Demographic Decline and the Dynamics of Asset Bubbles: Implications for South Korea's Asset Markets

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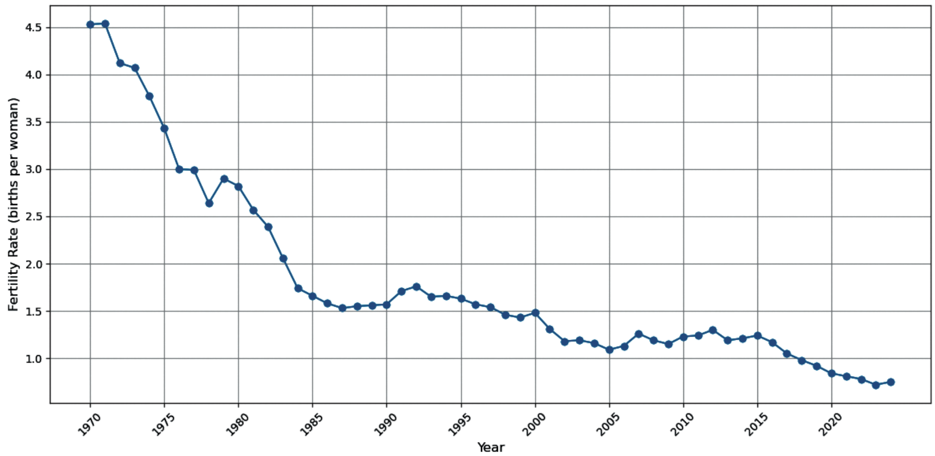
This paper examines the dynamics of asset bubbles in an overlapping generations economy experiencing population decline. We incorporate a stochastic Markov process to model uncertain demographic transitions, including the possibility of stabilization. Our analysis shows that declining population levels endogenously undermine bubble sustainability by raising per capita participation costs in asset markets. Even when the eventual collapse lies far in the future, forward-looking agents rationally anticipate it, leading to a gradual erosion of asset valuations. These findings uncover a novel mechanism through which demographic fundamentals—particularly persistent low fertility and population aging—can destabilize financial markets, offering new insights into asset pricing in an era of demographic decline.

Keywords: asset bubbles, population decline, fertility rates, demographic transition, bubble collapse

1. Introduction

South Korea has entered a demographic emergency, with fertility rates reaching unprecedented lows and raising serious concerns about long-term growth and sustainability. In 2024, the total fertility rate (TFR) rose slightly to 0.75 births per woman—still the lowest among OECD countries—up from 0.72 in 2023 but far below the replacement level of 2.1 (see Figure 1). This prolonged period of ultra-low fertility has triggered a demographic decline: the number of births fell to approximately 238,300 in 2024, while deaths have outnumbered births for five consecutive years. While the impending population shrinkage driven by persistently low fertility poses numerous challenges—including labor shortages and rising welfare costs—it may also have significant implications for asset prices.

This paper focuses on the impact of population decline on asset bubbles. This focus is motivated by the persistently high residential property prices in South Korea, where real estate constitutes a major component of household assets. Residential property prices have risen sharply in recent years, especially in the late 2010s (see Figure 2). These trends have sparked ongoing debates about the presence of speculative bubbles



The figure illustrates the steep decline in fertility over the past five decades, highlighting the country's demographic transition and current challenges.

Source: Statistics Korea (KOSTAT).

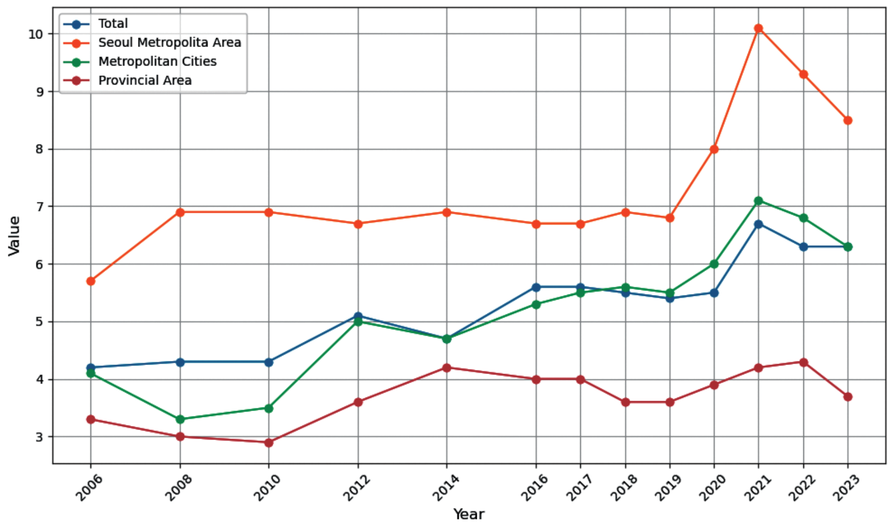
Figure 1. Total fertility rate in South Korea, 1970–2024

in the housing market. While property prices in Seoul have remained relatively robust, those in surrounding metropolitan and provincial areas have begun to decline. This divergence has renewed concerns over whether a shrinking population may exert downward pressure on elevated housing prices.

Our study approaches this issue through the lens of asset bubbles, examining how demographic contraction may affect asset price dynamics. Although the motivation for this study is rooted in the case of South Korea's population trends and real estate market, our theoretical findings are broadly applicable to general classes of assets across different markets.

