Consumer Behavior in a Money Economy

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I. Introduction

The classical standard theory of consumer behavior may be said to be the theory of consumer behavior in a barter rather than money economy. Though money prices and money income enter the model through the budget constraints, the budget equation in the traditional model does not constitute an appropriate definition of choice alternatives in a money economy.

The budget equation in the classical model, $\sum_{i=1}^{n} p_i x_i = y$, admits "as feasible trades every possible combination of commodities traded in the economy; i.e., any commodity, whether a good or money, can be offered directly in trade for every other commodity."[1, p. 3] This is possible only in an economy which would be regarded as a barter rather than money economy. In a money economy, some commodities——at least, one——cannot be directly traded with all other commodities. This distinguishing characteristic of a money economy is not reflected in the classical theory of consumer behavior.

Furthermore, consumers in the classical model have been assumed not to derive any utility from holdings of money balances. This seems to be due to the presupposition that money yields, by being spent on commodities, only indirect services to the consumer. Thus the standard theory has supposed that all of the utility from money could be reflected in the direct utility from commodities which enter the consumer's utility

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function.

Patinkin has correctly argued, however, that the holding of money yields direct services such as security against incoveniences caused by uncertainty about payment and receipt timing.[5]

This paper intends to examine how the classical standard theory of consumer behavior should be revised to describe the consumer behavior in a money rather than barter economy. This intention will be carried out by drawing implications from the reformulation of the utility function and budget equation so as to be appropriate for a money economy. The reformulation is mainly due to Clower [1], but his implications from the reformulation are not concise but concerned mainly with macroeconomic aspects. We are here concerned with the microeconomic implications from the reformulation.

II. Formulation of the Problem

Suppose a Euclidean space, E, of dimension n+2, and let $x \in E$ be denoted by $x=(x_1, \cdots, x_n, x_{n+1}, x_{n+2})$, where the first n components represent the quantities demanded of the n commodities (denoted by d_i, $i=1, \cdots, n$), and the n+1-th and n+2-th components represent the real quantities of money demanded for precautionary purposes and for transactions purposes, respectively (denoted by M/p and m/p, respectively).(1) Moreover, define the bundle space, $\dot{E}=\{x_i \geqq 0, \quad i=1, \cdots, n+2\}$. An element of E will be called a bundle. The assumptions listed below hold throughout the paper.

Assumption (A.1): For all x, $y \in \dot{E}$, one and only one of the relations $x\ P\ y$, $y\ P\ x$, $x\ I\ y$ holds, where the relation "P" on $\dot{E}$ is read "is preferred to" and "I" is read "is indifferent from".

Assumption (A.2): There exists a real valued utility function, denoted $u(x)$, $x \in E$, which has continuous second partials u_{ij}, $i,j=1, \cdots, n+2$, and $u(x)>u(y)$ if and only if $x\ P\ y$ for all $x,y \in E$:

$$(1.1) \qquad u=u(d_1, \ldots\ldots, d_n, M/p, m/p).$$

Note that the price level, denoted by p, is a function of prices of commodities and that a unit of M/p or m/p is defined such that the unit price of M/p or m/p is p:

(1) M and m denote the nominal quantities of money demanded for precautionary and transactins purposes, respectively, and p the price level.

(1.2) $$p=p(p_1,\ldots\ldots,p_n),\ \partial p/\partial p_j>0 \text{ for all } j=1,\ldots\ldots,n.$$

Assumption (A. 3): The consumer always prefers to get more of a commodity or money, i.e., $u_i(x)>0,\ i=1,\cdots,n+2.$

Suppose that the consumer receives a fixed quantities of n commodities, denoted by s_i, $i=1,\cdots,n$, and of money, denoted by $\bar{M}$, that may be used to purchase a bundle of d_i, $i=1,\cdots,n$, M/p and m/p. Moreover, suppose that only money can be used as the medium of exchange.(2)

Then the consumer is constrained so that all the goods (including M) demanded for purchase must be backed by a readiness to supply money in exchange and that all the goods offered for sale must involve a demand for money in exchange. The former condition, called as the "expenditure constraints" by Clower [1] can be expressed by (1. 3), and the latter, called as the "income constraints," by (1. 4):

(1.3) $$\sum_{i=1}^{a} p_i(d_i-s_i)+M-\bar{M}\leqq 0$$

or

$$\sum_{i=1}^{a} p_i d_i+M\leqq\sum_{i=1}^{a} p_i s_i+\bar{M} \qquad \text{for } d_i\geqq s_i;$$

