Estimating Needs and Demand for Housing in Developing Countries by Simple Models:

With an Application for Korea

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I. Introduction

This paper presents some simple models for estimating the *needs* and *demand* for housing, in particular in developing countries, taking account of a limited availability of data and applies these models to empirical data for the Third Five Year Plan Period of the Republic of Korea, 1972～1976. Section II gives an estimate of needs at the national level, based on estimates of the growth in the number of households, of the future need of replacing part of the housing stock, and of the insufficiency in existing housing provisions to be overcome in a certain period of time. Section III gives the same estimate but now distinguishing between urban and rural areas and taking account of rural-urban migration. Section IV disaggregates the housing needs according to income classes. A projection of the future effective demand for houses is made in Section V

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taking account of the distribution of income and its change over time. In Section VI all estimates are compared among themselves and to the Korean Plan figures over the same period, and some conclusions are drawn. It should be emphasized that the calculations indicate the dimension of the housing problem, but are only a tiny contribution to the task of solving it.

In the remainder of this section some arguments are briefly summarized why in addition to future demand also future needs for housing are estimated.(1) In general, economists are accustomed to think in terms of demand rather than in terms of needs for (private) goods, under the assumptions that needs express themselves as demand via the willingness to pay for the goods and that a free market mechanism will provide for supply meeting these needs expressed in demand. There are, however, a few doubts possible about the validity of these assumptions:

(1) In many developing countries the income distribution is rather skew. At the low income tail of these distributions conditions of poverty prevail at which people have to exist below a subsistence level. In such a situation there is ample reason to consider to which degree certain basic human needs, such as those for housing, cannot express themselves sufficiently in demand because of too low a income and thus remain unsatisfied.

(2) The phenomenon of rapid urbanization constitutes a special factor in the increased need for housing, alongside the growth of population. There is reason to suppose an insufficiently rapid reaction to this phenomenon by the forces which some decades ago determined the provision of housing. In so far as the urbanization process itself cannot be decelarated or stopped, a conscious overall policy, also with respect to housing, appears necessary. The assessment of the quantitative aspects of such a policy can begin best by a direct assessment of needs.

(3) In welfare economics characteristics of certain goods, production processes and market forms are discussed, mostly under the title of externalities, which make the outcome of pure market processes deviate from what would be desirable or efficient, even apart from income distribution considerations (so-called Pareto optimality), see e.g. Bator [1], Chakravarty [4], Lin [11]. Such externalities appear to occur in the area of housing, see a.o. Wegelin [15].

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(1) A more extensive discussion of these arguments and of urbanization in developing countries can be found in De Kruijk and Waardenburg [8].

II. Estimation of Housing Needs at the Level of National Aggregates: HNM 1

This section presents the housing needs model 1 (HNM 1), which is a simple model to be extended in later sections. HNM 1 estimates the needs for the extension of housing during a medium-term planning period at the level of national aggregates, investigates the relative importance of the determining factors and evaluates the actual house-building program.

1. The Model: HNM 1

The model consists of seven equations [2] without interdependencies. "Need" is used for the yearly flow of housing units to be built, "gap" indicates the insufficiency (or surplus) in the housing stock, based on comparing the actual (ε_t) and a normative (ε^*) housing units/households ratio (see (2.8)). Central to the model is the translation of this 'gap' into a flow, which is part of the needs, by specifyig a period (Z years) and a time path for abolishing this gap (see (2.7) and Appendix 4).

The population (P_t in year t) increases by an exogenously determined growth rate (α_t), which may gradually change over time reflecting possible changes in death and birth rates:

$$P_t=(1+\alpha_t)\ P_{t-1}. \qquad (2.1)$$

The number of households (H_t in year t) is as a first approximation proportional to the population, but the proportionality factor (the inverse of the average household size (δ_t) may gradually change over time under the influence of sociological factors:

$$H_t=\frac{1}{\delta_t}P_t. \qquad (2.2)$$

Total annual housing needs (n_t) consist of three flows: (a) needs due to the annual increase in the number of households (n_{h_t}), (b) replacement needs (n_{d_t}), and (c) needs arising from abolishing the housing gap (n_{g_t}):

$$n_t=n_{h_t}+n_{d_t}+n_{g_t}. \qquad (2.3)$$

The needs in year t corresponding to the increase in households in year t is

(2) Flow variables are indicated by small roman types, stock variables by capital roman types t_0: base year, T: number of the planning period, t runs from t_0+1 to t_0+T.

