Nash Implementation under Allocative Constraints[(1)]

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In exchange economies, we investigate social choice rules that can be implemented in Nash equilibrium under some allocative constraints. Allocative constraints can represent standard normative requirements such as efficiency and fairness and are formulated by a fixed set of allocations from which outcome functions (of game forms) can take values. We show that an extended notion of Maskin's monotonicity[Maskin(1977, 1999)] is a necessary and sufficient condition for Nash implementation under allocative constraints.

Keywords: Nash implementation, Social choice rules, Exchange economies, Monotonicity

1. Introduction

The pioneering study by Maskin(1977, 1999) and subsequent studies by Williams(1986), Repullo(1987), Saijo(1988), Moore and Repullo(1990), Yamato(1992) etc. have provided necessary and sufficient conditions for implementing social choice rules as Nash equilibrium outcomes. Their conditions reduce the seemingly daunting task of checking Nash implementability of a social choice rule to a simple task of checking a "monotonicity" condition. Maskin's(1977, 1999) monotonicity says that when an alternative is chosen at a profile of preferences, it should be chosen at any other profile of preferences where the position of the alternative in individual preferences improves unanimously for all agents (meaning the set of alternatives inferior to the chosen alternative expands). To prove sufficiency of this condition (together with an additional, yet very mild condition, known as "no-veto-power"), Maskin constructs a generic game form that can be used to implement any social choice rule satisfying his monotonicity. Unfortunately, in numerous economic environments, the outcome function of Maskin's game form (as well as the other generic

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game forms used by subsequent works) may take some undesirable values; punishments may be too harsh for some agents or the allocations may be extremely biased. The main objective of this note is to investigate Nash implementation under allocative constraints that exclude some undesirable consequences of a game from happening.

Considering (pure) exchange economies with social endowment, allocative constraints are described as a range-restriction of outcome functions over a fixed set of allocations. For example, given any normative criterion α for allocations, there may be allocations that never satisfies criterion α for any economy, a profile of preferences. Excluding such allocations, define a range-restriction as the set of allocations that can "potentially" satisfy criterion α and consider those outcome functions (of game forms) that take values only from this set. Examples are *Pareto efficiency* and standard fairness requirements such as *no-envy* (no one prefers someone else's bundle to his), *no-domination* (no vector domination between individual bundles), *egalitarian equivalence* (existence of an egalitarian allocation that is indifferent for all agents), the *equal division lower bound* property (everyone should be at least as well off as in the equal division), etc.(2) Our main results show that a necessary and sufficient condition for Nash implementation with the range-restriction of Y is the combination of an extended notion of Maskin's monotonicity and the obvious condition that the social choice rule under consideration should select allocations from Y. Our monotonicity, *monotonicity on Y* imposes the same condition as Maskin's except focusing on allocations inside Y; when the position of a chosen allocation in individual preferences "within Y" improves unanimously, it should still be chosen. Using the main results, we provide an example where the "standard Nash implementation" of Walrasian rule necessarily depends on using some undesirable allocations and its Nash implementation with the range restriction based on fairness can be impossible.

When the range-restriction coincides with the set of feasible allocations, Nash implementation with the range-restriction is simply the standard Nash implementation (using feasible outcome functions) by Maskin(1977, 1999) and the subsequent works. When there is no restriction at all, the range-restriction Y coincides with the entire allocation space and Nash implementation in this case coincides with Nash implementation with "possibly infeasible"

(2) See Thomson(2008) for an extensive treatment on fair allocation rules in economic environments.

outcome functions, as studied by numerous works on market games: e.g. Hurwicz(1979), Schmeidler(1980), Dubey(1982), Simon(1984), Silvestre(1985), Benassy(1986), etc. Our main results provide corollaries for these special cases.