(1.4) $$\sum_{i=a+1}^{n} p_i(d_i-s_i)+m\leqq 0$$

or

$$\sum_{i=a+1}^{n} p_i d_i+m\leqq\sum_{i=a+1}^{n} p_i s_i \qquad \text{for } d_i\leqq s_i.$$

Assumption (A.4): Given a price-income(3) pair, the consumer will maximize his utility function (1. 1) subject to his budget equations (1. 3) and (1. 4).(4)

The necessary conditions for a regular constrained maximum are

(1.5) $$\begin{aligned} &u_i-\lambda p_i=0,\ i=1,\ldots\ldots,a,\\ &u_i-\mu p_i=0,\ i=a+1,\ldots\ldots,n,\\ &u_{n+1}-\lambda p=0,\\ &u_{n+2}-\mu p=0,\\ &\sum_{i=1}^{a} p_i d_i+M=\sum_{i=1}^{a} p_i s_i+\bar{M} \end{aligned}$$

(2) In a money economy, as mentioned above, some other commodities——but not alll others ——than money can be used as media of exchange. This situation would, however, make the problem very difficult to be handled with. So we are considering a "pure money economy," in Clower's terminology, in which money is the only medium of exchange.

(3) Income here includes the initial stock of n commodities and the initial amount of money.

(4) In view of (A.3), the budget constrainsts become equations.

$$\sum_{i=a+1}^{n} p_i d_i + m = \sum_{i=a+1}^{n} p_i s_i,$$

where λ and μ are Lagrange multipliers. The sufficient conditions for a regular constrained maximum are (1.5) and

(1.6)
$$\begin{vmatrix} u_{11} & \cdots\cdots & u_{1k} & p_1 & 0 \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ u_{k1} & \cdots\cdots & u_{kk} & p_k & 0 \\ p_1 & \cdots\cdots & p_k & 0 & 0 \\ 0 & \cdots\cdots & 0 & 0 & 0 \end{vmatrix} (-1)^{k+1} > 0, \quad k=2, \ldots\ldots, a;$$

$$\begin{vmatrix} u_{11} & \cdots\cdots & & u_{1k} & p_1 & 0 \\ \cdot & & & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & p_a & \cdot \\ \cdot & & & \cdot & 0 & p_{a+1} \\ \cdot & & & \cdot & \cdot & \cdot \\ u_{k1} & \cdots\cdots & & u_{kk} & 0 & p_k \\ p_1 \cdots\cdots p_a & 0 & \cdots\cdots & 0 & 0 & 0 \\ 0 \cdots\cdots 0 & p_{a+1} & \cdots\cdots & p_k & 0 & 0 \end{vmatrix} (-1)^{k+1} > 0, \quad k=a+1, \ldots\ldots, n+2.$$

The system (1.5) can be written as n+4 implicit functions in 3n+5 arguments $(d_1, \cdots, d_n, M, m, p_n, s_1, \cdots, s_n, \overline{M}, \tilde{\lambda}, \mu)$. Furthermore, at the point $(\tilde{d}_1, \cdots, \tilde{d}_n, \widetilde{M}, \tilde{m}, \tilde{p}_1, \cdots, \tilde{p}_n, \tilde{s}_1, \cdots, \tilde{s}_n, \widetilde{\overline{M}}, \tilde{\lambda}, \tilde{\mu})$ in Euclidean 3n+5 space, they vanish and their Jacobian, in view of (1.6), is

$$J = \begin{vmatrix} U_{ij} & P_i \\ P_i & 0 \end{vmatrix} \neq 0, \quad i, j=1, \ldots\ldots, n+2,$$

where

$$U_{ij} = \begin{pmatrix} u_{11} & \cdots\cdots & u_{1,\,n+2} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ u_{n+2,\,1} & \cdots\cdots & u_{n+2,\,n+2} \end{pmatrix}, \quad p_i = \begin{pmatrix} p_1 & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ p_a & 0 \\ 0 & p_{a+1} \\ \cdot & \cdot \\ \cdot & \cdot \\ 0 & p_n \\ p & 0 \\ 0 & p \end{pmatrix}$$

and

$$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Moreover, the n+4 implicit functions have continuous first partials and consequently, there exist demand functions

(1.7) $$d_i = d_i(p_1, \ldots\ldots, p_n, s_1, \ldots\ldots, s_n, \bar{M})$$

in a neighborhood of $(\tilde{p}_1, \cdots, \tilde{p}_n, \tilde{s}_1, \cdots, \tilde{s}_n, \tilde{\bar{M}})$ which are unique and possess continuous first partials in this neighborhood.

