trivial fiscal policy, the model may not have stable equilibrium at all. See, for example, Leeper (1991) and Bhattarai, Lee, and Park (2016).

[12] At each point on a grid of (ϕ_π, ϕ_y) space, we check whether equilibrium is determinate or not, and find a boundary between the region where equilibrium is determinate and the region where equilibrium is indeterminate.



(a) With the CES aggregator ($\psi = 0$)



(b) With the Kimball-type aggregator ($\psi = 1.1$)

Figure 1: Boundary between the determinacy and indeterminacy region: 2 sectors with equal sizes

Notes: The values of ϕ_π and ϕ_y in the region to the left and to the right of each line lead to equilibrium indeterminacy and determinacy, respectively. The size of the two sectors is identical at 0.5. Trend inflation is annualized net trend inflation. Trend inflation is greater than its upper bound for $(\alpha_1, \alpha_2) = (0.1, 0.9)$ in panel (a-2) and for $(\alpha_1, \alpha_2) = (0.2, 0.8)$ and $(\alpha_1, \alpha_2) = (0.1, 0.9)$ in panel (a-3) so an equilibrium does not exist for them.

- Specification II: featuring the CES aggregator ($\psi = 0$) and positive trend inflation ($\bar{\Pi} > \tilde{\Pi} > 1$) with $\tilde{\Pi}$ close to 1;[13] and

- Specification III: featuring the Kimball-type aggregator ($\psi = 1.1$).

Figure 1 reports the result. In each panel of the figure, (ϕ_π, ϕ_y) in the region to the left and to the right of each line are associated with equilibrium indeterminacy and determinacy, respectively. Let us first look at panel (a-1), which shows the boundaries for Specification I. The boundary between the determinacy and indeterminacy region tilts to the left as the price change frequency becomes more heterogeneous, which means that the determinacy region expands. It follows that, as the degree of heterogeneity in the price change frequency increases, monetary policy can achieve equilibrium determinacy with weaker responses to inflation and output as long as it responds to output.

Interestingly, the result is reversed as trend inflation rises. Panels (a-2) and (a-3) display the boundaries for Specification II with trend inflation 3% ($\bar{\Pi} = 1.03^{1/4}$) and 6% ($\bar{\Pi} = 1.06^{1/4}$), respectively. The aggregator still features CES. As the price change frequency of the two sectors moves farther apart, the boundary between the determinacy and indeterminacy region shifts to the right, which means that the determinacy region shrinks. The boundary is now upward-sloping for large values of ϕ_y and downward-sloping for small values of ϕ_y. In general, if trend inflation is high, monetary policy is required to respond much more strongly to changes in inflation and output with heterogeneous price stickiness than with the homogeneous price stickiness.

In Specification III, with the Kimball-type aggregator ($\psi = 1.1$), we get qualitatively similar results as in Specification I. Panels (b-1) to (b-3) present the boundaries between the determinacy and indeterminacy region for trend inflation 0%, 3%, and 6%, respectively. As in panel (a-1), the boundary tilts to the left and the determinacy region expands as the price change frequency becomes more heterogeneous. It follows that monetary policy may adjust nominal interest rates by less and still achieve equilibrium determinacy in response to a change in inflation and output in the heterogeneous-frequency economy than in the homogeneous-frequency economy.

4 Inspecting the mechanism

This section inspects the mechanism underlying the change in the condition for equilibrium determinacy due to heterogeneity of the price change frequency. We discuss the mechanism for the three cases identified in Section 3 in turn.

We especially use the generalized Taylor principle to provide intuitive explanations. It is a prescription for sound monetary policy that the nominal interest rate should respond to a change

[13]If trend inflation is $1 < \bar{\Pi} \leq \tilde{\Pi}$, the result is similar to that of Specification I. See the discussion below.

in inflation more than one-for-one in the long run. The literature has shown that it is a necessary and sufficient condition for equilibrium determinacy in a simple New Keynesian model with homogeneous price stickiness (see, for example, Woodford 2003). We numerically verify that it is also a necessary and sufficient condition for equilibrium determinacy in Specifications I and III in our baseline model.[14] Though it is not necessary and sufficient in Specification II, it at least characterizes the upward-sloping part of the boundary in panels (a-2) and (a-3) of Figure 1. As the upward-sloping part of the boundary separates the determinacy and indeterminacy region in the most of (ϕ_π, ϕ_y) space in Specification II, we turn to the generalized Taylor principle to explain the mechanism for Specification II as well.

In the log-linearized baseline model, the generalized Taylor principle can be mathematically written as

$$\frac{di}{d\pi} = \phi_\pi + \phi_y \frac{dy}{d\pi} > 1, \tag{23}$$

where di, $d\pi$, and dy are a long-run change in the nominal interest rate, inflation and output, respectively. Therefore, any differences in the boundary that are characterized by the generalized Taylor principle in the panels of Figure 1 are attributed to differences in $dy/d\pi$, which is the long-run output-inflation trade-off. In the appendix at the end of this paper, we show that it corresponds to a change in output due to a marginal increase in trend inflation in the steady state. This correspondence provides formal justification to the heuristic interpretation of $dy/d\pi$ and $di/d\pi$ in terms of long-run changes of the variables often found in the literature. We take advantage of this correspondence when inspecting the mechanism below.

The distribution of the price change frequency is characterized by the size of each sector: $f(\alpha)$ can be considered as the density function of the distribution. The weighted average of the price change frequency, denoted by $\bar{\alpha} = \int_0^1 \alpha f(\alpha)\, d\alpha$, is assumed to be the representative frequency of the homogeneous-frequency economy in the following discussions. Let $\bar{f}(\alpha)$ denote the density function of the frequency distribution of the homogeneous-frequency economy: $\bar{f}(\alpha) = 1$ for $\alpha = \bar{\alpha}$ and $= 0$ otherwise.

See the online appendix for details of the derivation in the following.

4.1 Specification I: CES aggregator and zero trend inflation

The long-run output-inflation trade-off can be analytically found for Specification I, which is first derived below. For now, let us assume that the price change frequency is less than 1 in all the sectors: $f(1) = 0$.

[14] Therefore, one can use the generalized Taylor principle to more easily find the boundaries between the determinacy and indeterminacy region. However, we numerically find the boundaries in all the figures in the paper in order to verify and make sure that the generalized Taylor principle holds in all those cases.

The price-updating firms of sector α sets the optimal price as

$$p^*_{\alpha,t} = \beta(1-\alpha)\mathbb{E}_t\left(p^*_{\alpha,t+1} + \pi_{t+1}\right) + [1-\beta(1-\alpha)]\,\omega\left(y_t - \left(\frac{1+\varphi}{\sigma+\varphi}\right)a_t\right), \tag{24}$$

where $p^*_{\alpha,t}$, y_t, a_t, and π_t are the log-deviations of $P^*_{\alpha,t}/P_t$, Y_t, A_t, and Π_t, respectively, from the steady state. The relative price of sector α evolves as

$$p^R_{\alpha,t} = (1-\alpha)\left(p^R_{\alpha,t-1} - \pi_t\right) + \alpha p^*_{\alpha,t}, \tag{25}$$

where $p^R_{\alpha,t}$ is the log-deviation of

$$P^R_{\alpha,t} = \Lambda_{\alpha,t} = \left[\frac{1}{f(\alpha)}\int_{\mathcal{I}_\alpha}\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

from its steady state value. We can rearrange Equations (24) and (25) to produce the sectoral Phillips curve

$$\pi_{\alpha,t} = \beta\mathbb{E}_t\pi_{\alpha,t+1} + g(\alpha)\left[\omega\left(y_t - \left(\frac{1+\varphi}{\sigma+\varphi}\right)a_t\right) - p^R_{\alpha,t}\right], \tag{26}$$

where

$$g(\alpha) = \frac{\alpha[1-\beta(1-\alpha)]}{(1-\alpha)}, \tag{27}$$

$\omega = (\sigma+\varphi)/(1+\theta\varphi)$, and $\pi_{\alpha,t}$ is the log-deviation of

$$\Pi_{\alpha,t} = \frac{P^R_{\alpha,t}P_t}{P^R_{\alpha,t-1}P_{t-1}} = \frac{\left[\frac{1}{f(\alpha)}\int_{\mathcal{I}_\alpha}(P_t(i))^{1-\theta} di\right]^{\frac{1}{1-\theta}}}{\left[\frac{1}{f(\alpha)}\int_{\mathcal{I}_\alpha}(P_{t-1}(i))^{1-\theta} di\right]^{\frac{1}{1-\theta}}},$$

from its steady state value. Using the fact that the weighted sum of the relative price is zero,

$$0 = \int_0^1 f(\alpha)\,p^R_{\alpha,t}d\alpha, \tag{28}$$

we can aggregate the sectoral Phillips curves (26) into the aggregate Phillips curve

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \omega\left[\int_0^1 f(\alpha)\,g(\alpha)\,d\alpha\right]\left(y_t - \left(\frac{1+\varphi}{\sigma+\varphi}\right)a_t\right) - \Theta_t, \tag{29}$$

where

$$\Theta_t = \int_0^1 f(\alpha)\,g(\alpha)\,p^R_{\alpha,t}d\alpha.$$

For reference, here is the Phillips curve in the homogeneous-frequency economy

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \omega g(\bar{\alpha}) \left(y_t - \left(\frac{1+\varphi}{\sigma+\varphi} \right) a_t \right). \tag{30}$$

As Carvalho (2006) notes, there are two important differences in (29) compared to (30). First, the coefficient on the marginal cost is bigger in the heterogeneous-frequency economy than in the homogeneous-frequency economy

$$\int_0^1 f(\alpha) g(\alpha) d\alpha > g(\bar{\alpha}),$$

since $g(\alpha)$ is strictly convex. Function $g(\alpha)$ also appears in the New Keynesian Phillips curve with homogeneous price stickiness. As the marginal cost varies today, the price-updating firms adjust their optimal prices by a factor of the weight on the marginal cost today in (24), which is $1-\beta(1-\alpha)$. The size of the factor is increasing in α due to a stronger (weaker) front-loading behavior by the price-updating firms with a low (high) price change frequency as emphasized by Christiano, Eichenbaum and Evans (2005). As α decreases, they are more worried that they may not be able to change their prices for a long period of time in the future when the marginal cost varies and put a smaller weight on the marginal cost today. This adjustment in the optimal price affects sectoral inflation by a factor of the relative size of the price-updating firms to the non-price-updating firms in sector α, which is $\alpha/(1-\alpha)$. This relative size effect is seen in the following equation

$$(1-\alpha)\pi_{\alpha,t} = \alpha \left(p^*_{\alpha,t} - p^R_{\alpha,t} \right), \tag{31}$$

which can be derived using (25) and the fact that $\pi_{\alpha,t} = p^R_{\alpha,t} - p^R_{\alpha,t-1} + \pi_t$. As a result, $g(\alpha)$ is strictly convex and increasing in α: sectoral inflation is disproportionately more sensitive to a change in the marginal cost today in high-frequency sectors than in low-frequency sectors.