2. Exchange Economies

We consider exchange economies with L goods and N agents (consumers) with $L \geq 2$ and $N \geq 3$. An *allocation* $z \equiv (z_i)_{i \in N} \in \mathbb{R}_+^{L \times N}$ is a list of individual bundles $z_i \in \mathbb{R}_+^L$.(3) Let $\Omega \in \mathbb{R}_+^L$ be the social endowment and $N \equiv \{1, \ldots, N\}$ be the set of agents. Assume $\Omega >> 0$.(4) Let $Z = \{z \in \mathbb{R}_+^{L \times N}: \Sigma_i z_i \leq \Omega\}$ be the set of feasible allocations.(5) Each agent $i \in N$ has a preference relation, a complete and transitive binary relation over allocations. Generic notation for agent i's preference relation is R_i and its corresponding strict and indifference relations are P_i and I_i, respectively. When agent i prefers z to z', we write $z\ R_i\ z'$; likewise, we use notation $z\ P_i\ z'$ and $z\ I_i\ z'$. Assume that each agent cares only about her own bundle, that is, for all z, z' and all $i \in N$, if $z_i = z_i'$, then z and z' are indifferent for agent i, $z\ I_i\ z'$. Assume also the other assumptions of *classical preferences*, namely, "continuity," "monotonicity," and "convexity." Let $\mathscr{R}$ be the set of all such preference relations. A *domain* $\mathscr{D}$ is a non-empty subset of $\mathscr{R}^N$. A *social choice rule* on $\mathscr{D}$ is a correspondence $\varphi: \mathscr{D} \to Z$ associating with each preferences profile in the domain a non-empty set of feasible alternatives. A well-known rule is *Walrasian rule* (from the equal-division) denoted by φ^W. This rule associates with each $R \in \mathscr{D}$ the set of *Walrasian equilibrium allocations* z such that for some $p \in \mathbb{R}_+^L$, $p \cdot z_i = p \cdot (\Omega/n)$ and $z\ R_i\ z'$ for all $z' \in \mathbb{R}_+^{L \times N}$ satisfying $p \cdot z_i' = p \cdot (\Omega/n)$ and for all $i \in N$. Since preferences are classical, Walrasian rule is non-empty valued. An allocation z is (Pareto) *efficient* at profile $R \in \mathscr{D}$ if z is feasible and there is no other feasible allocation that makes at least one agent better off without making anyone else worse off than at z. *Pareto rule* associates with each profile the set of efficient allocations. Next are standard fairness criteria. An allocation z satisfies *no-envy* at $R \in \mathscr{D}$ if for all $i, j \in N$, $z_i\ R_i\ z_j$. Allocation z satisfies *no-domination* if for all $i, j \in N$, $z_i \not\geq z_j$. Allocation z satisfies *egalitarian equivalence* at $R \in \mathscr{D}$ if there is $z_0 \in \mathbb{R}_+^L$ such that

(3) $\mathbb{R}_+$ is the set of non-negative real numbers.

(4) Given $x, x' \in \mathbb{R}^n$, $x >> x'$ means that $x_k > x_k'$ for all components $k = 1, \ldots, n$.

(5) Given $x, x' \in \mathbb{R}^n$, $x \geq x'$ means that $x_k \geq x_k'$ for all $k = 1, \ldots, n$.

for all $i \in N$, $z\ I_i\ (z_0, \ldots, z_0)$. Allocation z satisfies the *equal division lower bound* property at $R \in \mathscr{D}$ if for all $i \in N$, $z\ R_i\ \Omega/N$. For any of these fairness conditions, we can define the corresponding fair allocation rule selecting the set of all fair and feasible allocations.