We can observe that for a given $(p_1, \cdots, p_n, s_1, \cdots, s_n, M)$ the conditions (1.5) and (1.6) are invariant when the utility function u is replaced by the utility function $F(u)$, where F is a strictly increasing function of u with a continuous second derivative.(5) Therefore, the properties of the demand function d_i, $i=1, \cdots, n+2$, are not affected by the choice of a utility function, i.e., the choice of a utility indication has no empirical implications for the theory of comparative statics of consumer demand.

III. The Comparative Statics of Consumer Demand

Let us first consider the effect on the individual's demand for the various goods resulting from a partial change in his initial stock of commodities. Differentiating (1.5) with respect to the initial stock of the j-th commodity with all the prices, the initial stocks of other commodities and initial balances of money being held constant, we obtain

(2.1)
$$\begin{pmatrix} U_{ij} & P_i \\ P_i^{\mathrm{T}} & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial D_i}{\partial s_j} \\ -\dfrac{\partial \lambda}{\partial s_j} \\ -\dfrac{\partial \mu}{\partial s_j} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ p_j \\ 0 \end{pmatrix} \begin{matrix} \\ \\ \\ \cdots\cdots(n+3\text{-}th) \\ \\ \end{matrix} \qquad \text{for } j \leqq a, \quad i=1, \ldots\ldots, n+2;$$

$$\begin{pmatrix} U_{ij} & P_i \\ P_i^{\mathrm{T}} & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial D_i}{\partial s_j} \\ -\dfrac{\partial \lambda}{\partial s_j} \\ -\dfrac{\partial \mu}{\partial s_j} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ p_j \end{pmatrix} \begin{matrix} \\ \\ \\ \cdots\cdots(n+4\text{-}th) \end{matrix} \qquad \text{for } j > a, \quad i=1, \cdots\cdots n+2,$$

where

$$D_i = \begin{pmatrix} d_1 \\ \vdots \\ d_n \\ M/p \\ m/p \end{pmatrix}$$ and the other notations are the same as before.

Thus, solving (2.1), we obtain

(5) See Samuelson [6, p. 104].

$$(2.2)\quad \begin{aligned} &\frac{\partial d_i}{\partial s_j}=p_j A_{n+3,i}/A, \quad \text{for } j\leq a; \\ &\frac{\partial d_i}{\partial s_j}=p_j A_{n+4,i}/A, \quad \text{for } j>a, \end{aligned} \qquad i=1,\ldots\ldots,n+2,$$

where A is the determinant of the coefficient matrix, and $A_{n+3,i}$ and $A_{n+4,i}$ are the cofactors of the elements in the n+3-th and n+4-th rows and the i-th column in A, respectively.

If we consider the effect of equiproportional infinitesimal change in the initial stocks of all the n commodities, (2.2) may be written as follows:

$$(2.3)\quad \begin{aligned} &\frac{\partial d_i}{\partial y_1}=A_{n+3,i}/A; \qquad \frac{\partial d_i}{\partial y_2}=A_{n+4,i}/A; \\ &\frac{\partial d_i}{\partial Y}=(A_{n+3,i}+A_{n+4,i})/A, \end{aligned}$$

where $Y=y_1+y_2$, $y_1=\sum_{i=1}^{a} p_i s_i$ and $y_2=\sum_{i=a+1}^{n} p_i s_i$.

From (1.5), we can note that y_1 and $\bar{M}$ appear as a sum on the right-hand side of the budget equation for the goods demanded to purchase. Thus, given a set of prices, any increase in y_1 can always be compensated by a decrease in $\bar{M}$ of the same absolute value as the increase in y_1. Furthermore, an increase in $\bar{M}$ will have exactly the same effect on the consumer's demand for n commodities, M/P and m/p as an equal increase in y_1. That is,

$$(2.4)\quad \partial d_i/\partial y_1=\partial d_i/\partial \bar{M}=\partial d_i/\partial(y_1+\bar{M})=\frac{y_1}{y_1+\bar{M}}\frac{\partial d_i}{\partial y_1}+\frac{\bar{M}}{y_1+\bar{M}}\frac{\partial d_i}{\partial \bar{M}}=A_{n+3,i}/A.$$