Second, an aggregate of the sectoral relative prices, Θ_t, influences the dynamics of aggregate inflation when the price change frequency is heterogeneous across sectors.[15] It emerges in (29) since the sectoral relative price affects sectoral inflation in (26). The real marginal cost is in terms of the final good that aggregates the intermediate goods across all the sectors while sectoral inflation is the rate of a change in the price level of the intermediate goods of a particular sector only. Therefore, any deviations of the sectoral price level from the aggregate price level, or fluctuations in the sectoral relative price $p^R_{\alpha,t}$, would have to be taken into account when determining a response of sectoral inflation to a change in the real marginal cost.[16]

[15] Carvalho (2006) dubs this term an endogenous shifter. A similar term is also observed in the aggregate Phillips curve derived in Aoki (2001). In Aoki (2001) and Carvalho (2006), the term is written in terms of sectoral relative output. Using the demand curve, Θ_t in (29) can be also written in terms of sectoral relative output.

[16] The sectoral relative price in (26) is reminiscent of the terms of trade in the Phillips curve for domestic inflation in an open economy model where the marginal cost includes imported goods. See, for example, Galí (2015).

For example, suppose that the nominal marginal cost and the aggregate price level rise by the same speed in tandem so that the real marginal cost does not move. If the sectoral price level of a sector is fixed, however, the marginal cost increases relative to the sectoral price level, which puts an upward pressure on sectoral inflation. The negative sign on $p^R_{\alpha,t}$ in (26) captures this upward pressure since the sectoral relative price declines.

The sectoral relative prices do not cancel out through aggregation despite (28). This is because the sectoral relative price affects sectoral inflation through the marginal cost so its coefficient is $g(\alpha)$. A fall (rise) in the sectoral relative price puts an upward (downward) pressure on sectoral inflation. Since $g(\alpha)$ is strictly convex and increasing in α, however, the upward and downward pressures across sectors do not cancel out, which has an important implication for the long-run trade-off between inflation and output as explained below.

Based on these two differences, we can explain why the determinacy region expands in the heterogeneous-frequency economy compared to the homogeneous-frequency economy and also why the boundary tilts more to the left with a bigger degree of heterogeneity as seen in panel (a-1) of Figure 1. Let us turn to the generalized Taylor principle (23), which holds for both the heterogeneous-frequency economy and the homogeneous-frequency economy. The long-run output-inflation trade-off is given as

$$\left.\frac{dy}{d\pi}\right|_{\{f(\alpha)\}} = \frac{1-\beta}{\omega\int_0^1 f(\alpha)\, g(\alpha)\, d\alpha}\left(1+\frac{1}{1-\beta}\frac{d\Theta}{d\pi}\right), \tag{32}$$

in the heterogeneous-frequency economy and

$$\left.\frac{dy}{d\pi}\right|_{\{\bar{f}(\alpha)\}} = \frac{1-\beta}{\omega g(\bar{\alpha})}, \tag{33}$$

in the homogeneous-frequency economy.

Among the two differences in the Phillips curve discussed above, an increase in the coefficient on the marginal cost makes $dy/d\pi$ smaller with the other terms fixed. That is, aggregate inflation becomes more responsive to changes in the marginal cost in the heterogeneous-frequency economy. Thus, a bigger change in inflation is associated with the same change in output and $dy/d\pi$ becomes smaller.

On the contrary, the second difference turns out to make $dy/d\pi$ bigger. This is because the dispersion of the sectoral relative prices across sectors has a disproportionately larger effect on aggregate inflation in high-frequency sectors than in low-frequency sectors. In response to a positive shock that leads the price-updating firms to raise their prices, the sectoral relative price rises in high-frequency sectors but falls in low-frequency sectors. Since the sectoral relative price in high-frequency sectors has a bigger influence on aggregate inflation than in low-frequency sectors, Θ_t rises and dampens an increase of aggregate inflation after the shock. In other words, aggregate inflation becomes less sensitive to changes in the marginal cost and thus $dy/d\pi$ increases.

For example, consider the simple two-sector case of panel (a-1) of Figure 1: there are two sectors with equal sizes but with different price change frequencies. Suppose that a permanent shock to aggregate nominal spending hits the economy and price-updating firms want to raise their prices. In the heterogeneous-frequency economy, unlike the homogeneous-frequency economy, the sectoral relative prices diverge across the two sectors after the shock hits: the sectoral relative price rises in the high-frequency sector while it declines in the low-frequency sector. This is simply because there are more price-updating firms in the high-frequency sector in a given period than in the low-frequency sector by the law of large numbers.[17] In the log-linearized model, the sectoral relative price of the high-frequency sector becomes positive while that of the low-frequency sector becomes negative. However, as discussed above, $g(\cdot)$ is strictly convex and increasing in α so Θ_t becomes positive. It follows that a positive change in inflation is associated with a positive change in the sectoral relative price dispersion and $d\Theta/d\pi > 0$.

Note that the long-run trade-off (32) can be further simplified after eliminating $d\Theta/d\pi$ as follows

$$\left.\frac{dy}{d\pi}\right|_{\{f(\alpha)\}} = \frac{1-\beta}{\omega} \int_0^1 \frac{f(\alpha)}{g(\alpha)} d\alpha. \tag{34}$$

Since $1/g(\alpha)$ is also convex, indeed $dy/d\pi$ is bigger in the heterogeneous-frequency economy than in the homogeneous-frequency economy. That is, an increase in $dy/d\pi$ due to $d\Theta/d\pi$ dominates its decrease due to the convexity of $g(\alpha)$. Also, $dy/d\pi$ gets bigger as the price change frequency becomes more heterogeneous.[18]

The expression of $dy/d\pi$ in (34) shows that there is an interaction between strategic complementarity in price-setting and heterogeneity of the price change frequency.[19] In the baseline model, strategic complementarity in price setting arises because of the firm-specific labor market assumption. A stronger strategic complementarity in price setting or a stronger real rigidity resulting from smaller ω (larger φ) implies a bigger long-run trade-off given the distribution of the price change frequency. This is because price-updating firms adjust their prices by less in response to changes in the marginal cost with a stronger real rigidity. Therefore, the effect of heterogeneity in the price change frequency on the condition for equilibrium determinacy is amplified.

Lastly, we propose a way to formally define an increase in the degree of heterogeneity of the price

[17] Because of the strategic complementarity in price setting generated by firm-specific labor market, the price-updating firms do not fully respond to the shock. The price-updating firms in the low-frequency sector would change their prices more than those in the high-frequency sector because they have a smaller chance of changing their prices again in the future. Overall, however, the sectoral relative price increases in the high-frequency sector and decreases in the low-frequency sector because of the size effect.

[18] If there is a sector with fully flexible prices ($\alpha = 1$) in the heterogeneous-frequency economy, $dy/d\pi$ can be computed in the same way by recognizing that $1/g(1) = 0$.

[19] Carvalho (2006) calls this interaction a strategic interaction.

change frequency and establish a proposition that summarizes its implication for the condition of equilibrium determinacy. Consider a distribution of the price change frequency characterized by density f. Suppose that, for each α, there exists a random variable γ with density h_α such that

$$\int_{-1}^{1} \gamma h_\alpha(\gamma)\, d\gamma = 0,$$

and $0 \leq \alpha + \gamma \leq 1$. Then, the distribution of the price change frequency obtained by $\alpha + \gamma$ is a mean-preserving spread of f. It perturbs f to increase heterogeneity of the price change frequency among firms while keeping the mean fixed. Let us denote this mean-preserving spread by $\tilde{f}$. Suppose that $\tilde{f}$ is not identical to f. Since $1/g$ is strictly convex, we can establish the following proposition. The proof is a straightforward application of the Jensen's inequality.

Proposition. *Consider the baseline model with the CES aggregator and zero trend inflation and suppose that a distribution of the price change frequency is characterized by density f. For its mean-preserving spread $\tilde{f}$, the long-run output-inflation trade-off is bigger with $\tilde{f}$ than with f. Therefore, the determinacy region with $\tilde{f}$ expands (nests and is not the same as) the one with f.*

4.2 Specification II: CES aggregator and positive trend inflation

Panels (a-2) and (a-3) in Figure 1 show that, if the aggregator features CES but trend inflation is sufficiently greater than zero, the determinacy region shrinks and monetary policy has to respond more strongly to changes in inflation and output to achieve equilibrium determinacy in the heterogeneous-frequency economy than in the homogeneous-frequency economy. The reason for this reversal of the result is that the long-run output-inflation trade-off turns negative with sufficiently greater than zero trend inflation as discussed by Kiley (2007), Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), and Ascari and Sbordone (2014). If trend inflation is positive but close to zero, the long-run trade-off does not become negative and the result is qualitatively similar to that of Specification I. However, the threshold above which the long-run trade-off turns negative is very small.[20] Thus, we do not consider such a case separately.

Note that the price of intermediate good firms, whether they can adjust their prices or not, is always identical to the aggregate price in the steady state if trend inflation is zero in symmetric equilibrium. Now consider a marginal increase in trend inflation from zero in the steady state. The relative price of the firms that do not reset their prices is continually eroded by small but positive trend inflation while the relative price of the price-updating firms is raised because of the firms' concern on a deterioration of their relative prices by positive trend inflation in the future. It

[20]The threshold is about 0.2% in terms of annual net trend inflation in the two-sector model we consider. We present how the boundary between the determinacy and indeterminacy region shifts when trend inflation is 0.1%, lower than the threshold, in the online appendix.

follows that, because of resulting changes in the demand, output of the non-price-updating firms increases while that of the price-updating firms declines. The latter is smaller with positive but low trend inflation than the former. As a result, aggregate output increases.