A *game form* G is composed of a strategy space $S \equiv \times_{i \in N} S_i$ and an outcome function g: $S \to \mathbb{R}_+^{L \times N}$, where for all $i \in N$, S_i is the set of i's strategies. For all $R \in \mathscr{D}$, let $NE(G, R)$ be the set of Nash equilibria of the (complete information) game defined by game form G and the profile of preferences R. A rule φ on $\mathscr{D}$ is *Nash implementable* if there is a game form $G \equiv (S, g)$ such that for all $R \in \mathscr{D}$, the set of Nash equilibrium outcomes of the game given by G and R coincides with the set of allocations rule φ recommends at R, namely, $g(NE(G, R)) = \varphi(R)$. It is *feasibly Nash implementable* if the range of the outcome function is in the feasibility set Z. A *range-restriction* is a non-empty set of allocations $Y \subseteq \mathbb{R}_+^{L \times N}$. A rule φ is *Nash implementable under the range-restriction of* Y if φ is Nash implementable by a game form of which the outcome function has the range in Y.

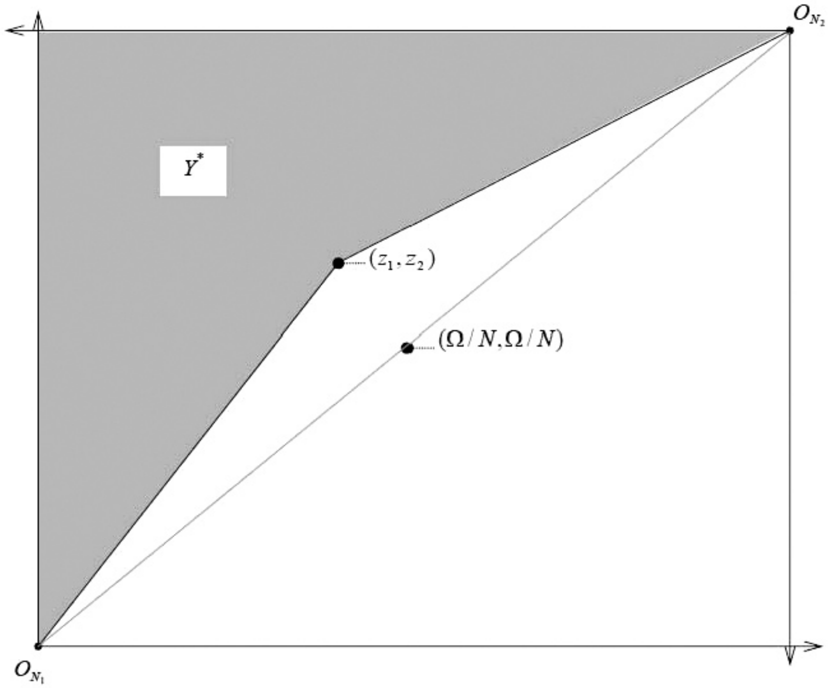
The rage-restriction associated with *efficiency* is the set of all "potentially" *efficient* allocations on domain $\mathscr{D}$, that is, $Y_P \equiv \{z \in Z$: z is *efficient* at some $R \in \mathscr{D}\}$. Likewise, we define range-restriction associated with each fairness condition defined above as follows. For *no-envy*, let $Y_{NV} \equiv \{z \in Z$: z is envy-free at some $R \in \mathscr{D}\}$. For *no-domination*, let $Y_{ND} \equiv \{z \in Z$: for all $i, j \in N$, $z_i \ngeq z_j\}$. For egalitarian equivalence, let $Y_{EE} \equiv \{z \in Z$: z satisfies *egalitarian equivalence* at some $R \in \mathscr{D}\}$. For the *equal-division lower bound property*, let $Y_{ED} \equiv \{z \in Z$: z satisfies the *equal division lower bound property* at some $R \in \mathscr{D}\}$.

All these range-restrictions are not too severe to generate no "conflict of interests" among agents. Formally, range-restriction Y exhibits *conflict of interest* among two agents if there is no allocation in Y that is preferred to all other allocations in Y for both agents.

Throughout the paper we assume that for all $R \in \mathscr{D}$ and all $i \in N$, range-restriction Y exhibits conflict of interest among at least two agents other than i.