In this paper, we develop a discrete-time overlapping generations (OLG) model with an infinite horizon, in which a pure bubble asset is traded across generations. The asset has no intrinsic value but may be priced positively due to resale motives, following the seminal contributions of Samuelson (1958) and Tirole (1985).

Our key innovation lies in introducing a stochastic demographic process that



The chart compares the national average with disaggregated data for the Seoul metropolitan area, other metropolitan cities, and provincial regions.

Source: Statistics Korea (www.index.go.kr).

Figure 2. Regional trends in the Price to Income Ratio (PIR) for residential properties across South Korea, 2006–2023

governs the evolution of the population, which in turn influences the sustainability of the bubble. This leads to stochastic bubbles with random prices, in the spirit of Blanchard (1979), Blanchard and Watson (1982), and Weil (1987). However, in contrast to sunspot-driven or purely exogenous collapses, the bubble in our model bursts endogenously when population size falls below a critical threshold. We focus on a class of equilibria in which the bubble can survive only if the population remains sufficiently large to diffuse the fixed cost of participating in the bubble market. This mechanism highlights a novel demographic channel through which population decline can destabilize asset price dynamics.

The demographic process is modeled as a Markov chain with two regimes: a shrinking population with a negative constant growth rate and a stabilized population with zero growth. This specification is motivated by the stylized facts presented in Figure 1. Transitions between these regimes are probabilistic, reflecting the empirical reality of uncertain demographic transitions in aging societies.

We assume that population decline is persistent, with a small probability each period that this trend may reverse, resulting in demographic stabilization. This structure captures the uncertainty faced by agents: future asset resale values are not only uncertain due to potential bubble collapse, but this collapse is endogenously driven by demographic fundamentals. Specifically, the bubble bursts when the population falls below a critical threshold, thereby linking the sustainability of the bubble directly to the trajectory of demographic change.

Agents live for two periods and derive utility from consumption when young and when old. They may purchase the bubble asset when young, facing both its price and a fixed participation cost that is shared among all buyers. The participation cost can be motivated by various setup costs, including those related to transactions, information processing, and the maintenance of trading infrastructure. As the population declines, the per capita burden of this cost increases, making participation in the bubble market less attractive. Ultimately, the bubble becomes unsustainable when the population falls below a critical threshold. We derive this threshold analytically and show that it is determined by fundamental parameters such as endowments, preferences, and participation costs.

The model yields a rich set of implications for equilibrium asset prices and intertemporal allocations. First, we characterize the conditions under which a stationary bubble can exist in a stable population. We show that even when a bubble is viable in autarky—i.e., when the autarky interest rate is below the population growth rate—the presence of participation costs makes the existence condition more stringent. Second, we analyze the dynamic behavior of bubble prices when the population is still shrinking. We derive recursive equilibrium conditions for bubble prices in economies that are a given number of periods away from reaching the sustainability threshold, showing that the price and size of the bubble decline as the threshold nears.

A central finding of our analysis is that the size of the bubble is positively related to the level of the population, both in stable and shrinking demographic regimes. In particular, we prove that the equilibrium bubble size increases monotonically with the population size, holding all else constant. This result reflects the intuition that a larger investor base lowers the per capita participation cost, thereby supporting higher valuations for the bubble asset. Conversely, demographic decline systematically undermines the scope for sustaining bubbles, and the anticipation of a future collapse feeds back into current valuations through forward-looking behavior.

Our framework provides new insights into how demographic risks can influence financial fragility. While much of the literature has emphasized the role of credit conditions, investor behavior, and technological shocks in driving bubbles, we show that purely demographic forces can play a decisive role in the emergence and collapse of asset price booms. In particular, the anticipation of population decline—and the resulting increase in the cost of sustaining the bubble—can lead to gradual price erosion even in the absence of immediate shocks. This intertemporal transmission mechanism is a direct consequence of rational expectations in an OLG setting and offers a new perspective on asset price dynamics in aging economies.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature related to this study. Section 3 presents the model setup, including the demographic structure and the asset market. Section 4 characterizes the equilibrium under both stable and declining population regimes, and derives the critical conditions for bubble sustainability. Section 5 examines the relationship between bubble size and population growth, highlighting how equilibrium prices adjust in response to demographic trends. Section 6 concludes.

2. Related Literature

This paper contributes to the literature on asset bubbles in OLG economies and extends it by incorporating demographic uncertainty into bubble sustainability.

The theoretical foundation of bubbles in OLG settings originates with Samuelson (1958), who demonstrated that intrinsically worthless assets—such as fiat money—can improve allocations in dynamically inefficient economies. Tirole (1985) later formalized the concept of rational bubbles, identifying the conditions under which bubbles can arise and persist in general equilibrium. More specifically, the framework developed by Diamond (1965), which laid the foundation for many OLG capital accumulation models, shows that an economy can reach a steady state in which capital is overaccumulated—i.e., where the marginal product of capital falls below the rate of population growth. This dynamic inefficiency opens the door for bubbles to enhance welfare by reallocating resources intergenerationally. In Tirole's (1985) model, bubbles emerge precisely because of this dynamic inefficiency and are ruled out when the economy is dynamically efficient. Thus, the existence of bubbles in OLG models is closely tied to the underlying macroeconomic environment, particularly the relationship between growth and interest rates.