To illustrate the mechanism, we compute changes of steady state output by the non-price-updating and price-updating firms, respectively, in the baseline model with equally-sized two sectors and compare them in Figure 2. Note that, since the final good producers are perfectly competitive, aggregate output can be written as

$$Y_t = \int_0^1 \frac{P_t(i)\, Y_t(i)}{P_t} di = \lim_{K \to \infty} Y_t^{(K)},$$

where

$$Y_t^{(K)} = \sum_{k=0}^{K} \left\{ \int_0^1 f(\alpha) \left[\alpha (1-\alpha)^k \left(\frac{P^*_{\alpha,t-k}}{P_t} \right) Y_{\alpha,t|t-k} \right] d\alpha \right\}, \tag{35}$$

with $Y_{\alpha,t|t-k}$ being the period-t demand faced by the sector-α firms that have last updated their prices in period $t-k$. Their measure is $\alpha(1-\alpha)^k$ in sector α. Hence, in period t, $Y_t^{(0)}$ is output by the price-updating firms and $Y_t^{(\infty)} - Y_t^{(0)}$ is output by the non-price-updating firms. We assume that $(\alpha_1, \alpha_2) = (0.3, 0.7)$ and compute the change in $Y_t^{(K)}$ for $K = 0, 1, 2, \cdots$ in the steady state by raising trend inflation by 0.1%p from 0% and 3%, respectively. In panel (a) of Figure 2 where trend inflation rises from 0% to 0.1%, output declines by the largest for the price-updating firms. Output by the non-price-updating firms with $k = 1$ also declines but it increases enough for $k > 1$ so that aggregate output eventually rises.

Figure 3 shows how steady state output and the long-run output-inflation trade-off varies depending on the level of trend inflation. To compute the quantities, we again consider the simple two-sector example where the two sectors have an identical size. Steady state output increases as trend inflation rises up to a certain point in panel (a) of the figure. It follows that the long-run trade-off remains positive up to the the same threshold. Up to the threshold, the long-run trade-off becomes bigger as the price change frequency becomes more heterogeneous for the same reasons as in Specification I. See panel (b) of Figure 3.

The result is however overturned if trend inflation rises more beyond the threshold. Suppose now that trend inflation is already sufficiently high and consider a marginal increase in trend inflation in the steady state. The relative price of the firms that do not reset their prices is more eroded by the marginal increase in trend inflation while the relative price of the price-updating firms is raised more because of the firms' concern on a further deterioration of their relative prices by the marginal increase in trend inflation. The price-updating firms are worried that their relative prices deteriorate more quickly by trend inflation and put smaller weight on the marginal cost today compared to the future. In other words, they become more forward-looking so less responsive to changes in the marginal cost today. Unlike in a case with a very small positive trend inflation, the

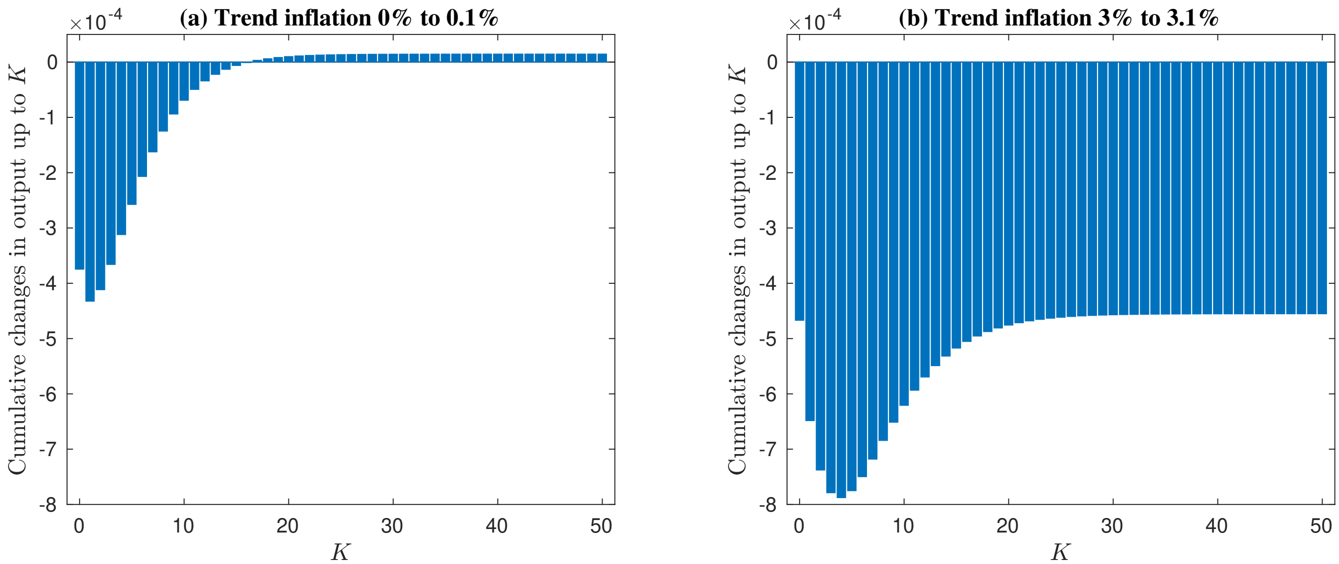


Figure 2: Cumulative changes in output for an increase in trend inflation in the steady state: 2 sectors, CES aggregator

Notes: The size of the two sectors are assumed to be identical. The frequencies are $(\alpha_1, \alpha_2) = (0.3, 0.7)$. Trend inflation is an annualized net rate of changes in the aggregate price level. The cumulative change in output is the change in $Y_t^{(K)}$ when trend inflation rises by 0.1%p.

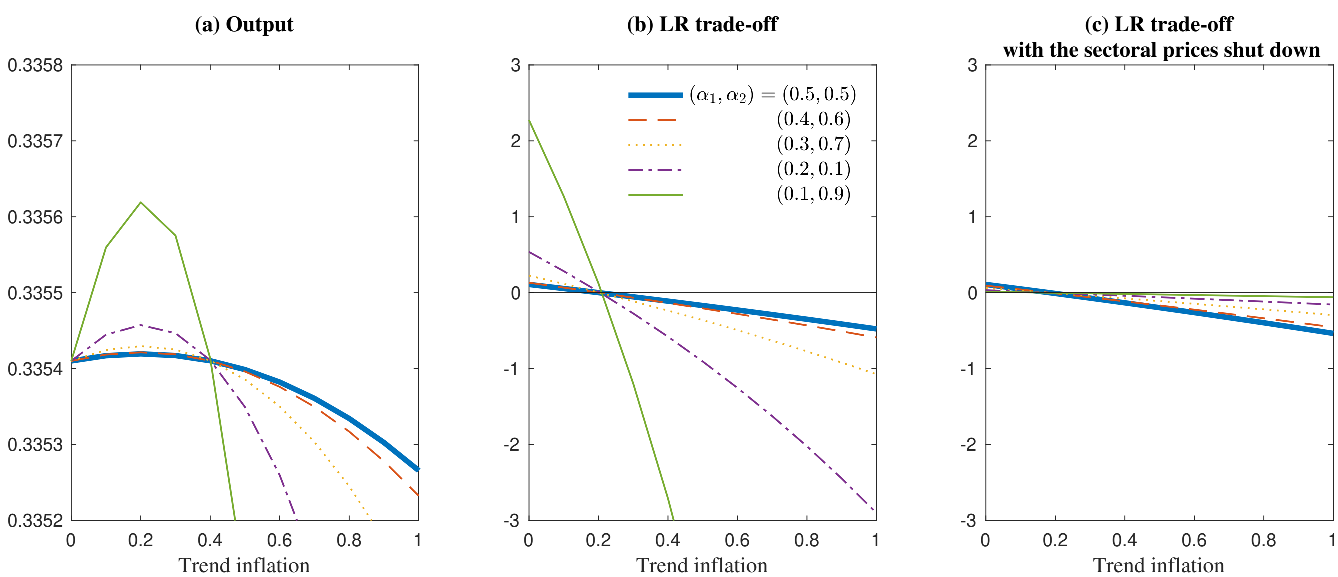


Figure 3: Output and the long-run output-inflation trade-off in the steady state: 2 sectors, CES aggregator

Notes: The size of the two sectors are assumed to be identical. Trend inflation is an annualized net rate of changes in the aggregate price level. In panel (c), we drop the aggregate of the sectoral relative price in the Phillips curve and compute the long-run trade-off. The exact expression is presented in the online appendix.

optimal prices are raised by so much, which pushes down the demand for the price-updating firms so much.

This mechanism is shown in panel (b) of Figure 2 where trend inflation rises from 3% to 3.1%. Output of the price-updating firms drops and so does output of the non-price-updating firms for $k = 1, \cdots, 4$. Output by the non-price-updating firms for $k > 4$ increases but not sufficiently. It follows that aggregate output decreases. Therefore, the long-run output-inflation trade-off turns negative as in panels (a) and (b) of Figure 3.

The long-run trade-off becomes bigger negative as the price change frequency becomes more heterogeneous due to the sectoral price dispersion. We can see this by comparing panels (b) and (c) of Figure 3. In panel (c) of the figure, we shut down the influence of the sectoral relative price dispersion on aggregate inflation and compute the long-run trade-off between inflation and output.[21] When we ignore the effect of the sectoral relative price dispersion on aggregate inflation, as the price change frequency gets more heterogeneous, the coefficient on the marginal cost becomes larger, which makes the magnitude of the long-run trade-off smaller. The difference between panels (b) and (c) of the figure is attributed to the role of the sectoral relative price dispersion. As the price change frequency becomes more heterogeneous, the sectoral relative price dispersion tends to make aggregate inflation less responsive to changes in the marginal cost today as in Specification I, which increases the magnitude of the long-run trade-off.

If we shut down the strategic interaction between real rigidity and heterogeneity of the price change frequency by setting $\varphi = 0$, the reversal effect of positive trend inflation shown in panels (b) and (c) of Figure is not amplified by real rigidity. We present the result in the online appendix and discuss it in the sensitivity analysis in Section 4.4.

4.3 Specification III: Kimball-type aggregator

In Specification III featuring the Kimball-type aggregator, as in Specification I, the boundary between the determinacy and indeterminacy region tilts to the left and the determinacy region expands in the heterogeneous-frequency economy compared to the homogeneous-frequency economy. This is because the kinked demand curve that results from the Kimball-type aggregator constrains the effect of trend inflation so the long-run output-inflation trade-off remains positive even with high trend inflation.