For reasonably rich domains $\mathscr{D}$, the above examples of range-restrictions satisfy this *conflict-of-interest assumption*. In particular, when $\mathscr{D}$ includes all classical preferences, one can easily show that $Y_P = Z$ and $Y_{NV} = Y_{ND} = \{z \in Z$: there is no i, j with $i \neq j$ such that $z_i \gg z_j\}$. Then it can be easily checked that for any Y of these range-restrictions, for all $i, j \in N$, there is no $z \in Y$ that is a best allocation over Y both for R_i and for R_j.

Next are examples of two domains and another range-restriction that satisfies the conflict-



<Figure 1>

of-interest assumption.

Example 1. Let $Y_{PNV} \equiv Y_P \cap Y_{NV}$. Let $\mathscr{D}_1$ be the set of profiles $R \in \mathscr{R}^N$ such that for all $i \in N$, R_i is "strongly" monotonic and for all efficient allocations z at R, $z_i = 0$ or $z_i >> 0$. For example, when R consists of Cobb-Douglas preferences, $R \in \mathscr{D}_1$. Then $Y_{PNV} = \{z \in Z$: for all i, j with $i \neq j$, $z_i \ngeq z_j$, and $z_i >> 0\}$. For all $R \in \mathscr{D}_1$ and all $i, j \in N$ with $i \neq j$, one can easily check that there is no allocation in Y_{PNV} that is preferred to any other allocation in Y_{PNV} by both agents, i and j. Hence, Y_{PNV} exhibits conflict of interest among any two agents, which implies that the conflict-of-interest assumption holds for Y_{PNV}.

Example 2. Assume $L = 2$. Let N be partitioned into two non-empty subsets N_1 and N_2 with at least two agents. Assume that all agents in N_1 have the same "homothetic" and "strictly" convex preferences and all agents in N_2 also have the same homothetic and strictly convex preferences. Then at all efficient and envy-free allocations z, all agents of the same type consume the same consumption bundle; all such allocations z can be represented by its *type allocation* (z_1, z_2), where z_i is the bundle for type i agents in N_i for all $i = 1, 2$. Let Y^* be a set of type allocations that lies above the diagonal line of the Edgeworth box, as illustrated in <Figure 1>. Let $\mathscr{D}_2$ be the set of profiles of homothetic and strictly convex preferences $R \in \mathscr{D}_1$ such that (i) for all k =1, 2 and all $i, j \in N_k$, $R_i = R_j$ and (ii) for all efficient type allocations (z_1, z_2) at R, $(z_1, z_2) \in Y^*$. Thus good 2 is relatively more important for type 1 agents than for type 2 agents so that all efficient type allocations are always in Y^* above the diagonal line of the Edgeworth box. It can be easily shown that for any two agents from each type set, $i \in N_1$ and $j \in N_2$, Y_{PNV} exhibits conflict of interest among i and j, which implies that the conflict-of-interest assumption holds for Y_{PNV} (recall that each type set has at least two agents).

3. Results

Our main results provide a necessary and sufficient condition for Nash implementation under a range-restriction. The condition is a modification of Maskin's monotonicity[Maskin (1977, 1999)] through using range-restriction $Y \subseteq \mathbb{R}_+^{L\times N}$. For all preferences $R_i \in \mathscr{R}$ and all allocations z, let $LC(R_i, z; Y) \equiv \{z' \in Y\colon z\ R_i\ z'\}$ be the intersection of the "lower-contour set" at z and Y.

Monotonicity on *Y*. For all $R, R' \in \mathscr{D}$ and all $z \in \varphi(R)$, if for all $i \in N$, $LC(R_i, a; Y) \subseteq LC(R_i', a; Y)$, then $z \in \varphi(R')$.

When $Y = Z$, *monotonicity on Y* coincides with Maskin's monotonicity[Maskin(1977, 1999)]. When $Y = \mathbb{R}_+^{L\times N}$, *monotonicity on Y* coincides with Gevers' monotonicity[Gevers (1986)].

We first show that *monotonicity on Y* is a necessary condition for Nash implementation under the range-restriction of Y.