Let us now examine the effect of a partial change in the price of one of the commodities. Differentiating (1.5) with respect to p_j with all the other prices, initial stocks of n commodities and initial money balances held constant, we obtain

$$(2.5)\quad \begin{pmatrix} U_{ij} & P_i \\ P_i^{\mathrm{T}} & 0 \end{pmatrix} \begin{pmatrix} \partial D_i/\partial p_j \\ -\partial\lambda/\partial p_j \\ -\partial\mu/\partial p_j \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \lambda \\ 0 \\ \vdots \\ 0 \\ \lambda\partial p/\partial p_j \\ \mu\partial p/\partial p_j \\ -(d_j-s_j)-\dfrac{M}{p}\dfrac{\partial p}{\partial p_j} \\ -\dfrac{m}{p}\dfrac{\partial p}{\partial p_j} \end{pmatrix} \begin{matrix} \\ \\ \\ \ldots(j\text{-th}) \\ \\ \\ \ldots(n\text{-th}) \\ \\ \\ \\ \\ \end{matrix} \quad \text{for } j\leq a;$$

$$\begin{bmatrix} U_{ij} & P_i \\ P_i & 0 \end{bmatrix} \begin{bmatrix} \partial D_i/\partial p_j \\ -\partial\lambda/\partial p_j \\ -\partial\mu/\partial p_j \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \mu \\ 0 \\ \vdots \\ 0 \\ \lambda\partial p/\partial p_j \\ \mu\partial p/\partial p_j \\ -\frac{M}{p}\frac{\partial p}{\partial p_j} \\ -(d_j-s_j)-\frac{m}{p}\frac{\partial p}{\partial p_j} \end{bmatrix} \begin{matrix} \\ \\ \\ \ldots(j\text{-th}) \\ \\ \\ \ldots(n\text{-th}) \\ \\ \\ \\ \\ \end{matrix} \quad \text{for } j>a,$$

$$(i=1,\ldots\ldots,n+2).$$

Solving (2.5), we obtain

$$\partial d_i/\partial p_j = \frac{1}{A}\left\{\lambda A_{ji}+\lambda\frac{\partial p}{\partial p_j}A_{n+1,i}+\mu\frac{\partial p}{\partial p_j}A_{n+2,i}-\left[(d_j-s_j)+\frac{M}{p}\frac{\partial p}{\partial p_j}\right]A_{n+3,i}-\frac{m}{p}\frac{\partial p}{\partial p_j}A_{n+4,i}\right\}$$

(2.6) $\quad$ for $j\leq a$;

$$\partial d_i/\partial p_j = \frac{1}{A}\left\{\mu A_{ji}+\lambda\frac{\partial p}{\partial p_j}A_{n+1,i}+\mu\frac{\partial p}{\partial p_j}A_{n+2,i}-\frac{M}{p}\frac{\partial p}{\partial p_j}A_{n+3,i}-\left[(d_j-s_j)+\frac{m}{p}\frac{\partial p}{\partial p_j}\right]A_{n+4,i}\right\}$$

for $j>a$,

$(i=1,\ldots\ldots,n+2)$.

Now we want to show, by decomposing (2.6), that the effect of the j-th price change on the demand for the i-th good is consisted of the substitution effect, income effect and real balance effect. The substitution effect describes the effect of a price change accompanied by a variation in the initial stock of commodities and/or a variation in the initial balance of money that enables the consumer to maintain the same utility level.

If we conduct a conceptual experiment to minimize the total expenditure on n commodities, M and m subject to a givn level of utility, we obtain

$$(\partial d_i/\partial p_j)_{u=\text{const.}} = \frac{1}{A}\left(\lambda A_{ji}+\lambda\frac{\partial p}{\partial p_j}A_{n+1,i}+\mu\frac{\partial p}{\partial p_j}A_{n+2,i}\right) \quad \text{for } j\leqslant a;$$

(2.7)

$$(\partial d_i/\partial p_j)_{u=\text{const.}} = \frac{1}{A}\left(\mu A_{ji}+\lambda\frac{\partial p}{\partial p_j}A_{n+1,i}+\mu\frac{\partial p}{\partial p_j}A_{n+2,i}\right) \quad \text{for } j>a,$$

$$(i=1,\ldots\ldots,n+2).$$

The first term of (2.7) is the familiar one, the substitution effect in the classical Slutsky equation. (2.7) tells us that the consumer in a money economy is subject not only to the traditional substitution effect, $\lambda A_{ji}/A$, but also to the *indirect* substitution effect which arises from the fact that a change in the price of a commodity necessarily changes the price level, the price of the n+1-th good(M/p) and of the n+2-th good(m/p).