The relative price of the firms that do not reset their price is again continually eroded by an increase in trend inflation. However, firms raise their prices by less when they get a chance to update their prices in Specification III than in Specification II. This is because of real rigidity due to the kink in the demand curve: as the relative price is raised, the demand for their products

[21] The exercise is similar to dropping $d\Theta/d\pi$ in Equation (32). The exact expression is presented in the online appendix.

becomes more elastic and a decline in the demand and profit accelerates. In other words, the price-updating firms do not become too much forward-looking even with high trend inflation unlike those with the CES aggregator in Specification II. Therefore, output by the price-updating firms does not decline as much. Since output by the non-price-updating firms expands, it follows that aggregate output rises as trend inflation increases regardless of the current level of trend inflation. This mechanism is illustrated in panels (a-1) and (a-2) of Figure 4 where we present changes in $Y_t^{(K)}$ due to a marginal increase in trend inflation when the current level of trend inflation is 0% and 3%, respectively. Output by the price-updating firms drops by most but output by the non-price-updating firms rises sufficiently so that aggregate output eventually increases. This occurs for both trend inflation 0% and 3%.

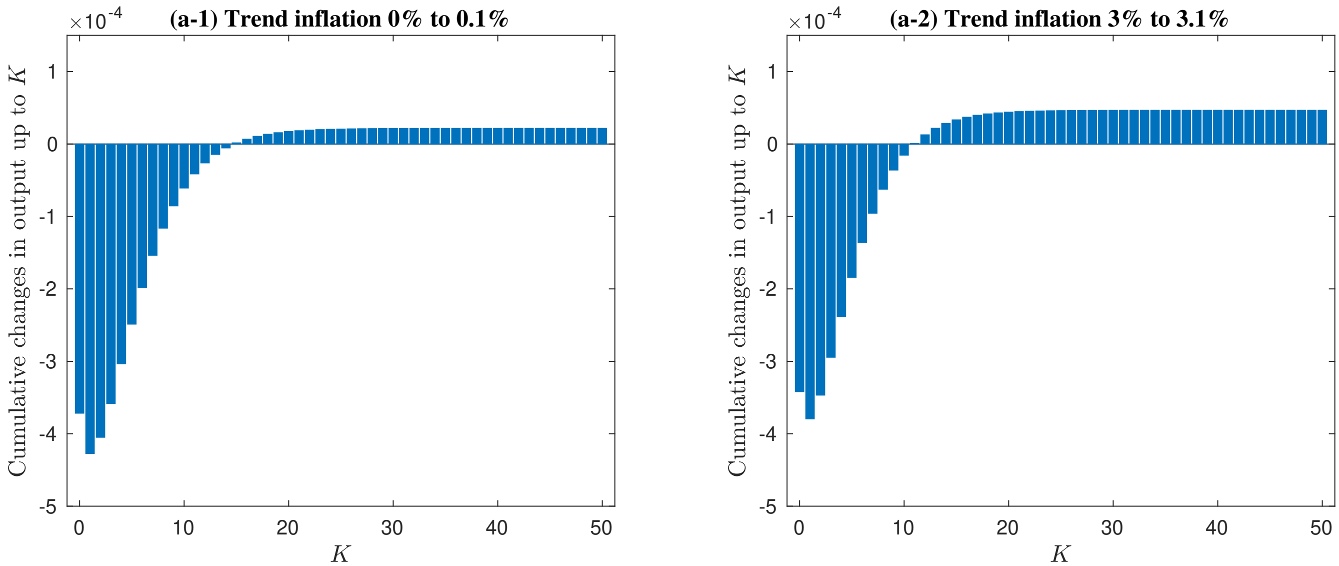
As a result, as Figure 5 shows, aggregate output increases and the long-run output-inflation trade-off remains positive. The long-run trade-off becomes bigger positive in Figure 5 as the price stickiness becomes more heterogeneous. First, as trend inflation rises, the relative prices of the non-price-updating firms are more quickly eroded. So, they produce more and aggregate output rises more with higher trend inflation as can be seen in panels (a-1) and (a-2) of Figure 4. Second, as the price change frequency becomes more heterogeneous with their average kept constant, more non-price-updating firms have not had chances to update their prices for longer periods of time since they last reset their prices.[22] So, their relative prices are more eroded by trend inflation and their output rises more. It follows that aggregate output increases. Panels (b-1) and (b-2) of Figure 4 compare the cumulative changes in output following a marginal increase in trend inflation between a less heterogeneous-frequency economy with $(\alpha_1, \alpha_2) = (0.4, 0.6)$ and a more heterogeneous-frequency economy with $(\alpha_1, \alpha_2) = (0.2, 0.8)$. The non-price-updating firms produce more and aggregate output eventually rises more in panel (b-2) than in panel (b-1).

4.4 Sensitivity analysis

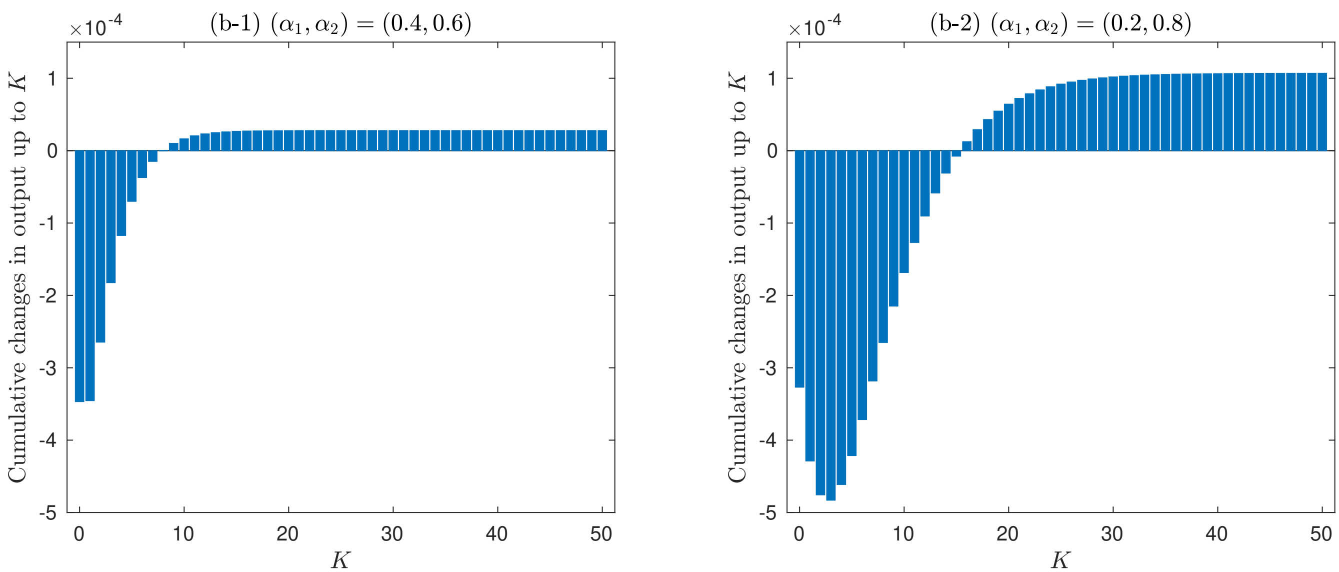
We carry out several exercises for sensitivity analysis and report the results in this section. All the exercises discussed below are done in the baseline model with two sectors.

First, we check how the relative size of the two sectors is associated with the effect of heterogeneity in the price change frequency on the condition for equilibrium determinacy. For this purpose, we fix the price change frequency at $(\alpha_1, \alpha_2) = (0.3, 0.7)$ and compute the boundaries between the determinacy and indeterminacy region for different relative sizes of the two sectors. The results are reported in Figure 6, where the boundary for the homogeneous-frequency economy with $(\alpha_1, \alpha_2) = (0.5, 0.5)$ is also presented for comparison. Note that the relative size does not

[22]In other words, the distribution of the non-price-updating firms, over the time since they last reset their prices, becomes more skewed to the right. Recall that the measure of the firms for whom it has elapsed k periods since the last reset of the price is $\alpha (1-\alpha)^k$ for $k = 0, 1, 2, \cdots$ in sector α. The measure quickly rises as α declines except for very small values of k.



(a) Frequencies $(\alpha_1, \alpha_2) = (0.3, 0.7)$



(b) Trend inflation 3% to 3.1%

Figure 4: Cumulative changes in output for an increase in trend inflation in the steady state: 2 sectors, Kimball-type aggregator

Notes: The size of the two sectors are assumed to be identical. In panels (a-1) and (a-2), trend inflation is raised by 0.1%p from 0% and 3%, respectively, while the frequencies are $(\alpha_1, \alpha_2) = (0.3, 0.7)$. In panels (b-1) and (b-2), the frequencies are $(0, 4, 0.6)$ and $(0.2, 0.8)$, respectively, while trend inflation is raised from 3% to 3.1%. Trend inflation is an annualized net rate of changes in the aggregate price level. The cumulative change in output is the change in $Y_t^{(K)}$ when trend inflation rises by 0.1%p.