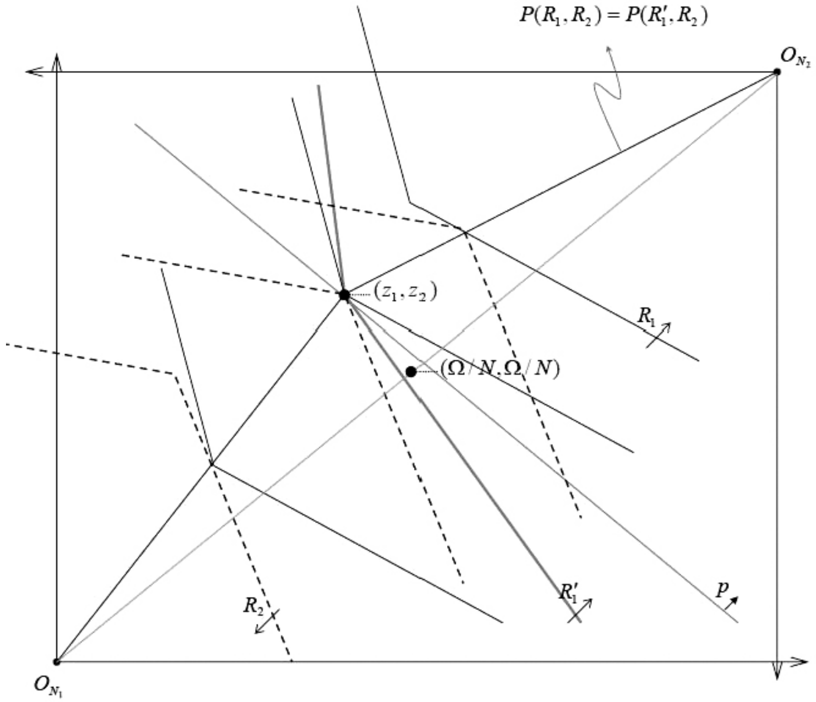
Theorem 1. *If a rule is Nash implementable under the range-restriction of* $Y \subseteq \mathbb{R}_+^{L\times N}$, *then it is a subcorrespondence of Y and satisfies monotonicity on Y.*

Proof. Let $Y \subseteq \mathbb{R}_+^{L\times N}$ and φ be a rule on $\mathscr{D}$. Assume that φ is Nash implementable under the range-restriction of Y. Then there is a game form $G \equiv (S, g)$ such that $g(S) \subseteq Y$ and for each $R \in \mathscr{D}$, $\varphi(R) = g(NE(G, R))$. Since $g(S) \subseteq Y$, φ is a subcorrespondence of Y. In order to prove monotonicity on Y, let $R, R' \in \mathscr{D}$ and $z \in \varphi(R)$. Suppose that for all $i \in N$, $LC(R_i, z; Y) \subseteq LC(R'_i, z; Y)$. Let $s \in NE(G, R)$ be a Nash equilibrium strategy profile such that $g(s) = z$. Then for all $i \in N$, $A_i(s_{-i}) \subseteq LC(R_i, z)$, where $A_i(s_{-i}) \equiv \{g(s'_i, s_{-i})$: $s'_i \in S_i\}$. Since $g(S) \subseteq Y$, then for all $i \in N$, $A_i(s_{-i}) \subseteq LC(R_i, z; Y)$. Hence for all $i \in N$, $A_i(s_{-i}) \subseteq LC(R'_i, z; Y)$. This implies s is a Nash equilibrium strategy profile at R' too, that is, $s \in NE(G, R')$. Therefore, since $\varphi(R') = g(NE(G, R'))$, $g(s) = z \in \varphi(R')$. ■

Remark 1. It is evident from the proof that this result does not rely on either $N \geq 3$ or the conflict-of-interest assumption.