Blanchard (1979) and Blanchard and Watson (1982) introduced stochastic bubble dynamics through Markov processes, while Weil (1987) demonstrated that bubbles can be supported if agents are sufficiently confident in their future resale value. Han and Wang (2025) extend this framework by showing that multiple bubbles can coexist in a stationary environment, where the bubbles interact through both competitive and complementary channels. In particular, a portfolio composed of multiple bubbles exhibits greater stability—not only due to standard diversification effects but also through a "compensation effect," whereby the collapse of one bubble is partially offset by the continued appreciation of surviving ones. This multi-bubble structure introduces a new dimension to bubble resilience and highlights the potential for endogenous portfolio-based risk sharing among speculative assets.

While much of the early literature focused on pure bubbles—assets with no intrinsic value—recent developments have extended the theory to encompass assets with positive cash flows. These advances broaden the relevance of bubble dynamics to a wider class of real-world assets. For a comprehensive discussion, see the survey by Hirano and Toda (2024). In particular, Hirano and Toda (2025) establish a "bubble necessity theorem," demonstrating that when long-run economic growth exceeds the growth rate of dividends, and the counterfactual autarky interest rate lies below the dividend growth rate, all equilibria necessarily feature bubbles with non-negligible sizes relative to the scale of the economy. This result significantly expands the scope of bubbly equilibria, suggesting that bubbles can emerge not only in pure assets but also in productive assets such as stocks and real estate. Accordingly, the framework of bubble dynamics is applicable beyond intrinsically worthless assets—such as cryptocurrencies (assuming no intrinsic utility from exchange convenience or collateral use)—and may extend to any financial instrument where expectations of future resale value are key to current pricing.

The macroeconomic implications of demographic change have been extensively studied in the literature, and many of these insights are directly relevant to our analysis. In their seminal paper, Cutler et al. (1990) examine the broad economic consequences of demographic transitions, including their effects on saving behavior, fiscal sustainability, and international capital flows. Weil (1997) provides a comprehensive survey of how demographic changes influence long-run growth, saving, and capital accumulation. Abel et al. (1989) demonstrate that under stochastic population growth, dynamic inefficiency cannot be assessed solely based on the average population growth rate, complicating welfare evaluations across generations. Krueger and Ludwig (2007) show that in aging societies, asset-rich households—

typically older generations—are negatively affected by declining returns to capital. Lee (2016) discusses the risk of an “asset price meltdown,” wherein aging populations begin liquidating their assets in retirement, potentially driving down asset prices. This phenomenon may be amplified by falling returns to capital as the capital-to-labor ratio increases. Finally, Eggertsson, Mehrotra, and Robbins (2019) argue that demographic pressures in aging economies may help explain persistently low interest rates, driven by a structural shortfall in investment demand relative to savings.

While the specific mechanisms vary, these studies collectively support the view that demographic change—especially aging and population decline—has a substantial impact on asset prices, often exerting downward pressure. This paper approaches the issue from a novel angle by focusing on the implications of demographic transitions for stochastic bubbles, offering a new perspective on how population dynamics can endogenously undermine asset price sustainability.

As previously discussed, Weil (1987) demonstrates that stochastic bubbles can be sustained in an overlapping generations (OLG) economy, provided agents have sufficient confidence in the asset’s intergenerational resale value. Building on this insight, our paper revisits Weil’s framework by relaxing the assumption of constant population growth. We introduce time-varying demographic dynamics and analyze how changes in the population growth rate influence asset valuation and equilibrium outcomes over time. In particular, we derive modified equilibrium conditions and show that the trajectory of population growth plays a critical role in determining the sustainability of asset bubbles.

Whereas most existing stochastic bubble models—such as Weil (1987) and Han and Wang (2025)—-rely on sunspot shocks or exogenous disturbances to generate bubble collapse, our framework introduces a purely demographic mechanism. Specifically, we show that population decline alone can endogenously undermine the sustainability of asset bubbles. By linking demographic fundamentals to agents’ intertemporal expectations, our model offers a novel channel through which long-run population risk translates into financial fragility, even in settings governed by rational behavior and forward-looking decision-making.

3. Model

3.1. Basic Setup

We consider a discrete-time overlapping generations (OLG) model with an infinite horizon. We study an exchange economy in which agents consume their endowments but may trade across generations through financial markets.

Each agent lives for two periods—young and old—and derives utility from consumption in both periods. Agents receive an endowment of e_y when young and e_o when old. Preferences are represented by the utility function:

$$U(c_{y,} c_o) = u(c_y) + \beta u(c_o),$$

where c_y denotes consumption when young, c_o denotes consumption when old, and $\beta > 0$ is the subjective discount factor. The utility function u satisfies the standard properties of a concave utility function with the Inada conditions: $u' > 0$, $u'' < 0$, and $lim_{c\to 0}\, u'(c) = \infty$.

The size of generation t, denoted N_t, evolves according to a time-varying population growth process:

$$N_{t+1} = (1 + n_t)N_t,$$

where n_t is the population growth rate in period t. We assume that the population is shrinking at a constant rate $\eta < 0$ in each period, but this declining trend may permanently stop with probability q, in which case the population level remains constant thereafter (i.e., $n_t = 0$).

Formally, the transition is independent across time, and the population dynamics follow the transition matrix:

$$\Pi = \begin{bmatrix} 1-q & q \\ 0 & 1 \end{bmatrix},$$

where the first state corresponds to a shrinking population (growth rate η), and the second state corresponds to a constant population (growth rate zero).