proportional to the latter, the proportionality factor being the housing units/households ratio (ε_{t_0}) in the base year:

$$n_{h_t}=\varepsilon_{t_0}\ (H_t-H_{t-1}). \tag{2.4}$$

The needs for replacement is as a first approximation assumed to be proportional to the existing housing stock in the base year ($\varepsilon_{t_0}H_{t_0}$), the proportionality factor being the average drop out rate (λ) of the existing housing units (approximately constant):

$$n_{d_t}=\lambda\varepsilon_{t_0}H_{t_0}. \tag{2.5}$$

The expression for the needs arising from abolishing the housing gap rests on two assumptions, viz. (a) that it is possible to determine a minimum norm (ε^*) for an acceptable or satisfactory housing units/households ratio, and (b) that it is possible to specify a time path of yearly norms (ε^*_t) for the housing units/households ratio, which leads from ε_{t_0} to ε^*. This gives for each year a normative stock of houses needed ($\varepsilon^*_tH_t$). In deriving n_{g_t} from its yearly differences n_{h_t} should be deducted:

$$n_{g_t}=\varepsilon^*_tH_t-\varepsilon^*_{t-1}\ H_{t-1}-\varepsilon_{t_0}\ (H_t-H_{t-1}). \tag{2.6}$$

We refashion (2.6) to be able to impute n_{g_t} to increases in H_t and ε^*_t:

$$n_{g_t}=(\varepsilon^*_t-\varepsilon^*_{t-1})\ H_{t_0}+(\varepsilon^*_{t-1}-\varepsilon_{t_0})\ (H_t-H_{t-1})+(\varepsilon^*_t-\varepsilon^*_{t-1})\ (H_{t-1}-H_{t_0})$$
$$+(\varepsilon^*_t-\varepsilon^*_{t-1})\ (H_t-H_{t-1}). \tag{2.6'}$$

Here the first term is only the result of the increase in the norm ε^*_t in year t, while the other terms are cross-effects of the increases in the yearly norm and in the households, viz. the second term of the "past" increase in the norm and the "actual" increase in households, the third term of the "past" increase in households and the "actual" increase in the norm, and the fourth term of the "actual" increases in households and in the norm. Later on, when we will impute part of the total needs to the increase in households and to the increase in the norm (or to the gap bridging activity) respectively, we will add the second term to (2.4) and take the first and third term of (2.6′) together respectively, distributing the fourth term of (2.6′) proportionately over these two categories.

For the specification of the time path of ε^*_t we choose an arithmetical series for the period t_0 until t_0+Z inclusive, during which Z years the norm ε^* is to be reached:[3]

$$\varepsilon^*_t = (t-t_0)\frac{(\varepsilon^* - \varepsilon_{t_0})}{Z} + \varepsilon_{t_0}. \tag{2.7}$$

Some comments on the model are in order. The basic units in which we formulate the main variables of the model are housing units and not e.g. rooms, living space or value of the housing accomodations. Disadvantages of this choice are that housing units may be very different in quality and in size, and accordingly require different inputs if their supply is looked into. Advantages are that housing units can be easily connected with households (implicitly accounting for size differences), that they are empirically directly observable and that statistical material can be collected straightforwardly in terms of there units, while it is not impossible to formulate certain minimum standards for judging whether they are of acceptable quality.

In the model two explicit general norms are expressed, ε^* and Z which are quantitative in nature. Via the time path assumption yearly norms ε^*_t are derived from them. The qualitative norm which housing units are of acceptable quality is only implicitly expressed in the measurement or estimation of ε_{t_0} (or $\varepsilon_{t_0}H_{t_0}$), i.e. housing units below a certain standard (which may be very low) are simply treated as not existing in the model.