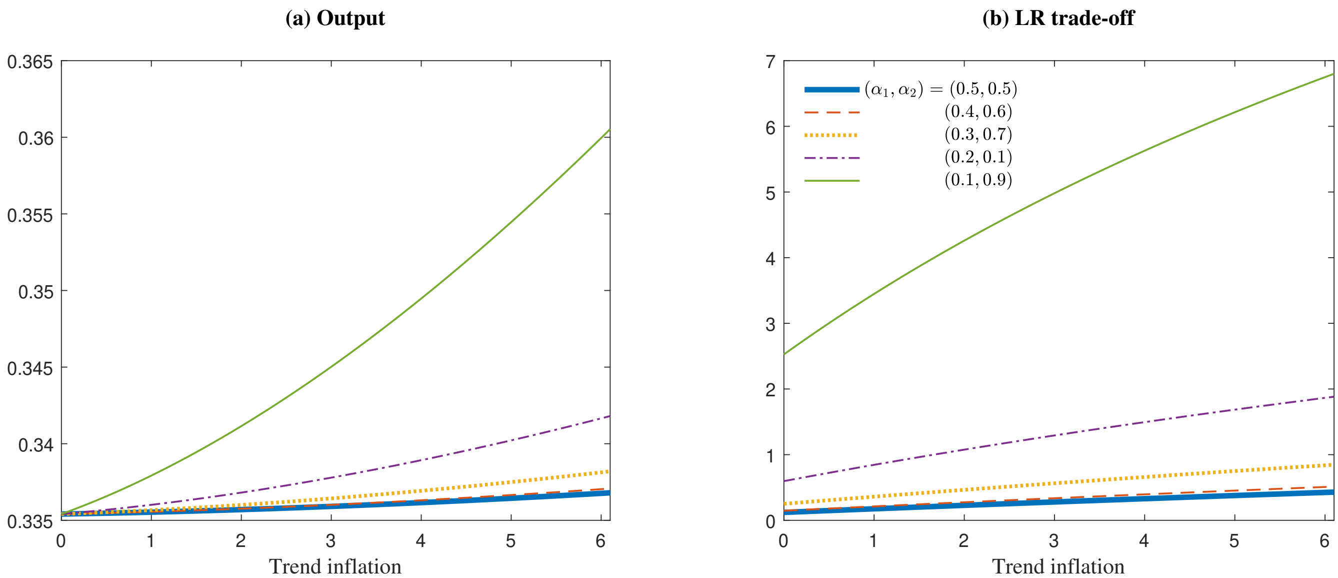


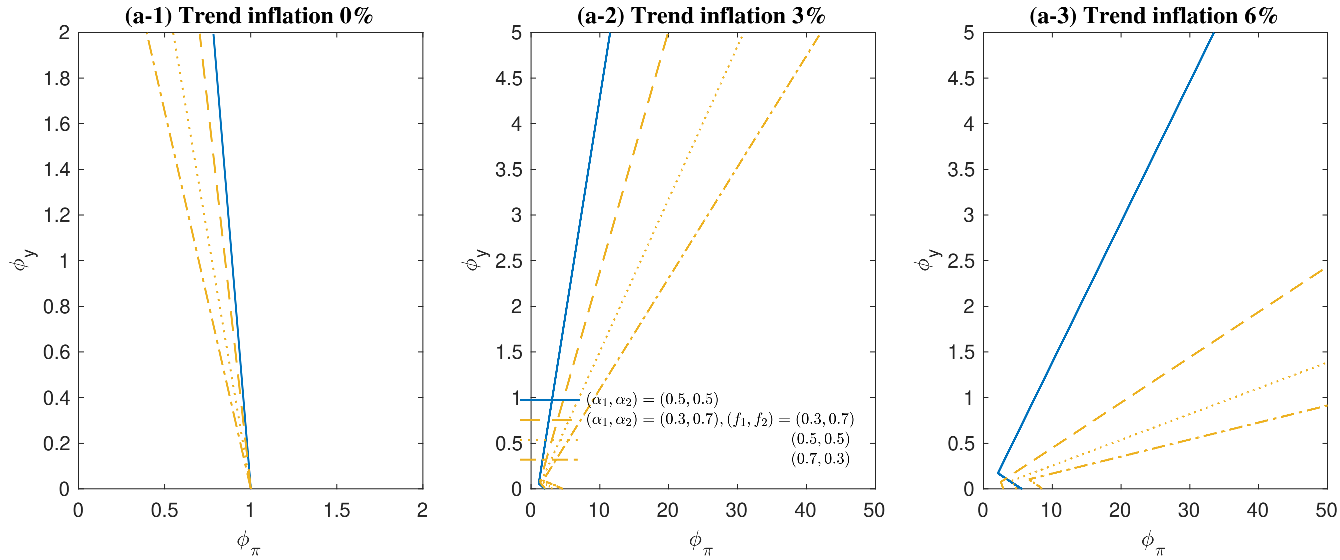
Figure 5: Output and the long-run output-inflation trade-off in the steady state: 2 sectors, Kimball-type aggregator

Notes: The size of the two sectors are assumed to be identical. Trend inflation is an annualized net rate of changes in the aggregate price level.

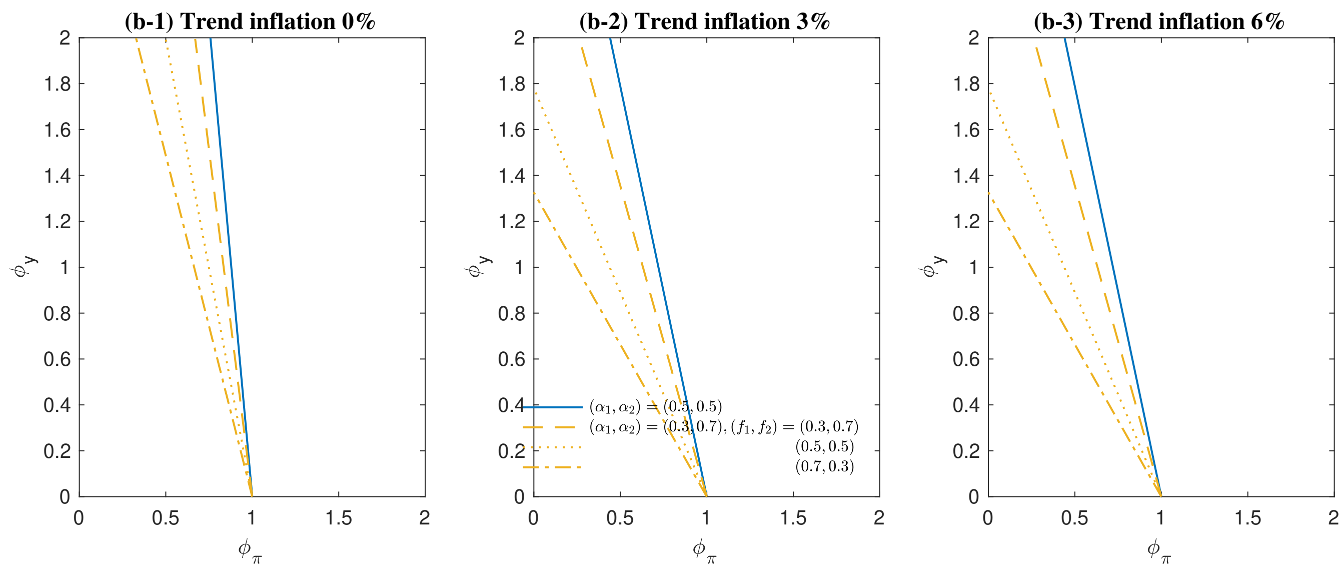
matter for the homogeneous-frequency economy. In panels (a-1) for Specification I and panels (b-1)-(b-3) for Specification III, as the low-frequency sector becomes bigger, the boundary shifts more to the left and the determinacy region expands more. This is because, as overall price stickiness increases, the long-run trade-off between output and inflation becomes larger. In Specification II, the long-run trade-off between output and inflation also becomes larger in magnitude but the sign is negative. Therefore, in panels (a-2) and (a-3) of Figure 6, the boundary tilts more to the right and the determinacy region shrinks more as the relative size of the low-frequency sector increases.[23]

Then, we reveal the role of the strategic interaction between the real rigidity due to strategic complementarity in price setting and heterogeneity in the price change frequency by setting $\varphi = 0$ and shutting down the real rigidity that arises because of the firm-specific labor market. Since the real rigidity due to strategic complementarity in price setting makes aggregate inflation less responsive to changes in the marginal cost, it amplifies the effect of the heterogeneous price stickiness on the long-run trade-off between output and inflation. This is confirmed in Figure A.2 in the online appendix where the determinacy region shrinks compared to the baseline case with $\varphi = 1$. The result holds for different distributions of the price change frequency in Specification I and in Specification III. Interestingly, given the distribution of the price change frequency, the extent to which the determinacy region shrinks is inversely related with the level of trend inflation in Specification III. This is because the price elasticity of the demand decreases as the relative

[23]Because of changes in the downward-sloping part of the boundary, the determinacy region becomes slightly larger for small values of ϕ_y in panel (a-3) of Figure 6.



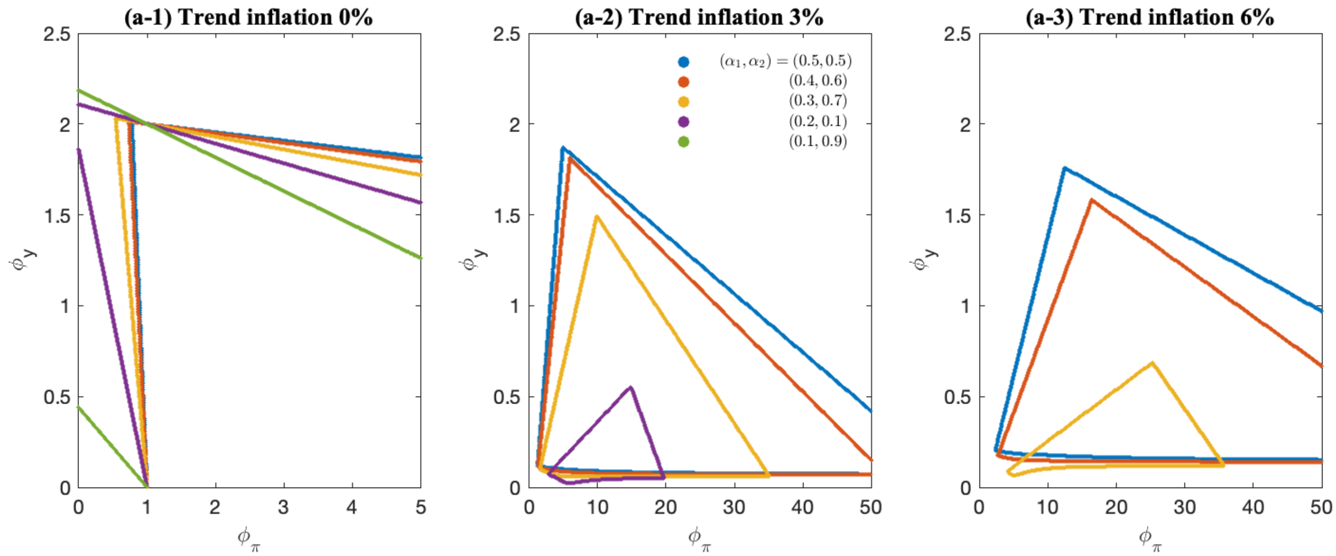
(a) With the CES aggregator ($\psi = 0$)