The classical model has taken into consideration only the *direct* substitution effect and neglected the indirect substitution effect by excluding money from the utility function. We can note from (2.7) that the sign of own substitution effect, $(\partial d_i/\partial p_i)_{u=\text{const.}}$, which is known to be negative in the classical model, is ambiguous because of the indirect effect. In general we can not deduce the sign of the indirect substitution effect, while its absolute value is greater, the greater the marginal effect of the commodity price on the price level.

Inserting (2.7), (2.4) and (2.3) in (2.6), we obtain

$$
\begin{aligned}
\partial d_i/\partial p_j &= (\partial d_i/\partial p_j)u=\text{const.} - \frac{\bar{M}}{y_1+\bar{M}}(d_j-s_j)\frac{\partial d_i}{\partial \bar{M}} - \frac{y_1}{y_1+\bar{M}}(d_j-s_j)\frac{\partial d_i}{\partial y_1} \\
&\quad - \frac{\partial p}{\partial p_j}\left(\frac{y_1}{y_1+\bar{M}}\frac{M}{p}\frac{\partial d_i}{\partial y_1}+\frac{m}{p}\frac{\partial d_i}{\partial y_2}\right) \quad \text{for } j\leqslant a; \\
(2.8)\qquad & \\
\partial d_i/\partial p_j &= (\partial d_i/\partial p_j)u=\text{const.} - \frac{\bar{M}}{y_1+\bar{M}}\frac{M}{p}\frac{p}{p_j}\frac{\partial d_i}{\partial \bar{M}} - (d_j-s_j)\frac{\partial d_i}{\partial y_2} \\
&\quad - \frac{\partial p}{\partial p_j}\left(\frac{y_1}{y_1+\bar{M}}\frac{M}{p}\frac{\partial d_i}{\partial y_1}+\frac{m}{p}\frac{\partial d_i}{\partial y_2}\right) \quad \text{for } j>a, \\
&\qquad\qquad (i=1,\ldots\ldots,n+2).
\end{aligned}
$$

The system (2.8) may be said to be an extension of the Slustky equation to a money economy. The second term of (2.8) can be interpreted as the real balance effect, which arises form the fact that a change in the j-th price and the price level change accompaying it are equivalent to a change in the real value of initial money balances. The last two terms of (2.8) may be called as the income effect, which arises conceptually from the fact that a change in the j-th price and the price level change accompanying it are equivalent to a change in the initial stocks of commodities at their previous prices.

Unlike the traditional model, the above results may be decomposed, as in the following table, to show that each effect consists not only of the direct effect, which was considered in the classical model (except for the real balance effect), but also of the indirect effect, which was neglected in the classical model.(6)

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(6) According to the way of interpretation Lloyd employed for his model [4], the substitution effect would be represented by the first term, $\lambda A_{ji}/A$, of (2.6), the income effect by the forth term, $-(d_j-s_j)A_{n+3,i}/A$, of (2.6) and the (derived) real balance effect by all the

		Direct effect		Indirect effect	
		$j \leqq a$	$j > a$	$j \leqq a$	$j > a$
Substitution effect		$\lambda \frac{A_{ji}}{A}$	$\mu \frac{A_{ji}}{A}$	$-\frac{\partial p}{\partial p_j} \frac{\lambda A_{n+1,i} + \mu A_{n+2,i}}{A}$	
Income effect	$i=1, \cdots$ $\cdots, a, n+1$	$-(d_j - s_j) \frac{y_1}{y_1 + \bar{M}} \frac{\partial d_i}{\partial y_1}$	0	$-\frac{\partial p}{\partial p_j} \left(\frac{y_1}{y_1 + \bar{M}} \frac{M}{p} \frac{\partial d_i}{\partial y_1} + \frac{m}{p} \frac{\partial d_i}{\partial y_1} \right)$	
	$i=a+1,$ $\cdots, n, n+2.$	0	$-(d_j - s_j) \frac{\partial d_i}{\partial y_2}$	$-\frac{\partial p}{\partial p_j} \left(\frac{y_1}{y_1 + \bar{M}} \frac{M}{p} \frac{\partial d_i}{\partial y_1} + \frac{m}{p} \frac{\partial d_i}{\partial y_2} \right)$	
Real balance effect		$-(d_j - s_j) \frac{\bar{M}}{y_1 + \bar{M}} \frac{\partial d_i}{\partial \bar{M}}$	0	$-\frac{\partial p}{\partial p_j} \frac{\bar{M}}{y_1 + \bar{M}} \frac{M}{p} \frac{\partial d_i}{\partial \bar{M}}$	

The sign of each effect appearing in the above table is, for $i=j$, ambiguous except for the direct substiution effect. Even if we assume the non-inferiority of the i-th good (including M/p and m/p), the slope of the demand curve is still undetermined since we have ambiguity about the sign of indirect subtitution effect.