Equation (2.5) would be correct if: (a) in year t_0 the housing stock would be distributed evenly over building years, (b) the time profile of drop out rates would be non-stochastic and the same for each stock of a building year, and (c) the drop out rate for future building years would equal 0 within the planning period considered. If (a) or (b) are violated, λ could be made time-dependent (λ_t); if (c) is violated building activities should appear in (2.5), making n_{d_t} clearly endogenous in the model.[4]

The adding up of the components of n_t in (2.3) does not only imply that housing units are not differentiated according to quality and size, but also that needs arising from different sources are treated in the same way. When consider-

(3) One could make a different assumption, less simple than that of an arithmetical series, about the time path of ε^*_t, or one in terms of the time path of n_{g_t} or n_t, but then the mathematical formulation of the exercises with the model becomes more complicated or the needs in year t become dependent on demographic assumptions beyond the planning period, if Z is larger than it. In Appendix 4 some further discussion around the specification of the time path for reaching ε^* is given.

(4) In view of this latter possibility we will count n_{d_t} as an endogenous variable.

ing the provision of housing units to satisfy these needs (which is not done in this paper), a differentiation of the existing housing stock as well as of the newly arising needs will be useful, as upgrading below standard housing units and renovating obsolete housing units may be, as far as possible, more attractive than building only new ones.

2. Solutions of the Model in Policy and in Analytical Form

The distinction between the analytical form and policy form of a model is essential in the discussion of its solutions.(5) In the *policy form* Z is the *planned target* (now written $\bar{Z}$) given the norm ε^*, i.e. the number of years after the year t_0 in which the housing gap

$$N_{g_t}=(\varepsilon^*-\varepsilon_t)H_t \tag{2.8}$$

is to be reduced to 0 (ε_t being the actual housing units/households ratio in year t). Then n_t is to be determined as a function of $\bar{Z}$. The variables in this form of the model can be classified as follows:

exogenously determined variables:

Z(now written $\bar{Z}$), H_{t_0}, P_{t_0}, ε^*;

lagged endogenous variables (given at time period t):

P_{t-1}, H_{t-1};

endogenous variables:

P_t, H_t, n_t, n_{h_t}, n_{d_t}, n_{g_t}, ε^*_t.

The model is determinate, having seven endogenous variables in seven independent equations for each year. The variable n_t in which we are after all interested representing the total annual housing needs during the plan period, can be expressed in the exogenously given and the lagged endogenous variables by using (2.3)～(2.6), and eliminating H_t by (2.1)～(2.2) and ε^*_t by (2.7):

$$\begin{aligned} n_t=&\underset{(1)}{\varepsilon^*_{t-1}\alpha_t H_{t-1}}+\underset{(2)}{\varepsilon^*_{t-1}\gamma_t H_{t-1}}+\underset{(3)}{\lambda\varepsilon_{t_0}H_{t_0}}+\underset{(4)}{\frac{(\varepsilon^*-\varepsilon_{t_0})}{\bar{Z}}H_{t-1}} \\ &+\underset{(5)}{\varepsilon^*_{t-1}\alpha_t\gamma_t H_{t-1}}+\underset{(6)}{\frac{(\varepsilon^*-\varepsilon_{t_0})}{\bar{Z}}(\alpha_t+\gamma_t+\alpha_t\gamma_t)H_{t-1}}. \end{aligned} \tag{2.9}$$

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(5) In the analytical form of a model certain instrument variables are exogenously given and the resulting values for other variables, among which aim variables, can be calculated; in the policy form of a model the values of certain aim variables are given as targets, and the necessary or implied values of instrument variables are calculated (cf. Tinbergen[13]).

in which

$$\gamma_t = \frac{1/\delta_t - 1/\delta_{t-1}}{1/\delta_{t-1}}$$

is the annual growth rate of households due to the decrease in household size, and (2. 7) determines ε^*_{t-1} The six terms in this equation represent the annual needs due to population increase (1), smaller households (2), obsolescence (3), abolishing part of the housing gap during year t (4), and two small cross terms (5) and (6) respectively of the effects (1) and (2) and of the effects (1), (2) and (4) respectively, which in the numerical calculations will be distributed over the terms concerned proportionally to the values of these terms.