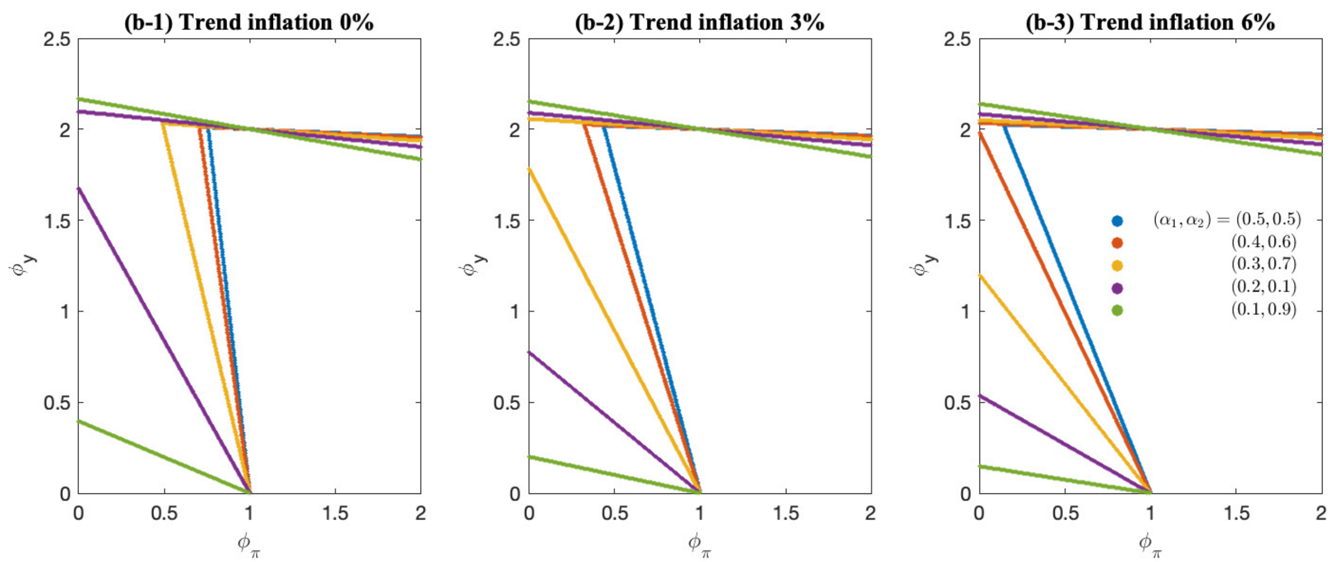
(b) With the Kimball-type aggregator ($\psi = 1.1$)

Figure 6: Boundary between the determinacy and indeterminacy region: 2 sectors with varying sizes

Notes: The values of ϕ_π and ϕ_y in the region to the left and to the right of each line lead to equilibrium indeterminacy and determinacy, respectively. The relative size of the two sectors in the homogeneous-frequency benchmark with $(\alpha_1, \alpha_2) = (0.5, 0.5)$ does not matter. Trend inflation is annualized net trend inflation.



(a) With the CES aggregator ($\psi = 0$)



(b) With the Kimball-type aggregator ($\psi = 1.1$)

Figure 7: Boundary between the determinacy and indeterminacy region: 2 sectors, forward-looking monetary policy rule

Notes: The values of ϕ_π and ϕ_y in the region between the lines of the same color lead to equilibrium determinacy and those in the region outside the lines of the same color lead to indeterminacy. The monetary policy rule is forward-looking as in (36). The size of the two sectors is identical at 0.5. Trend inflation is annualized net trend inflation. Trend inflation is greater than its upper bound for $(\alpha_1, \alpha_2) = (0.1, 0.9)$ in panel (a-2) and for $(\alpha_1, \alpha_2) = (0.2, 0.8)$ and $(\alpha_1, \alpha_2) = (0.1, 0.9)$ in panel (a-3) so an equilibrium does not exist for them.

price falls in Specification III. As trend inflation rises, the price set by the non-price-updating firms are continually eroded, which pushes down the price elasticity of the demand for their goods. Therefore, the additional effect of the real rigidity is relatively small with high trend inflation. This is especially severe for a case with very heterogeneous frequencies.

In another exercise, we increase ψ that determines the curvature of the demand curve implied by the Kimball-type aggregator from 1.1 to 8, which is the value chosen by Levin, Lopez-Salido, Nelson, Yun (2008). The demand curve exhibits a sharper kink and it follows that the real rigidity operates more strongly than in the baseline model with $\psi = 1.1$. Hence, the boundary tilts more to the left and the determinacy region expands more than in the baseline model with $\psi = 1.1$, which is shown in Figure A.3 in the online appendix. Interestingly, when such a high value of ψ is combined with as high trend inflation as 6%, the effect of heterogeneity of the price change frequency on the long-run trade-off of output and inflation is non-monotonic. That is, more heterogeneous price change frequencies lead the boundary to shift to the right and the determinacy region shrink. This is due to a strong non-linearity in the Kimball-type aggregator and the resulting demand curve. However, this happens only when the price change frequency is $(\alpha_1, \alpha_2) = (0.1, 0.9)$ in the equally-sized two-sector example and thus very heterogeneous. Note that even in such a case, the determinacy region still expands compared to the one of the homogeneous-frequency economy.

Lastly, we replace the contemporaneous monetary policy rule (21) with the following forward-looking monetary policy rule

$$\frac{I_t}{\bar{I}} = \mathbb{E}_t \left(\frac{\Pi_{t+1}}{\bar{\Pi}} \right)^{\phi_\pi} \mathbb{E}_t \left(\frac{Y_{t+1}}{\bar{Y}} \right)^{\phi_y}, \tag{36}$$

and compute the boundaries between the determinacy and indeterminacy region again. It is well known for a homogeneous-frequency New Keynesian model that an overreaction of monetary policy to expected inflation and output would be conducive to equilibrium indeterminacy. Figure 7 shows that it is also the case in a heterogeneous-frequency model. However, as in a homogeneous-frequency model, the extent of overreaction required to induce indeterminacy is quite extreme compared to empirical estimates of (ϕ_π, ϕ_y) in the literature. In an empirically more relevant space of (ϕ_π, ϕ_y), for example if $\phi_y < 1$, an increase in the degree of heterogeneity in price change frequencies expands the determinacy region in Specifications I and III but shrinks it in Specification II.

5 Empirical distribution of price stickiness

So far we used a simple two-sector NK model to illustrate how heterogeneity in the price change frequency alters the condition for equilibrium determinacy and to inspect the mechanism generating the changes. In this section, we investigate changes in the equilibrium determinacy condition in

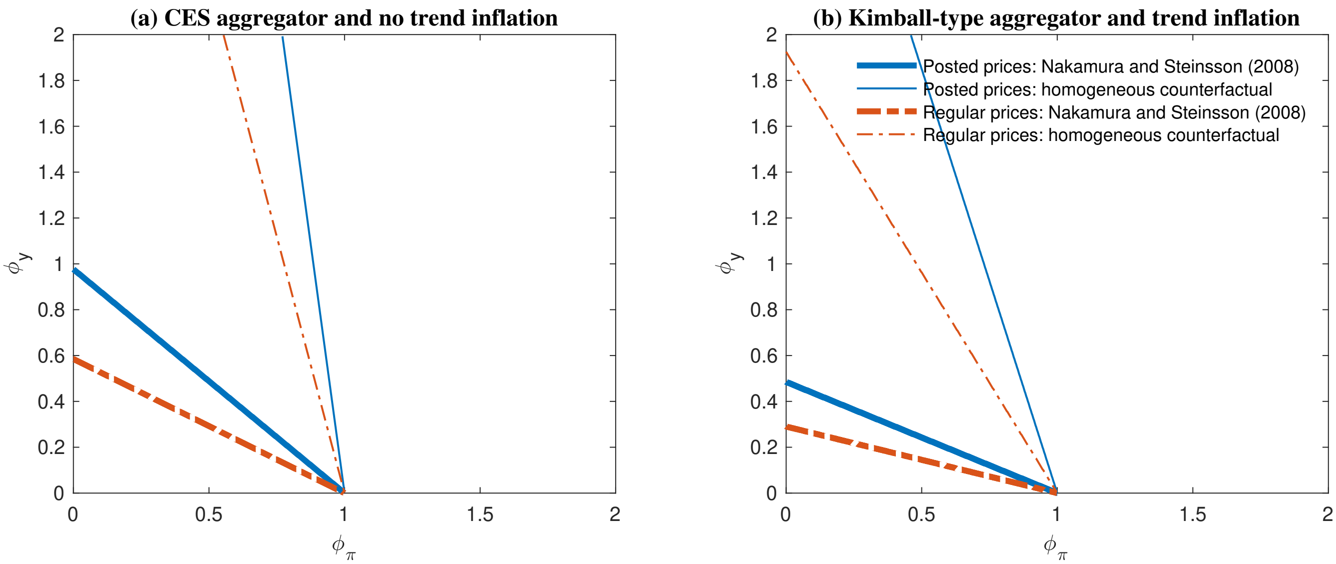


Figure 8: Boundary between the determinacy and indeterminacy region: 272 sectors, price change frequency estimates by Nakamura and Steinsson (2008), contemporaneous monetary policy rule

Notes: The values of ϕ_π and ϕ_y in the region to the left and to the right of each line lead to equilibrium indeterminacy and determinacy, respectively. The model has 272 sectors with price change frequencies estimated by Nakamura and Steinsson (2008). The weighted average of the price change frequency is used for the homogeneous counterfactual model in both panels. In panel (a), the price elasticity of demand is constant ($\psi = 0$) and trend inflation is assumed zero ($\bar{\Pi} = 1$) while in panel (b), the price elasticity of demand is not constant ($\psi = 1.1$) and annual net trend inflation is set at 2.54% ($\bar{\Pi} = (1.0254)^{1/4}$), which is the average CPI inflation rate on the sample period 1998-2005 of Nakamura and Steinsson (2008). The monetary policy rule is a log-linearized version of (21).

a realistic model with more disaggregated sectors. The model is parameterized with the empirical distribution of price change frequencies estimated by Nakamura and Steinsson (2008). Their estimates are based on the data of posted prices including temporary sales and regular prices excluding temporary sales in 272 categories of goods and services collected by BLS for CPI. Therefore, the realistic model has 272 sectors. Parameters other than those in the monetary policy rule and the price change frequency takes identical values as in the baseline model described in Section 3.1.

As Figure 8 shows, it is found that heterogeneity in price stickiness expands the determinacy region. This is the case with the frequency estimates based on both posted and regular price changes. The result holds in Specification I with the CES aggregator and zero trend inflation and also in Specification III with the Kimball-type aggregator and trend inflation at the historical level.[24] The same mechanism as for the simple two-sector model also operates for the 272-sector model: Proposition 1 applies for Specification I; and for Specification III featuring the Kimball-type aggregator, we present in the online appendix two figures equivalent to Figures 4 and 5 to confirm.

A prominent explanation for the Great Inflation in the US, led by Taylor (1999) and Clarida,

[24] Over the sample period 1998-2005 for Nakamura and Steinsson (2008), the average rate of CPI inflation was 2.54% per annum. Since this is above the upper bound for trend inflation with the CES aggregator, we do not consider Specification II featuring the CES aggregator and positive trend inflation.