This assumption reflects the demographic realities faced by many advanced economies, where fertility rates remain persistently below replacement levels, leading to population decline. While the downward trend may continue, it could also stabilize due to recovered fertility rates or an influx of foreign populations. Although it is possible for the population to rebound and grow again, we focus on the case where the population stabilizes at a reduced level following a period of decline.

The economy features a single tradable asset—a pure bubble asset with no intrinsic or fundamental value—which is available in a fixed supply of one unit. The bubble asset is intrinsically worthless but may be valued for its resale potential, depending on agents' expectations about future prices. Following the frameworks of Blanchard (1979), Blanchard and Watson (1982), and Weil (1987), we assume that agents perceive the price of this asset as stochastic. However, unlike these existing studies in which the bubble's dynamics are driven by extraneous forces (e.g., sunspots), we assume that the collapse is endogenously triggered by the unsustainability of the bubble due to a declining population.

This formulation captures the idea that demographic changes—such as persistent population shrinkage—can undermine the continuation of the bubble and ultimately lead to its collapse.

We further assume that purchasing the bubble asset incurs a fixed aggregate participation cost of ζ. As a result, the per capita cost in period t is given by:

$$\zeta \cdot \frac{1}{N_t}$$

This cost represents the participation expenses associated with establishing trading activities, including transaction fees, information processing costs, intermediation charges, and asset management expenses. The assumption of a fixed aggregate cost reflects the idea that these setup costs are independent of the number of investors. For simplicity, we assume that this cost is borne entirely by buyers, while sellers face no such expenses.

As the investor base grows, the per capita cost declines. Conversely, as the population shrinks, the burden of maintaining the bubble asset becomes increasingly concentrated, raising the cost borne by each investor.

We impose the following parametric restriction to focus on economically interesting situations in which a bubble can exist even in the absence of participation costs:

Assumption 1. *The parameters β, e_y, and e_o satisfy the following inequality:*

$$1 < \frac{\beta u'(e_o)}{u'\left(e_y\right)}.$$

This is the standard condition for the existence of a bubble, which is equivalent to the growth rate of the economy exceeding the autarky interest rate. A more detailed explanation will be provided in Subsection 4.1.

We assume that the rest of the economy is deterministic: preferences and endowments are fixed and known. Hence, the only source of uncertainty stems from population dynamics, specifically regarding the continued existence of the bubble asset.

3.2. Agent's Optimization Problem

Each agent lives for two periods and seeks to maximize expected lifetime utility, taking into account the possibility of trading a bubble asset and the uncertainty surrounding its future value due to population dynamics.

In generation t, when young, the agent receives an endowment e_y, consumes c_{yt}, and may purchase a_t units of the bubble asset at price P_t. In addition to the purchase price, the agent incurs a proportional share of the participation cost, amounting to ζ/N_t. When old, the agent consumes c_{ot}, financed by an endowment e_o and the resale value of the bubble asset, which may depend on whether the bubble survives. The individual budget constraints are given by:

$$c_{yt} + P_t a_t + \frac{\zeta}{N_t} = e_y \tag{1}$$

$$c_{ot} = e_o + \tilde{P}_{t+1} a_t \tag{2}$$

where $\tilde{P}_{t+1}$ denotes the (possibly stochastic) price of the bubble asset in period $t + 1$, which is zero if the bubble collapses.

The agent maximizes expected utility:

$$\max_{a_t} u(c_{yt}) + \beta E_t \left[u(c_{ot}) \right]$$

subject to the budget constraints above and the belief that the bubble may collapse with some probability governed by population dynamics.

3.3. Market Clearing Condition

The market-clearing condition is given by:

$$N_t a_t = 1,$$

which implies that the holdings of generation t young agents are:

$$a_t = \frac{1}{N_t}. \tag{3}$$

4. Equilibrium

We focus our analysis on a stationary equilibrium in which prices remain constant in the absence of further changes in the economy.

4.1. Stationary Equilibrium under Stable Population

Let us first explore the stationary equilibrium after population shrinkage has ceased (i.e., $n_t = 0$). In this case, the environment becomes deterministic, and the price of the bubble asset stabilizes. Thus, we have $P_t = \widetilde{P}_{t+1} = P$, where P is the constant per-unit price of the bubble asset in the steady state.

The first-order condition for the young agent's optimization problem is:

$$Pu'\left(e_y - Pa - \frac{\zeta}{N}\right) = \beta Pu'(e_o + Pa), \tag{4}$$

where the left-hand side reflects the marginal utility cost of acquiring the bubble asset, and the right-hand side captures the expected discounted marginal utility benefit from selling it when old.

In the steady state where the population is constant at N, Equation (3) implies that the steady-state level of asset holdings becomes:

$$a = \frac{1}{N}. \tag{5}$$

Using Equations (4) and (5), we obtain:

$$1 = \frac{\beta u'\left(e_o + P \cdot \frac{1}{N}\right)}{u'\left(e_y - (P+\zeta) \cdot \frac{1}{N}\right)}. \tag{6}$$

Equation (6) in turn implies that the bubble is sustainable in the steady state (i.e., $P > 0$) if and only

$$1 = \frac{\beta u'\left(e_o + P \cdot \frac{1}{N}\right)}{u'\left(e_y - (P+\zeta) \cdot \frac{1}{N}\right)} < \frac{\beta u'(e_o)}{u'\left(e_y - \frac{\zeta}{N}\right)}. \tag{7}$$

There exists a unique positive solution to Equation (6) if and only if the condition in Equation (7) is satisfied. This condition ensures that the marginal benefit of purchasing the bubble asset exceeds its marginal cost at the margin. As a result, a stationary bubble equilibrium can be sustained under a stable population, provided that the participation cost is sufficiently small. This existence result follows directly from the continuity and strict concavity of the utility function, which guarantee a unique interior solution to the optimality condition.