Clower has claimed several propositions from the essentially same model of consumer behavior. His first proposition is that "substitution effects of changes in price are asymmetrical unless both commodities are either offered for sale or demanded for purchase."[1, p.7] This proposition is correct so far as it is concerned with the direct substitution effects. If, however, we take into consideration the indirect substitution effect, too, the proposition does not seem to be true, since, as we can see from the above table, the total substitution effect is asymmetric between any goods mainly because of the indirect effect.

Secondly, he claimed that "a change in the initial money balance has no 'income' effect on goods for sale," [*Ibid.*] This proposition does not seem to be appropriately asserted in view of (2.4), where we cannot establish

remaining terms of (2.6). He included to the real balance effect all the indirect effects derived from the price level change accompanying the j-th price change and decomposed again the real balance effect into the substitution effect and income effect which occur conceptually between the i-th commodity and money since the relative price of money has been changed. This definition of the real balance effect is not the usual one—the real balance effect is defined to measure the influence on demand of a change in the real value of the initial money holdings with other things—in our model, prices and initial real stocks of commodities—being held constant.[5, p. 21] According to this usual definition of the real balance effect, the (derived) substitution effect between the i-th commodity and money in Lloyd model cannot be included in the real balance effect and a part of the (original) income effect arising from a change in the relative price of the j-th good should be included in the real balance effect, as in our analysis.

$A_{n+3,1}=0$. Instead, we may assert from the above table that a change in the price of a commodity offered for sale has no direct real balance effect. This unfamiliar result seems to be due to the dichotomization of budget constraints in a money economy.

Clower's third proposition is that "changes in initial endowments of goods have no 'income' effect on commodities that are demanded for purchase." [*Ibid.*] This proposition seems also to be incorrectly formulated in view of (2.2), where usually $A_{n+3,i} \fallingdotseq 0$. From the above table, instead, we can see that a change in the price of goods demanded for purchase (offered for sale) has no direct income effect for those goods that are offered for sale (demanded for purchase). These are also consequences of dichotomizing budget constraints into expenditure and income branches.

Some more qualifications of the classical results(7) can be obtained from our model. First, the classical result that some of the cross-price elasticities of demand are negative is confirmed only for the goods (including M/p) demanded for purchase. (See Appendix 1.)

Secondly, we cannot confirm the established classical proposition that the sum of the compensated cross-price elasticities, over all the prices of the goods even including money, is equal to zero. (See Appendix 2.)

Thirdly, we cannot confirm the classical result that all commodities cannot be complements for each other. (See Appendix 6.)

In this section we have been interested mainly in finding those results that are cotradictory to the classical standard theory. The following results from the standard model, however, are valid also for the consumer behavior in a money economy.

1. The demand functions for commodities and for real money income (m/p) and real reservation balances(M/p) are homogeneous of degree zero in money prices and initial money balances. (See Appendix 3.)

2. The sum, over all the quantities of goods including money, of the income elasticities weighted by total expenditure proportions equals unity. (See Appendix 4.)

3. The sum, over all the quantities of goods including money, of the compensated cross-price elasticities weighted by total expenditure proportions equals zero. (See Appendix 5.)

(7) See Henderson and Quandt[2] for the classical results.

Appendix 1.

From the budget equation (1.3),

$$\sum_{i=1}^{a} p_i d_i + M = \sum_{i=1}^{a} p_i s_i + \bar{M} \quad \text{for } d_i \geqslant s_i$$

$$\sum_{i=1}^{a} p_i \frac{\partial d_i}{\partial p_j} + d_j + p \cdot \frac{\partial (M/p)}{\partial p_j} + \frac{M}{p} \frac{\partial p}{\partial p_j} = s_j \quad \text{for } j=1, \ldots\ldots, n,$$

$$\sum_{i=1}^{a} p_i \frac{p_j}{p_j} \frac{d_i}{d_i} \frac{\partial d_i}{\partial p_j} + d_j + p \frac{p_j}{p_j} \frac{M/p}{M/p} \frac{\partial (M/p)}{\partial p_j} + \frac{M}{p} \frac{\partial p}{\partial p_j} = s_j$$