Example 3. Consider the domain $\mathscr{D}_1$ and the range restriction Y_{PNV} in Example 1. On $\mathscr{D}_1$, Walrasian allocations are composed of interior consumption bundles and Walrasian rule satisfies Maskin's monotonicity. Here we show that Walrasian rule also satisfies monotonicity on Y_{PNV}. In order to show this, let $R \in \mathscr{D}_1$ and z be a Walrasian equilibrium allocation at R with equilibrium price $p \in \mathbb{R}^L_{++}$. Let $R' \in \mathscr{D}_1$ be such that for all $i \in N$, $LC(R_i, z; Y_{PNV}) \subseteq LC(R'_i, z; Y_{PNV})$. Note that for all $i \in N$, $z\ R_i\ z'$; for all $z' \in \mathbb{R}^{L\times N}_+$ with $z'_i \in [0, \Omega]$ and $p \cdot z'_i = p \cdot (\Omega/n)$. Let $i \in N$. For all $z'_i \in [0, \Omega]$ with $p \cdot z'_i = p \cdot (\Omega/n)$, there is $z''_{-i} \in \mathbb{R}^{L\times(N-1)}_+$ such that $(z'_i, z''_{-i}) \in Y_{PNV}$ (for example, for all $j \neq i$, let $z''_j \equiv (\Omega - z'_i)/(n-1)$). Thus, $(z'_i, z''_{-i}) \in LC(R_i, z; Y_{PNV})$ and so $(z'_i, z''_{-i}) \in LC(R'_i, z; Y_{PNV})$, which implies $z\ R'_i\ z'$. Therefore, z is also a Walrasian equilibrium allocation at R' too.

Example 4. Consider the domain $\mathscr{D}_2$ and the range restriction Y_{PNV} in Example 2. On $\mathscr{D}_2$, Walrasian allocations from the equal division are composed of interior consumption bundles and Walrasian rule satisfies Maskin's monotonicity. However, Walrasian rule violates monotonicity on Y_{PNV}. To show this, let $R \in \mathscr{D}_2$ be a profile of preferences such that z is an Walrasian equilibrium allocation at R and the set of efficient type-allocations at R is given by the line segment connecting O_{N_1}, (z_1, z_2), and O_{N_2} in <Figure 2>. Let p be an equilibrium price vector supporting equilibrium allocation z at R. Now let $R' \in \mathscr{D}_2$ be such that $R'_{N_2} = R_{N_2}$ and the set of efficient type-allocations at R' is the same as at R, and $\Omega/n\ P'_1\ z_1$ as illustrated in <Figure 2>. Clearly, then z cannot be a Walrasian equilibrium at R' since $\Omega/n\ P'_1\ z_1$. However,



<Figure 2>

$LC(R_1, z; Y_{PNV}) \subseteq LC(R'_1, z; Y_{PNV})$. This shows that Walrasian rule violates monotonicity on Y_{PNV}.

This example shows that restricting outcomes on Y_{PNV}, it is impossible to Nash implement Walrasian rule on $\mathscr{D}_2$. Therefore, in order to Nash implement Walrasian rule on this domain, it is necessary for the outcome function to take some "unfair" outcomes, which are either "never efficient" or "never envy-free."

We next show that the converse of Theorem 1 also holds.

Theorem 2. *If a rule on $\mathscr{D}$ is a subcorrespondence of $Y \subseteq \mathbb{R}_+^{L\times N}$ and satisfies monotonicity on Y, then it is Nash implementable under the range-restriction of Y.*

Proof. Let $Y \subseteq \mathbb{R}_+^{L\times N}$. Let φ be a rule on $\mathscr{D}$ that is a subcorrespondence of Y and satisfies monotonicity on Y. To prove Nash implementability under the range-restriction of Y, we consider the following game form, a modification of the game form used by Maskin(1977, 1999).

Construction of a game form G: For all $i \in N$, let $S_i \equiv \mathscr{D} \times Y \times \mathbb{Z}$ be the set of agent i's strategies, where $\mathbb{Z}$ is the set of integers. Denote its generic element by $s_i \equiv (R^i, a^i, t^i)$. Let $S \equiv S_1 \times \ldots \times S_n$. Let g: $S \to \mathbb{R}_+^{L\times N}$ be the outcome function defined by the following three states.