Galí, and Gertler (2000), is that US monetary policy was not aggressive enough to rising inflation during the episode so that the US economy was exposed to self-fulfilling expectations-driven fluctuations. The estimates of the monetary policy rule by Clarida, Galí, and Gertler (2000) indeed imply equilibrium indeterminacy in the standard New Keynesian model with homogeneous price change frequencies. Coibion and Gorodnichenko (2011) reinvestigates the empirical evidence on equilibrium determinacy in the US economy using a model where positive trend inflation leads to the breakdown of the Taylor principle. So their model is a homogeneous-frequency version of Specification II of our paper. They conclude that a decline in trend inflation as well as changes in the other components of the monetary policy rule made equilibrium determinacy a more likely outcome since early 1980s. While Clarida, Galí, and Gertler (2000) and Coibion and Gorodnichenko (2011) estimate a forward-looking monetary policy rule where the central bank reacts to expectations of inflation, Carvalho, Nechio and Tristao (2019) estimate a contemporaneous monetary policy rule where the central bank reacts to contemporaneous variables. They reach the same conclusion that the monetary policy reaction to inflation was not strong enough in the 1960s and 1970s.[25,26]

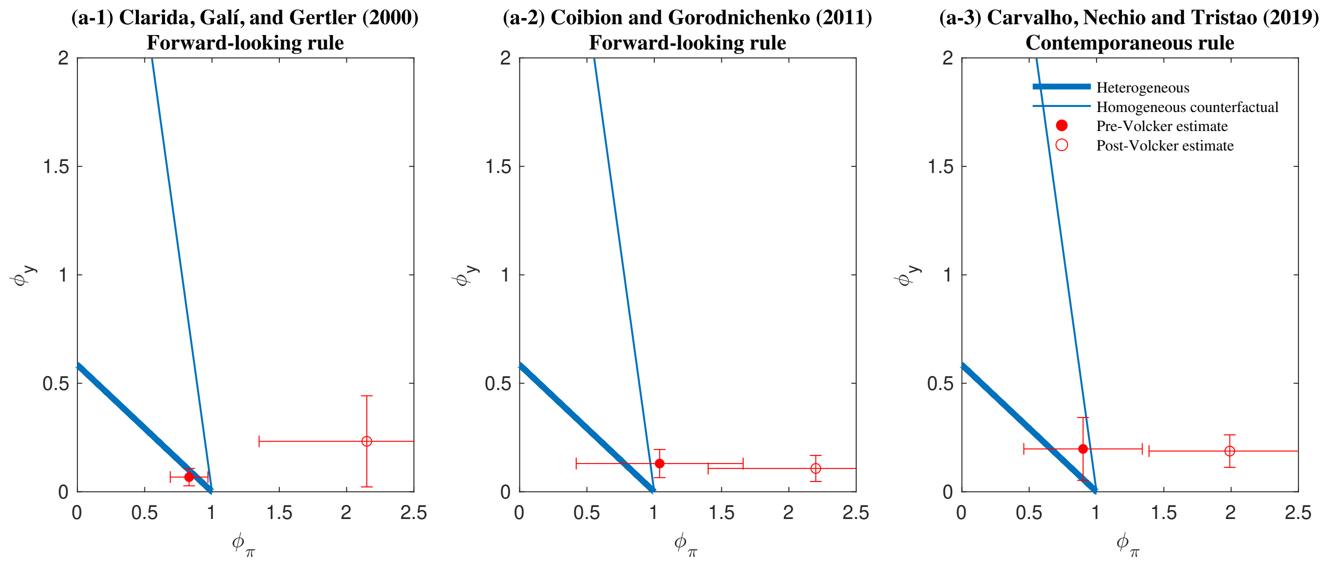
These previous studies assumed the homogeneity of the price change frequency. We now investigate whether their estimates of the monetary policy rule lead to equilibrium determinacy or indeterminacy if heterogeneity in the price change frequency is taken into account. We again do so in Specifications I and III using the empirical distribution of price change frequencies estimated by Nakamura and Steinsson (2008) based on the regular price changes. Specification II featuring the CES aggregator and assuming positive trend inflation is not considered since the historical level of trend inflation in the 1960s and 1970s violates the upper bound implied by the price change frequency estimates by Nakamura and Steinsson (2008).[27] There are 272 sectors in the economy. Other than those in the monetary policy rule and the price change frequency, the model is parameterized in the same way as the baseline model.

Specifically, the baseline specification of the monetary policy rule by Clarida, Galí, and Gertler

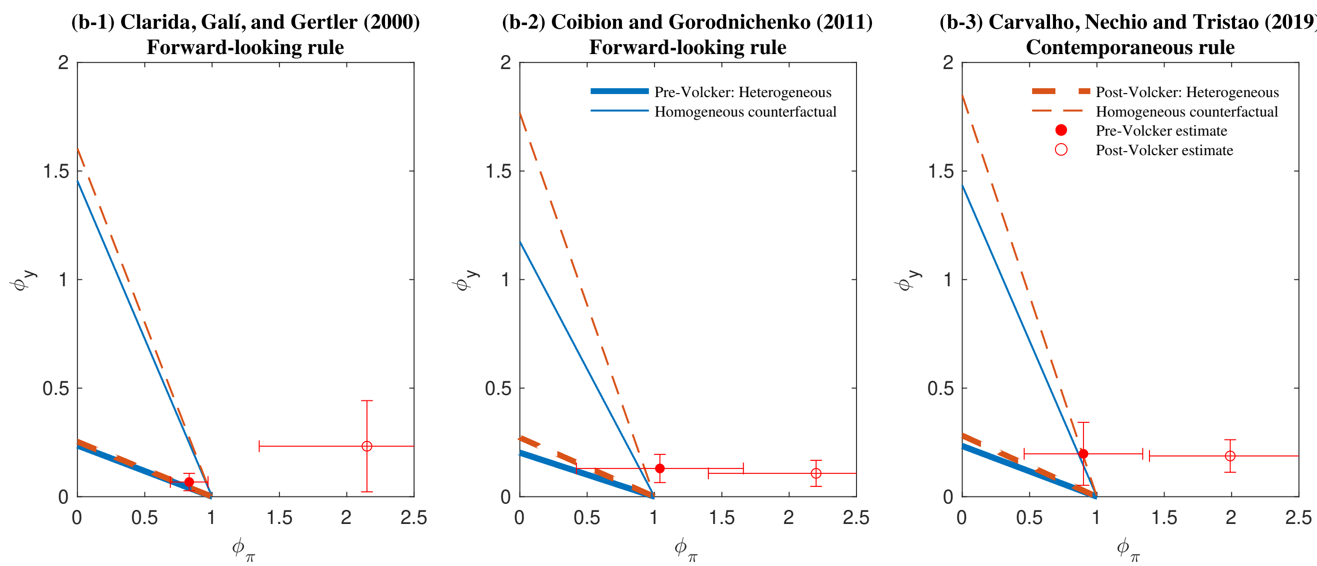
[25] While Clarida, Galí, and Gertler (2000) estimate the monetary policy rule using instrumental variables to avoid a potential endogeneity bias, Coibion and Gorodnichenko (2011) estimate the monetary policy rule by ordinary least squares (OLS). Coibion and Gorodnichenko (2011) argue that OLS estimation is likely to be adequate for several reasons and provide some supporting evidence. Carvalho, Nechio and Tristao (2019) support Coibion and Gorodnichenko (2011) in this regard by showing that monetary policy shocks explain only a small fraction of the variance of the variables in the monetary policy rule so the endogeneity bias that arises due to OLS estimation is small.

[26] These three studies estimate the monetary policy rule independently from a structural model. Another strand of the literature, pioneered by Lubik and Schorfheide (2004), estimates a New Keynesian model under equilibrium determinacy and indeterminacy, respectively, and favors the indeterminacy regime in terms of a data fit. Since their estimates are model-dependent, we do not compare their estimates here.

[27] The upper bound is not violated in the homogeneous-frequency economy if the representative frequency is set to the weighted average of the price change frequency.



(a) Specification I: the CES aggregator ($\psi = 0$) and zero trend inflation



(b) Specification III: the Kimball-type aggregator ($\psi = 1.1$) and positive trend inflation

Figure 9: Boundary between the determinacy and indeterminacy region: 272 sectors, frequency estimates by Nakamura and Steinsson (2008) based on regular prices

Notes: Panels show the boundaries between the determinacy and indeterminacy region computed using the monetary policy rule specified by each study and their point estimates of ϕ_π and ϕ_y. The values of ϕ_π and ϕ_y in the region to the left and to the right of each line lead to equilibrium indeterminacy and determinacy, respectively. The heterogeneous-frequency economy has 272 sectors with price change frequencies estimated by Nakamura and Steinsson (2008) based on regular price changes. The representative frequency for the homogeneous-frequency counterfactual is the weighted average of the price change frequency. Trend inflation is 0% ($\bar{\Pi} = 1$) with the CES aggregator. Trend inflation for the Kimball-type aggregataor in Specification III is the empirical estimate for each period by each study described in the main text. The error bars indicate the two-standard error confidence interval for ϕ_π and ϕ_y. *Note that they are not joint statistics but marginal ones.*

(2000) can be written in our notation as

$$i_t = (1 - 0.68)\, i_{t-1} + (1 - 0.68) \left(0.83 \times \mathbb{E}_t \pi_{t+1} + \frac{0.27}{4} \times y_t \right),$$

in the pre-Volcker period (1960Q1-1979Q2), and

$$i_t = (1 - 0.79)\, i_{t-1} + (1 - 0.79) \left(2.15 \times \mathbb{E}_t \pi_{t+1} + \frac{0.93}{4} \times y_t \right),$$

in the Volcker-Greenspan period (1973Q3-1996Q4), where the numbers are their coefficient estimates. Average inflation in each subsample is estimated to be 4.24% and 3.58%, respectively. The best-fit specification of the monetary policy rule by Coibion and Gorodnichenko (2011) can be written in our notation as

$$i_t = 1.34 i_{t-1} - 0.44 i_{t-2} + (1 - 1.34 + 0.44) \left[1.04 \times \left(\frac{\mathbb{E}_t \pi_{t+1} + \mathbb{E}_t \pi_{t+2}}{2} \right) + \frac{0.52}{4} \times y_t + 0.00 \times \Delta y_t \right],$$

on the period prior to 1979 (1969-1978), and

$$i_t = 1.05 i_{t-1} - 0.13 i_{t-2} + (1 - 1.05 + 0.13) \left[2.20 \times \left(\frac{\mathbb{E}_t \pi_{t+1} + \mathbb{E}_t \pi_{t+2}}{2} \right) + \frac{0.43}{4} \times y_t + 1.56 \times \Delta y_t \right],$$

on the period post 1982 (1983-2002). Again, the numbers are their coefficient estimates. Coibion and Gorodnichenko (2011) set trend inflation to be 6% and 3% in the pre-1979 period and in the post-1982 period, respectively, in analysis. Lastly, the specification of the monetary policy rule estimated by Carvalho, Nechio and Tristao (2019) and their coefficient estimates are

$$i_t = 0.80 \times i_{t-1} + (1 - 0.80) \left(0.90 \times \pi_t + \frac{0.79}{4} \times y_t \right),$$

with average inflation 4.34%, in the pre-Volcker period (1960Q1-1979Q2), and

$$i_t = 0.55 \times i_{t-1} + (1 - 0.55) \left(1.99 \times \pi_t + \frac{0.75}{4} \times y_t \right),$$

with average inflation 2.75%, in the Volcker-Greespan period (1979Q3-2005Q4).[28]

In order to check whether these estimated monetary policy rules lead to equilibrium determinacy or not and also to see how far they are from the boundary between the determinacy and indeterminacy region, we take each of the estimated monetary policy rules in the baseline model of

[28] Carvalho, Nechio and Tristao (2019) use two lags of the interest rate but report only the sum of the coefficients on the two lags. The estimates above are these sums. In our exercise with Specifications I and III, aggregating the two lags into one lag or using the two lags separately does not matter in terms of the condition for equilibrium determinacy.