Lemma 1. *There exists a unique positive solution $P > 0$ to Equation (6):*

$$1 = \frac{\beta u'\left(e_o + \frac{P}{N}\right)}{u'\left(e_y - \frac{P+\zeta}{N}\right)},$$

if and only if the following inequality is satisfied:

$$1 < \frac{\beta u'(e_o)}{u'\left(e_y - \frac{\zeta}{N}\right)}.$$

Proof. Define the function

$$F(P) = \frac{\beta u'\left(e_o + \frac{P}{N}\right)}{u'\left(e_y - \frac{P+\zeta}{N}\right)}.$$

This function is continuous and strictly decreasing on the interval $(0, Ne_y - \zeta)$, due to the strict concavity of u. We have:

$$F(P) = \frac{\beta u'(e_o)}{u'\left(e_y - \frac{\zeta}{N}\right)} \quad as\ P \to 0+$$

and

$$F(P) = 0 \ as \ P \rightarrow (Ne_y - \zeta) -$$

Hence, by the Intermediate Value Theorem, $F(P) = 1$ has a unique solution $P^* \in (0, Ne_y - \zeta)$ if and only if $F(0) > 1$, which gives the stated condition. QED

Note that the condition in Equation (1) becomes more stringent as N decreases. Specifically, it raises the marginal utility of consumption when young, tightening the equilibrium condition. We summarize our findings in the following result:

Proposition 2. *The bubble is sustainable if and only if Equation (1) is satisfied. Furthermore, the viability of the bubble decreases as the stable population size N becomes smaller.*

The proposition highlights the negative impact of population shrinkage on the sustainability of bubbles. As the fixed cost of maintaining the bubble asset remains unchanged, a smaller number of investors implies a higher per capita burden. This increased cost makes the bubble less viable in steady state, reducing the likelihood of its existence.

Also, note that the presence of a participation cost makes the existence condition for a bubble more stringent than in standard models. This is illustrated by the following inequality:

$$\frac{\beta u'(e_o)}{u'\left(e_y - \frac{\zeta}{N}\right)} < \frac{\beta u'(e_o)}{u'(e_y)} = \frac{1}{1 + r_A},$$

where r_A denotes the autarky interest rate. This implies that bubble existence is not guaranteed even under Assumption 1.

Again, by the continuity and concavity of the utility function (see Lemma 1), there exists a unique solution $\overline{N}$ that satisfies:

$$1 = \frac{\beta u'\left(e_o\right)}{u'\left(e_y - \frac{\zeta}{N}\right)}. \tag{8}$$

The solution $\overline{N}$ represents the population threshold below which the bubble becomes unsustainable—i.e., the minimum population size above which the bubble is sustainable. This is a direct consequence of Proposition 2:

Corollary 3. *There exists a threshold $\overline{N}$ such that the bubble is sustainable if and only if $N \geq \overline{N}$; otherwise, it is unsustainable.*

4.2. Stationary Equilibrium under Shrinking Population

Let us now turn to the stationary equilibrium when the population is still on a declining trajectory (i.e., $n_t = \eta$). Among the possible equilibria, we focus on the class of stationary equilibria in which the bubble collapses whenever the population falls below the threshold level $\overline{N}$ defined in Corollary 3.

Under this setup, the environment becomes stochastic, and the price of the bubble asset may take two different values depending on the realized path of population growth in the future.

Assuming that Equation (1) holds, we define $P^0(N_t)$ to be the solution to Equation (6), which represents the price of the bubble given population size N_t under a stable population. We also define $P^\eta(N_t)$ as the price of the bubble given population size N_t under a shrinking population. In what follows, we solve for $P^\eta(N_t)$ explicitly.

Suppose that the population in period t is still shrinking (i.e., $n_t = \eta$). If the population stops shrinking in the subsequent period (i.e., $n_{t+1} = 0$), then $\widetilde{P}_{t+1} = P^0(N_t)$, since $N_t = N_{t+1} = N$. On the other hand, if the population continues to shrink (i.e., $n_{t+1} = \eta$), then $\widetilde{P}_{t+1} = P^\eta(N_t(1+\eta))$. Therefore, the agent's objective function becomes:

$$u\left(e_y - P_t^\eta(N_t)a - \frac{\zeta}{N_t}\right) + \beta\left[(1-q)u(e_o + P^\eta(N_t(1+\eta))a_t) + qu(e_o + P^0(N_t)a_t)\right]. \tag{9}$$

To compute the values of $P^{\eta}(N_t)$ at different levels of N_t, we begin by solving it at the threshold level $\overline{N}$, and then proceed recursively by moving away from the threshold in either direction.