State I: If for all $i \in N$, $s_i = (R, z, t) \in \mathscr{D} \times Y \times \mathbb{Z}$ and $z \in \varphi(R)$, then

$$g(s) \equiv z.$$

State II: If there exists $i \in N$ such that for all $j \neq i$, $s_j = (R, z, t) \in \mathscr{D} \times Y \times \mathbb{Z}$, $z \in \varphi(R)$, and $s_i = (R', z', t') \neq (R, z, t)$, then

$$g(s) \equiv \begin{cases} z', \text{ if } z' \in LC(R_i, z; Y), \\ z, \text{ if } z' \notin LC(R_i, z; Y). \end{cases}$$

State III: In all other cases,

$$g(s) \equiv z^h,$$

where $h \equiv \min\{i \in N: t^i \in \max\{t^1, \ldots, t^n\}\}$.

It is evident by definition that $g(S) \subseteq Y$ and so the game form meets the range-restriction of Y. Let $R \in \mathscr{D}$. In what follows, we show $\varphi(R) = g(NE(G, R))$.

Let $z \in \varphi(R)$. For all $i \in N$, let $s_i \equiv (R, z, t)$. Then state I applies and $g(s) = z$. If agent i chooses $s_i' = (R', z', t') \neq s_i$, then $g(s_i', s_{-i}) = z'$ when $z' \in LC(R_i, z, Y)$ or $g(s_i', s_{-i}) = z$ when $z' \notin LC(R_i, z, Y)$. In both cases, $g(s_i', s_{-i}) \in LC(R_i, z, Y)$ and so $g(s_i, s_{-i}) = z \; R_i \; g(s_i', s_{-i})$. Therefore, $z \in g(NE(G, R))$.

To prove the reverse inclusion, let $s \in NE(G, R)$ and $z \equiv g(s)$. We first show that State I applies at the Nash equilibrium strategy profile s. To show this, suppose to the contrary that either States II or III holds at s. Then there are at least two agents $i, j \in N$ who can attain any allocation in Y by deviating from s. This means that z is a best allocation over Y for both i and j, contradicting the conflict-of-interest assumption. Therefore, there exists $(\bar{R}, \bar{z}, \bar{t}) \in \mathscr{D} \times Y \times$

$\mathbb{Z}$, such that $\bar{z} \in \varphi(\bar{R})$ and for all $i \in N$, $s_i \equiv (\bar{R}, \bar{z}, \bar{t})$. Then $\bar{z} = z$ and $g(s) = z$. For each $i \in N$ and each $z' \in LC(\bar{R}_i, z, Y)$, i can attain z'. Then $z\, R_i\, z'$. Thus $LC(\bar{R}_i, z, Y) \subseteq LC(R_i, z, Y)$. Therefore, by *monotonicity on Y*, $z \in \varphi(R)$. ■

Remark 2. Unlike Theorem 1, this result relies crucially on both $N \geq 3$ and the conflict-of-interest assumption. The result does not hold for $N = 2$ as shown by Maskin(1977, 1999). Without the conflict-of-interest assumption, an additional no-veto-power condition[Maskin (1977, 1999)] is needed.

When $Y = Z$, Nash implementation under the range-restriction of Y simply means the standard Nash implementation (using feasible outcome functions) as considered by Maskin (1977, 1999). It follows from our main results:

Corollary 1. A rule is feasibly Nash implementable if and only if it satisfies monotonicity on Z.

When $Y = \mathbb{R}_+^{L\times N}$, Nash implementation under Y means Nash implementation without any range-restriction; so outcome functions may yield infeasible allocations [for instance, Hurwicz(1979), Schmeidler(1980)]. It follows from our main results:

Corollary 2. *A rule is Nash implementable (without any range restriction) if and only if it satisfies monotonicity on* $\mathbb{R}_+^{L\times N}$.

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