In the *analytical form* the problem is reversed: now the n_t is given exogenously (written $\bar{n}_t$) as the number of houses having been built or to be built(e.g. according to the plan) in year t, ε^*_t becomes ε_t and Z (written Z_t and interpreted as an evaluator of $\bar{n}_t$) is calculated endogenously from its definition at the number of years in which the gap N_{g_t} becomes 0 if ε_t would still belong to an arithmetical time path like in (2. 7). So (2. 7) is replaced by:

$$Z_t = (\varepsilon^* - \varepsilon_{t_0})/(\varepsilon_t - \varepsilon_{t-1}) \qquad (2.7')$$

As in year t already some years may have elapsed since t_o, which makes the definition of Z_t somewhat artificial, and as Z_t might be different for each year t, it is most meaningful to define a $Z_{1 \dots t}$ analogously to Z_t, but now as an aveage for a rnumber of years (T), viz. to evaluate the housing program $(\sum_{t=t_0}^{t_0+T} \bar{n}_t)$ for a planning period $t_0+1, \cdots, t_0+T$ in terms of Z_1

$$Z_{1 \cdot \cdot T} = \frac{(\varepsilon^* - \varepsilon_{t_0})}{(\varepsilon_{t_0+T} - \varepsilon_{t_0})/T} = \frac{(\varepsilon^* - \varepsilon_{t_0})\ H_{t_0+T} T}{\sum_{t=t_0+1}^{t_0+T} \bar{n}_t - \varepsilon_{t_0}\ (H_{t_0+T} - H_{t_0}) - \lambda \varepsilon_{t_0} H_{t_0} T} \qquad (2.10)^{(6)}$$

The first = holds by definition, the second = follows from adding up (2. 1)～(2. 6) for the years $t_0+1, \dots, t_0+T$ and solving $\varepsilon^*_{t_0+T}$ from the resulting equation.

A conclusion about the course of a present housing shortage or surplus over time can be drawn only with regard to the combined signs of $Z_{1 \dots T}$ and $(\varepsilon^* - \varepsilon_o)$ (See Appendix 1).

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(6) Notice that (2. 10) is "scale free" with respect to the measurement of the housing units, i.e. a different measure, if always proportional to that of a housing unit, would give the same result. However, e.g. living space, rooms or value need not be always proportional to housing units as a measure.

A second parameter can be calculated and used for assessing the actual housing production program ($\sum n_t$), viz. the housing units/households ratio resulting from the program at the end of the planning period (ε_{t_0+T}) compared to this ratio just before the planning period (ε_{t_0}), e.g.:

$$\tilde{\Delta}\varepsilon_T = \frac{\varepsilon_{t_0+T} - \varepsilon_{t_0}}{\varepsilon_{t_0}}$$

which indicates the relative change in the housing units/households ratio. It is zero for no change, positive for an improvement and negative for a deterioration of the housing situation.

3. Empirical Application of HNM 1 to the Republic of Korea, 1972～1976

(1) Data

First we use HNM 1 as a policy model, and apply it to figures of the Republic of Korea for its Third Five Year Plan Period 1972～1976.(7) Now $\bar{Z}$ is a policy variable to be determined exogenously; further we need data about the variables P_{t_0} and H_{t_0} and for calculating the coefficients α_t, δ_t, ε_0 and λ, and we must choose ε^*.

(A) policy data: $\bar{Z}$ and ε^*

We assume $\bar{Z}=35$ and $\varepsilon^*=1$, which implies that the number of housing units should equal the number of households in 35 years from year t_0, t_0 being 1970.

(B) demographic data: P_{t_0}, H_{t_0}, α_t and δ_t (or γ_t)

The changes of α_t and δ_t per year are assumed constant. Explicitly, we estimate

$$\alpha_t = 0.021 - 0.001\ (t - t_0),$$

and

$$\delta_t = 5.37 - 0.04\ (t - t_0) \longrightarrow \gamma_t = 0.00744 + 0.00006\ (t - t_0).$$

Moreover we use $H_{1970} = 5,920 \times 10^3$ and $P_{1970} = 31,793 \times 10^3$ (for the underlying data see Appendix 2).