4.2.1. Stationary Equilibrium around the Threshold Population

Now, suppose that the population in generation t is still above the threshold level $\overline{N}$ (i.e., $N_t \geq \overline{N}$), but would fall below the threshold in the next period if it continues to shrink (i.e., $N_t(1+\eta) < \overline{N}$). In this case, the bubble collapses with probability $1-q$, and Equation (9) simplifies to:

$$u\left(e_y - P^{\eta}(N_t)a_t - \frac{\zeta}{N_t}\right) + \beta\left[(1-q)u(e_o) + qu(e_o + P^0(N_t)a_t)\right], \tag{10}$$

where the resale value is zero with probability $1-q$ due to the collapse of the bubble when the population drops below the sustainability threshold.

The agent maximizes the objective function in Equation (10), subject to the budget constraints given in Equations (1) and (2). The first-order condition for this problem, combined with the market-clearing condition in Equation (3), yields

$$P^{\eta}(N_t)u'\left(e_y - \frac{P^{\eta}(N_t)}{N_t} - \frac{\zeta}{N_t}\right) = \beta q P^0(N_t)u'\left(e_o + \frac{P^0(N_t)}{N_t}\right). \tag{11}$$

Since $P^0(N_t)$ is known, we can solve Equation (11) for $P^{\eta}(N_t)$. By the continuity and concavity of the utility function (see Lemma 1), a unique solution for $P^{\eta}(N_t)$ exists. We summarize our findings in the following lemma:

Lemma 4. *There exists a unique price $P^{\eta}(N_t)$ when the population is still shrinking and is one period away from reaching the threshold level $\overline{N}$.*

4.2.2. Stationary Equilibrium away from the Threshold Population

Now, suppose that the population in generation t is above the threshold level $\overline{N}$

(i.e., $N_t > \overline{N}$), and also would not fall below the threshold in the next period even if it continues to shrink (i.e., $N_t(1+\eta) \geq \overline{N}$). But it would fall below the threshold in the next period after the next period if it continues to shrink in two consecutive periods (i.e., $N_t(1+\eta)^2 < \overline{N}$).

For notational convenience, we denote $P_\tau^\eta(N_t)$ as the price of the bubble under a shrinking population when the population is expected to reach the threshold level $\overline{N}$ in τ periods. In particular, $P_1^\eta(N_t)$—whose existence is guaranteed by Lemma 4—corresponds to the case in which the threshold is reached in the next period and is determined by the solution to Equation (11).

In this case, the objective function in Equation (9) simplifies to:

$$u\left(e_y - P_2^\eta(N_t)a_t - \frac{\zeta}{N_t}\right) + \beta\left[(1-q)u(e_o + P_1^\eta(N_t(1+\eta))a_t + qu(e_o + P^0(N_t)a_t)\right], \quad (12)$$

The agent maximizes the objective function in Equation (12), subject to the budget constraints given in Equations (1) and (2). The first-order condition for this problem, combined with the market-clearing condition in Equation (3), yields

$$\begin{aligned} &P_2^\eta(N_t)u'\left(e_y - \frac{P_2^\eta(N_t)}{N_t} - \frac{\zeta}{N_t}\right) \\ &= \beta\left[(1-q)P_1^\eta(N_t(1+\eta))u'(e_o + P_1^\eta(N_t(1+\eta))a_t) + qP^0(N_t)u'\left(e_o + \frac{P^0(N_t)}{N_t}\right)\right] \end{aligned} \quad (13)$$

Given $P_1^\eta(N_t(1+\eta))$ and $P^0(N_t)$ from the previous subsections, we can solve Equation (13) for $P_2^\eta(N_t)$. By the continuity and concavity of the utility function (see Lemma 1), there exists a unique solution for $P_2^\eta(N_t)$. We summarize this result in the following lemma:

Lemma 5. *There exists a unique price $P_2^\eta(N_t)$ when the population is declining and the economy is two periods away from crossing the threshold population level $\overline{N}$.*

We can now generalize this result to any N_t that is τ periods away from the threshold population level $\overline{N}$. By applying the argument recursively, the first-order condition for the agent's problem, combined with the market-clearing condition in Equation (3), yields

$$\begin{aligned} &P_\tau^\eta(N_t)u'\left(e_y - \frac{P_\tau^\eta(N_t)}{N_t} - \frac{\zeta}{N_t}\right) \\ &= \beta\left[(1-q)P_{\tau-1}^\eta(N_t(1+\eta))u'\left(e_o + \frac{P_{\tau-1}^\eta(N_t(1+\eta))}{N_t}\right) + qP^0(N_t)u'\left(e_o + \frac{P^0(N_t)}{N_t}\right)\right]. \end{aligned} \tag{14}$$

The unique existence of the solution for $P_\tau^\eta(N_t)$ is guaranteed by mathematical induction, based on the results established in Lemmas 4 and 5. We summarize this result in the following proposition:

Proposition 6. *There exists a unique price $P_\tau^\eta(N_t)$ when the population is still shrinking and is τ periods away from crossing the threshold level $\overline{N}$.*

5. Bubble Size and Population Growth Dynamics

In this section, we examine how the size of the bubble is influenced by population growth. For notational convenience, let $B_\tau^\eta(N_t)$ denote the size (or market capitalization) of the bubble under a declining population, where the population is expected to reach the threshold level $\overline{N}$ in τ periods. Similarly, let $B^0(N_t)$ represent the bubble size when the population remains stable at level N_t.

We first establish the relationship between the size of the bubble and the size of the population in an economy with a stable population.