$$\sum_{i=1}^{a} p_i d_i \varepsilon_{ij} + p_j d_j + p \cdot \frac{M}{p} \varepsilon_{n+1,j} + p_j \frac{M}{p} \frac{\partial p}{\partial p_j} = p_j s_j$$

$$\sum_{i=1}^{a} \alpha_i \varepsilon_{ij} + \alpha_{n+1} \varepsilon_{n+1,j} = -\frac{p_j (d_i - s_i)}{Y_1} - \frac{p_j}{Y_1} \frac{M}{p} \frac{\partial p}{\partial p_j} < 0$$

where $\alpha_1 = \frac{p_i d_i}{Y_1}$, $\alpha_{n+1} = \frac{M}{Y_1}$, $Y_1 = \sum_{i=1}^{a} p_i d_i + M$,

$\varepsilon_{ij} = \frac{p_j}{d_i} \frac{\partial d_i}{\partial p_j}$, $\varepsilon_{n+1,j} = \frac{p_j}{M} \frac{\partial M}{\partial p_j}$.

Similarly, from (1.4), we have

$$\sum_{i=a+1}^{n} \alpha_i \varepsilon_{ij} + \alpha_{n+2} \varepsilon_{n+2,j} = -\frac{p_j (d_i - s_i)}{Y_2} - \frac{p_j}{Y_2} \frac{m}{p} \frac{\partial p}{\partial p_j}, \quad d_i < s_i$$

$$j = 1, \ldots\ldots, n.$$

But we cannot establish

$$\sum_{i=a+1}^{n} \alpha_i \varepsilon_{ij} + \alpha_{n+2} \varepsilon_{n+2,j} < 0, \quad \text{since } d_i - s_i < 0.$$

Appendix 2.

$$\sum_j \frac{p_j}{x_i} \left(\frac{\partial x_i}{\partial p_j}\right)_{u=\text{const.}} = \frac{1}{x_i} \sum_j^n p_j \left(\frac{\partial x_i}{\partial p_j}\right)_{u=\text{const.}}$$

$$= \frac{1}{x_i} \sum_{j=1}^{n} p_j \left[\frac{1}{A} (\gamma A_{ji}) + \frac{\partial p}{\partial p_j} (\lambda A_{n+1,i} + \mu A_{n+2,i})\right]$$

$$= \frac{1}{x_i} \frac{1}{A} \left[\sum_{j=1}^{n} \gamma A_{ji} p_j + \frac{\partial p}{\partial p_j} (\lambda A_{n+1,i} + \mu A_{n+2,i} p_j)\right]$$

$$= \frac{1}{x_i} \frac{1}{A} \frac{\partial p}{\partial p_j} (\lambda A_{n+1,i} p_j + \mu A_{n+2,i} p_j) \neq 0 \text{ usually,}$$

where $\gamma = \lambda$ for $j \leqslant a$ and $\gamma = \mu$ for $j > a$.

Appendix 3.

$$\text{Max. } u(d_1, \ldots\ldots, d_n,\ M/p,\ m/p)$$

$$\text{subject to } \sum_{i=1}^{a} kp_i(d_i - s_i) + kM = k\bar{M}$$

$$\sum_{i=a+1}^{n} kp_i(d_i - s_i) + km = 0.$$

$$L = u(d_1, \ldots, d_n, M/p, m/p) - \lambda\left(\sum_{i=1}^{a} kp_i(d_i - s_i) + M - k\bar{M}\right)$$

$$-\mu\left(\sum_{i=a+1}^{n} kp_i(d_i - s_i) + m\right)$$

$$\frac{\partial L}{\partial d_i} = u_i - \lambda kp_i = 0,\ \ i = 1, \ldots\ldots, a\ \Rightarrow \frac{u_i}{u_a} = \frac{\lambda kp_i}{\lambda kp_a} = \frac{p_i}{p_a}$$

$$\frac{\partial L}{\partial d_i} = u_i - \mu kp_i = 0,\ \ i = a+1, \ldots, n \Rightarrow \frac{u_i}{u_n} = \frac{\mu kp_i}{ukp_n} = \frac{p_i}{p_n}$$

$$\frac{\partial L}{\partial M/P} = u_{n+1} - \lambda p = 0$$

$$\frac{\partial L}{\partial m/p} = u_{n+2} - \mu p = 0$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{a} kp_i(d_t - s_i) + kM = k\bar{M}$$

$$\frac{\partial L}{\partial \mu} = \sum_{i=a+1}^{n} kp_i(d_i - s_i) + km = 0$$

We have the same first-order conditions with (1.5) and, similarly, the same second-order conditions.