272 sectors, fix the estimates of the coefficients other than ϕ_π on inflation and ϕ_y on output, and numerically find the boundary by changing the values of ϕ_π and ϕ_y. Figure 9 presents the result.[29] The sample period does not exactly match between the three studies so, for simplicity, we call the two subperiods the pre-Volcker period and the post-Volcker period, respectively, in discussions below.

The point estimate for the pre-Volcker period by Clarida, Galí, and Gertler (2000) implies equilibrium indeterminacy in Specification I as shown in panel (a-1) while it implies equilibrium determinacy in Specification III as shown in panel (b-1). If the standard errors are taken into account, we cannot reject a possibility of equilibrium determinacy in both cases for the pre-Volcker period. Although it is likely that we reject equilibrium determinacy in the homogeneous-frequency counterfactual in both cases, the empirical evidence that monetary policy violated the generalized Taylor principle is assessed to be considerably weak in the heterogeneous-frequency economy. In panels (a,b-2) and (a,b-3), the point estimate for the pre-Volcker period by Coibion and Gorodnichenko (2011) and Carvalho, Nechio and Tristao (2019) imply equilibrium determinacy in both Specifications I and III. However, the standard errors are large so we cannot reject a possibility of equilibrium indeterminacy in Specification I for the pre-Volcker period in both studies. We are likely to reject such a possibility in Specification III for the same period based on marginal standard errors.

For the post-Volcker period, the point estimates of ϕ_π and ϕ_y for all the three studies imply equilibrium determinacy. Even with the sampling uncertainty taken into account, we can reject a possibility of equilibrium determinacy. Mavroeidis (2010), repeating the estimation by Clarida, Galí and Gertler (2000) using a method robust to identification failure, concludes that the view that policy remained inactive after 1979 cannot be rejected because of large identification-robust confidence sets. The boundaries in Figure 9 imply that even in the presence of weak identifiability, we can conclude that the Federal Reserve responded aggressively enough to inflation and satisfied the generalized Taylor principle in the post-Volcker period.

In summary, it can be concluded that once we consider a realistic price change frequency distribution, the evidence in favor of the possibility that the US economy was subject to equilibrium indeterminacy and suffered from self-fulfilling expectations-driven fluctuations is considerably weak. In other words, we do not have strong empirical evidence that the change in the monetary policy stance around the tenure of Paul Volcker as the chairperson of the Federal Reserve got the US economy out of equilibrium indeterminacy and led it to have a unique stable equilibrium.

[29] In panels of Specification I, the boundary is identical for the pre- and post-Volcker period since trend inflation is set to zero. Also, it turns out that the boundaries are identical across the three monetary policy rules we consider in Specification I. In panels (a,b-1) and (a,b-2) where the monetary policy rule is forward-looking only in terms of inflation, there is no additional boundary in the outer space of (ϕ_π, ϕ_y). See Section 6 for further analysis of the equilibrium determinacy condition with a forward-looking monetary policy rule in terms of both inflation and output.

Lastly, we point out that in panel (b-2) of Figure 9, the boundaries are not very different in both subsamples despite a big decline in trend inflation as opposed to the finding by Coibion and Gorodnichenko (2011) in a homogeneous-frequency model with the CES aggregator. This is because the Kimball-type aggregator weakens the effect of trend inflation on the equilibrium determinacy.

A caveat is that the sample period for Nakamura and Steinsson (2008) is 1998-2005. If the cross-sectional distribution of price stickiness was very different during the Great Inflation from the one estimated by Nakamura and Steinsson (2008), our conclusion about equilibrium indeterminacy could be wrong. There are two forces at work that might influence the boundary in opposite directions if we adopt the price change frequency estimates based on the data from the Great Inflation episode. First, inflation was high in the 1970s. We previously found that higher trend inflation pushes the boundary between the determinacy and indeterminacy regions further down in Specification III. Therefore, the boundary in the pre-Volcker period could have been lower than what we found in Figure 9 in Specification III. On the contrary, Nakamura, Steinsson, Sun and Villar (2018) estimate that the frequency has decreased sharply since the early 1980s on average. It means that back in the 1960s and 1970s, the boundary could have been higher due to more frequent price changes. With these two forces counteracting each other somewhat, we conjecture that the determinacy and indeterminacy regions were not dramatically different from those presented in Figure 9.

6 Conclusion

The present paper shows that monetary policy can achieve equilibrium determinacy with much weaker responses to inflation when price change frequencies are heterogeneous than not. The result holds both in a standard New Keynesian model with the CES aggregator where trend inflation is zero and in an extended model with a Kimball-type aggregator where trend inflation can take on a value at the historical level. The key mechanism is that the dispersion of the sectoral relative prices due to heterogeneity of price stickiness makes aggregate inflation less responsive to changes in the marginal cost. In other words, the long-run trade-off between output and inflation is larger in the heterogeneous-frequency economy than in the homogeneous-frequency economy.

Our result sheds a new light on the debate on the source of the Great Inflation and the subsequent transition to the Great Moderation in the postwar US economy. In a realistic model parameterized with the empirical price change frequency estimates, we could not reject a possibility that the equilibrium of the US economy was uniquely pinned down in the pre-Volcker period as opposed to the finding of the literature based on a homogeneous-frequency assumption.

Appendix

Here, we show that $dy/d\pi$ in the expression of the generalized Taylor principle (23) corresponds to a change in output due to a marginal increase in trend inflation in the steady state of the baseline model. Here, only a discrete distribution of the price change frequency is considered.

Consider the equilibrium equations (1), (9), (12) and (13)-(22). Let us drop the expectations and the time index of the variables to write them as the steady state equations. Denote the number of the endogenous variables by n. Let us collect all the endogenous variables other than the gross nominal interest rate $\bar{I}$ in $\bar{X} = \left[\bar{X}_1; \bar{\Pi}\right]$ where $\bar{X}_1$ includes $n-2$ endogenous variables other than gross inflation $\bar{\Pi}$ and $\bar{I}$. Collect $n-2$ equilibrium equations other than the consumption Euler equation (1) and the monetary policy rule (21) as

$$F\left(\bar{X}_1; \bar{\Pi}\right) = 0.$$

We do not include the monetary policy rule since it is written in terms of deviations of the variables from the steady state and is not used when solving for the steady state. The consumption Euler equation is dropped since it only determines the growth rate of output and is not used later on when computing $dy/d\pi$. It follows that, given $\bar{\Pi}$, $F : \mathbb{R}^{n-2} \supseteq Z \to \mathbb{R}^{n-2}$ where Z is the support of $\bar{X}_1$. That is, $\bar{\Pi}$ is considered as a parameter and we solve for $\bar{X}_1$ as a function of $\bar{\Pi}$. Let us denote the solution of $\bar{X}_1$ given $\bar{\Pi}$ as $\bar{X}_1\left(\bar{\Pi}\right)$. Note that F is differentiable with respect to $\bar{X}_1$ and $\bar{\Pi}$ and $D_1 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right)$ is invertible. By the implicit function theorem,

$$D\bar{X}_1\left(\bar{\Pi}\right) = -\left[D_1 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right)\right]^{-1} D_2 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right).$$

Especially, we can get $d\bar{Y}/d\bar{\Pi}$ from above.

To compute $dy/d\pi$ as a long-run change in output associated with a long-run change in inflation, we ignore the time index of the variables in the system of the log-linearized equilibrium equations. We also do not use the consumption Euler equation and the monetary policy rule. Therefore, we can log-linearize F with respect to X_1 and Π around their steady state $\bar{X}_1$ and $\bar{\Pi}$. Let us denote a log deviation of a variable from the steady state by its lower-case letter. It follows that

$$0 = F\left(X_1; \Pi\right) \approx D_1 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right) \operatorname{diag}\left(\bar{X}_1\right) x_1 + D_2 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right) \bar{\Pi}\pi,$$

where $\operatorname{diag}\left(\bar{X}_1\left(\bar{\Pi}\right)\right)$ is a diagonal matrix with $\bar{X}_1\left(\bar{\Pi}\right)$ on diagonal, and thus

$$\begin{aligned} dx_1 &= -\left[D_1 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right) \operatorname{diag}\left(\bar{X}_1\left(\bar{\Pi}\right)\right)\right]^{-1} D_2 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right) \bar{\Pi} d\pi \\ &= \left[\operatorname{diag}\left(\bar{X}_1\left(\bar{\Pi}\right)\right)\right]^{-1} \left\{-\left[D_1 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right)\right]^{-1} D_2 F\left(\bar{X}_1\left(\bar{\Pi}\right); \bar{\Pi}\right)\right\} \bar{\Pi} d\pi \\ &= \left[\operatorname{diag}\left(\bar{X}_1\left(\bar{\Pi}\right)\right)\right]^{-1} \times D\bar{X}_1\left(\bar{\Pi}\right) \times \bar{\Pi} d\pi, \end{aligned}$$

where diag $\left(\bar{X}_1\left(\bar{\Pi}\right)\right)$ and $\bar{\Pi}$ enter because of log-linearization (linear approximation in terms of x not X). Especially, for output, we have

$$\frac{dy}{d\pi} = \frac{d\bar{Y}}{d\bar{\Pi}} \times \frac{\bar{\Pi}}{\bar{Y}}.$$

In practice, to compute $dy/d\pi$, one can solve for $\bar{Y}$ as a function of $\bar{\Pi}$, compute $d\bar{Y}/d\bar{\Pi}$, and use the above relationship.

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