Proposition 7. *The equilibrium size of the bubble increases with the size of the population when $n_t = 0$.*

Proof. From Equation (4), the first-order condition for the agent's problem, combined with the market-clearing condition in Equation (3) and the definition of $B_\tau^\eta(N_t)$, yields

$$u'\left(e_y - B^0(N) - \frac{\zeta}{N}\right) = \beta u'(e_o - B^0(N)).$$

Then, define the function

$$F(B^0, N) = u'\left(e_y - B^0(N) - \frac{\zeta}{N}\right) = \beta u'(e_o - B^0).$$

When a bubble exists (i.e., $B^0(N) > 0$), the implicit function theorem implies:

$$\frac{dB^0}{dN} = -\frac{\partial F / \partial N}{\partial F / \partial B^0} = \frac{\frac{\zeta}{N^2} u''\left(e_y - B^0(N) - \frac{\zeta}{N}\right)}{u''\left(e_y - B^0(N) - \frac{\zeta}{N}\right) + \beta u''(e_o + B^0(N))} > 0.$$

The denominator is negative due to the concavity of the utility function ($u'' < 0$), and the numerator is also negative, implying that the derivative is positive. Therefore, under a stable population, the equilibrium bubble size increases with the population level. QED

We extend our findings establish the relationship between the size of the bubble and the size of the population in an economy with a stable population.

Proposition 8. *The equilibrium size of the bubble increases with the size of the population when* $n_t = \eta$.

Proof. We first start the proof when the economy is one period away from crossing the threshold level. From Equation (11), the first-order condition for the agent's problem, combined with the market-clearing condition in Equation (3) and the definition of $B_\tau^\eta(N_t)$, yields

$$B_1^\eta(N_t)u'\left(e_y - B_1^\eta(N_t) - \frac{\zeta}{N_t}\right) = \beta q B^0(N_t)u'(e_o + B^0(N_t)).$$

Define the function

$$F(B_1^\eta, N_t) = B_1^\eta u'\left(e_y - B_1^\eta - \frac{\zeta}{N_t}\right) = \beta q B^0(N_t)u'(e_o + B^0(N_t)).$$

When a bubble exists (i.e., $B_1^\eta(N_t) > 0$), the implicit function theorem implies:

$$\frac{dB_1^\eta}{dN_t} = -\frac{\partial F / \partial N_t}{\partial F / \partial B_1^\eta}.$$

We compute each derivative:

$$\frac{\partial F}{\partial N_t} = B_1^\eta \cdot u''\left(e_y - B_1^\eta - \frac{\zeta}{N_t}\right) \cdot \left(\frac{\zeta}{N_t^2}\right)$$
$$-\beta q\left[\frac{dB^0}{dN_t} \cdot u'(e_o + B^0(N_t)) + B^0(N_t) \cdot u''(e_o + B^0(N_t)) \cdot \frac{dB^0}{dN_t}\right],$$

and

$$\frac{\partial F}{\partial N_t} = u'\left(e_y - B_1^\eta - \frac{\zeta}{N_t}\right) - B_1^\eta \cdot u''\left(e_y - B_1^\eta - \frac{\zeta}{N_t}\right).$$

By Proposition 7, we have $\frac{dB^0}{dN_t} > 0$. Then, given the concavity of the utility function ($u'' < 0$), it follows that the numerator in the expression for $\frac{dB_1^\eta}{dN_t}$ is positive, while the denominator is negative. Hence,

$$\frac{dB_1^\eta}{dN_t} > 0.$$

Therefore, the size of the bubble one period before the threshold increases with the population level.

We now generalize the result to the case when the economy is $\tau \geq 1$ periods away from crossing the threshold. From Equation (14), the first-order condition for the agent's problem, combined with the market-clearing condition in Equation (3) and the definition of $B_\tau^\eta(N_t)$, yields

$$B_\tau^\eta(N_t)u'\left(e_y - B_\tau^\eta(N_t) - \frac{\zeta}{N_t}\right)$$
$$= \beta\Big[(1-q)B_{\tau-1}^\eta(N_t(1+\eta))u'\big(e_o + B_{\tau-1}^\eta(N_t(1+\eta))\big) + qB^0(N_t)u'(e_o + B^0(N_t))\Big].$$

Define the function

$$F(B_\tau^\eta, N_t) = B_\tau^\eta u'\left(e_y - B_\tau^\eta - \frac{\zeta}{N_t}\right)$$
$$-\beta\Big[(1-q)B_{\tau-1}^\eta(N_t(1+\eta))u'\big(e_o + B_{\tau-1}^\eta(N_t(1+\eta))\big) + qB^0(N_t)u'(e_o + B^0(N_t))\Big].$$

Applying the implicit function theorem, we obtain:

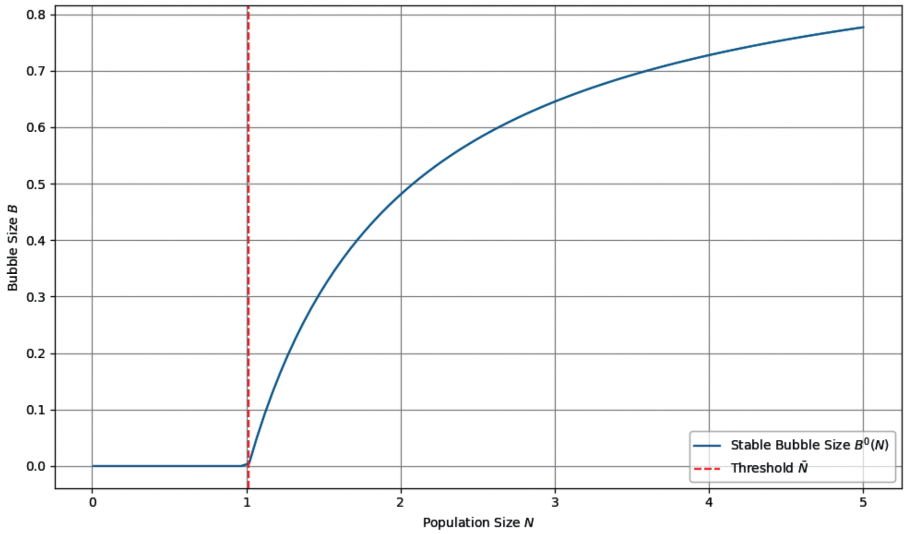
$$\frac{dB_\tau^\eta}{dN_t} = -\frac{\partial F / \partial N_t}{\partial F / \partial B_\tau^\eta}.$$

We compute the derivatives. The denominator is:

$$\frac{\partial F}{\partial B_\tau^\eta} = u'\left(e_y - B_\tau^\eta - \frac{\zeta}{N_t}\right) - B_\tau^\eta \cdot u''\left(e_y - B_\tau^\eta - \frac{\zeta}{N_t}\right).$$

The numerator is:

$$\frac{\partial F}{\partial N_t} = B_\tau^\eta \cdot u''\left(e_y - B_\tau^\eta - \frac{\zeta}{N_t}\right)\cdot\left(\frac{\zeta}{N_t^2}\right)$$
$$-\beta\left[\begin{array}{l}(1-q)\cdot\dfrac{dB_{\tau-1}^\eta(N_t(1+\eta))}{dN_t}\cdot\big(u'(e_o + B_{\tau-1}^\eta) + B_{\tau-1}^\eta\cdot u''(e_o + B_{\tau-1}^\eta)\big)\\ +q\cdot\dfrac{dB^0}{dN_t}\cdot\big(u'(e_o + B^0(N_t)) + B^0(N_t)\cdot u''(e_o + B^0(N_t))\big)\end{array}\right].$$



The curve is computed from the intertemporal optimality condition: $1 = \dfrac{\beta u'(e_0 + P)}{u'(e_y - (P+\zeta)/N)}$, assuming CRRA utility $u(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$, with parameters β = 0.95, e_y = 3.0, e_o = 1.0, ζ = 2, and γ = 2. The dashed red vertical line indicates the threshold value $\bar{N}$, below which no stable bubble can exist: $\bar{N} = \dfrac{\zeta}{e_y - \beta^{-1/\gamma} e_o}$.

Figure 3. Stable bubble price $B^0(N)$ as a function of population size N.

By induction, we assume that $\dfrac{dB^{\eta}_{\tau-1}}{dN_t} > 0$ and we know from Proposition 7 that $\dfrac{dB^0}{dN_t} > 0$. Given the concavity of the utility function ($u'' < 0$), the numerator is positive and the denominator is negative. Therefore,

$$\frac{dB^{\eta}_{\tau}}{dN_t} > 0.$$

This completes the inductive step and proves that the equilibrium bubble size $B^{\eta}_{\tau}(N_t)$ increases with the population level for any $\tau \geq 1$. QED

Therefore, Propositions 7 and 8 together show that, whenever a bubble exists, its size increases with the level of the population, regardless of the state of population growth. Figure 3 illustrates this relationship in the case of a stable population (n_t = 0).

This result reflects the economic intuition that a larger population increases aggregate demand for the bubble asset, thereby raising its equilibrium price and, consequently, its size. The finding holds under standard assumptions of concave utility and competitive equilibrium, and is robust to variations in the transition probabilities q and the growth rate η, as long as the bubble continues to exist. These insights suggest that demographic trends—such as population aging or long-term decline—can systematically reduce the scope for sustaining bubbles, with broader implications for understanding asset price dynamics in aging economies.

Finally, a key result from this section is that the size of the bubble today adjusts in anticipation of a potential collapse driven by demographic decline. In particular, even when the bubble has not yet burst, its equilibrium size already reflects agents' expectations about future population shrinkage and the resulting possibility of collapse. This forward-looking behavior is a natural consequence of rational expectations in an overlapping generations setting: agents internalize the risk that the economy may cross the demographic threshold beyond which the bubble cannot be sustained. As a result, the bubble begins to shrink well before the collapse actually occurs, highlighting the intertemporal transmission of demographic risks to current asset valuations.

6. Conclusion

This paper examines how demographic trends, particularly population decline, influence the formation and sustainability of rational bubbles in an overlapping generations economy. We show that the equilibrium size of the bubble increases with the level of the population, regardless of whether the economy is currently stable or on a path toward demographic collapse. Importantly, when agents anticipate that the population will fall below a critical threshold in the future, their expectations are reflected in the bubble's current valuation. As a result, bubbles begin to shrink before the actual collapse occurs, even in environments where a collapse is several periods away. This intertemporal adjustment underscores the sensitivity of asset prices to long-run demographic fundamentals and highlights the importance of incorporating forward-looking behavior into models of financial fragility. Our findings suggest that demographic shifts—such as aging and population decline—can serve as early warning signals for financial instability, even in the absence of immediate market distress.

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