Appendix 4.

From the budget equation (1.3),

$$\sum_{i=1}^{a} p_i d_i + M = \sum_{i=1}^{a} p_i s_i + \bar{M}$$

$$\text{Let } Y_1 = \sum_{i=1}^{a} p_i s_i + \bar{M}$$

$$\sum_{i=1}^{a} p_i \frac{\partial d_i}{\partial Y_1} + p \cdot \frac{\partial (M/P)}{\partial Y_1} = 1$$

$$\sum_{i=1}^{a} p_i \frac{d_i}{d_i}\ \frac{Y_1}{Y_1}\ \frac{\partial d_i}{\partial Y_1} + p\frac{M/P}{M/P}\ \frac{Y_1}{Y_1}\ \frac{\partial (M/P)}{\partial Y_1} = 1$$

$$\sum_{i=1}^{a} \alpha_i \eta_{i1} + \alpha_{n+1}\eta_{n+1,1} = 1,$$

where $\eta_{i1}=\frac{Y_1}{d_i}\frac{\partial d_i}{\partial Y_1}$ and $\eta_{n+1,1}=\frac{Y_1}{M/P}\frac{\partial (M/P)}{\partial Y_1}$.

Similarly, from (1.4),

$$\sum_{i=a+1}^{n}\alpha_i\eta_{i,2}+\alpha_{n+2,2}\eta_{n+2,2}=1,$$

where $\eta_{i2}=\frac{Y_2}{d_i}\frac{\partial d_i}{\partial Y_2}$, $\eta_{n+2,2}=\frac{Y_2}{m/P}\frac{\partial (m/P)}{\partial Y_2}$ and $Y_2=\sum_{i=a+1}^{n}p_is_i$.

If we consolidate the budget equations, we have

$$\sum_{i=1}^{n}\alpha_i\eta_i+\alpha_{n+1}\eta_2+\alpha_{n+2}\eta_2=1,$$

where $\alpha_i=\frac{p_id_i}{\sum_{i=1}^{n}p_id_i+M+m}$, $\eta_i=\frac{Y}{d_i}\frac{\partial d_i}{\partial Y}$, $i=1,\ldots\ldots,n+2;$

and $Y=Y_1+Y_2$.

Appendix 5.

$$u(d_1,\ldots\ldots,d_n,\ M/P,m/p)=u^0$$

$$\sum_{i=1}^{n}u_i\frac{\partial d_i}{\partial p_j}+u_{n+1}\frac{\partial (M/P)}{\partial p_j}+u_{n+2}\frac{\partial (m/p)}{\partial p_j}=0$$

$$\sum_{i=1}^{n}p_i\frac{\partial d_i}{\partial p_j}+p\cdot\frac{\partial (M/P)}{\partial p_j}+p\cdot\frac{\partial (m/p)}{\partial p_j}=0$$

(by the first-order condition.)

$$\sum_{i=1}^{n}p_i\frac{d_i}{d_i}p_j\frac{\partial d_i}{\partial p_i}+p\frac{M/P}{M/P}p_j\frac{\partial (M/P)}{\partial p_j}+p\cdot\frac{m/p}{m/p}p_j\frac{\partial (m/p)}{\partial p_j}=0$$

$$\sum_{i=1}^{n}\alpha_i\ \xi_{ij}+\alpha_{n+1}\ \xi_{n+1,j}+\alpha_{n+2}\xi_{n+2,j}=0,\ \ j=1,\ldots\ldots,n,$$

where $\alpha_i=\frac{p_id_i}{\sum_{i=1}^{n}p_id_i+M+m}$ and $\xi_{ij}=\frac{p_i}{d_i}\frac{\partial d_i}{\partial p_j}$

Appendix 6.

$$\varepsilon_{ij}=\xi_{ij}-\alpha_j\eta_i\quad \sum_j\varepsilon_{ij}=\sum_j\xi_{ij}-\eta_i\Sigma\alpha_j$$

But we cannot have $\sum_{j=1}^{n}\varepsilon_{ij}=-\eta_i$ since $\sum_j\xi_{ij}\neq 0$ usually.

Thus we cannot establish $\sum_j p_is_{ij}=0$.

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