(C) housing data: ε_0, λ

The total number of housing units (including unlicensed houses) in 1970 amounted to $4,443 \times 10^3$, while the total number of households was $5,920 \times 10^3$,

(7) As most data are given for 1970, we use 1970 as the base year, but present and compare the results only for 1972～1976.

so $\varepsilon_0=0.75$ (Yonsei University [16]).

No figures about the drop out rate of housing units are available. So we have to make a rather arbitrary assumption about it, viz. that it is 0.02 yearly: $\lambda=0.02$.

Unfortunately some results are rather sensitive for this choice, as will be discussed later on.

(2) Solution of the policy version

Using these data in (2.9) and taking the sum of n_t for the years 1972~1976 the total housing needs during the plan period are obtained:

$$\sum_{t=1972}^{1976} n_t = 1,290\times10^3 \text{ (see Table 1 in Section IV).}$$

The relative importance of the determining factors in total housing needs can be calculated from equation (2.9). This results in the following percentage contributions: (1) population increase=33 p.c., (2) smaller households=15 p.c., (3) replacements=34 p.s., (4) gap bridging=18 p.c.[(8)]

(3) Comparison of the results to the figures of the Plan: the analytical version

According to the Third Five Year Plan (Government of the Republic of Korea [12]) 800×10^3 houses will be built in the years 1971~1976, which means about $\frac{5}{6}\times800\times10^3=667\times10^3$ new houses during the plan period 1972~1976.

Now the model can be used in its analytical form, in which n_t is exogenous and $Z_{2\ldots T}$ is endogenous, and $Z_{2\ldots T}$ and $\tilde{\Delta}\varepsilon_T$(see Section II. 2) can be calculated as evaluating parameters (see for the results Table 1, first line).[(9)]

$Z_{2\ldots T}<0$ according to (2.10) while $\varepsilon^*>\varepsilon_0$, so the housing gap increases (cf. Appendix 1). In this case $\tilde{\Delta}\varepsilon_T$ is a more meaningful parameter, having the value $\tilde{\Delta}\varepsilon_T=-0.09$, as $\varepsilon_T=\varepsilon_{1976}=0.68$. Meanwhile the housing gap n_{g_t} at the norm $\varepsilon^*=1$ increases from $1,477\times10^3$ in 1970 to $2,167\times10^3$ in 1976 (see (2.8)).

While in the South Korean case annual housing needs according to HNM 1

(8) If λ would be 50 per cent lower (or higher) then the percentages of the factors (1)~(4) would become 36, 17, 26 and 20 (or 28, 14, 44 and 15). Population increase and replacements remain to account for the bulk of the housing needs during the plan period.

(9) As we assume $\bar{n}_{1971}=\frac{1}{5}\left(\sum_{t=1972}^{1976}\bar{n}_t\right)$, leaving out the year 1971 and calculating $Z_{2\ldots T}$ instead of $Z_{1\ldots T}$ according to (2.10) makes hardly any difference: only the growth of the households in 1971 is slightly different from the corresponding average during 1972~1976.

are twice the actual number of houses planned to be built, which gives a dim outlook for the housing situation, the program builds ca. 3.5~4 houses per 10^3 inhabitants per year, a figure well above that for most developing countries. (The U.N. Second Development Decade aims at an annual production of 10 houses per 10^3 inhabitants for developing countries, United Nations [14].)

III. Estimation of Housing Needs Distinguished into Urban and Rural Components: HNM 2

1. Introducton

This section follows the same program and uses as much as possible the same data as in the former section, but only after a distinction between rural and urban (>50,000 inhabitants per municipality) areas has been made. This distinction is introduced for the following reasons: (a) to analyse *where* geographically the housing needs arise, (b) to introduce explicitly rural-urban migration into the model, and (c) to investigate the quantitative importance of the migration factor for total needs as compared to that of the four factors dealt with already (natural population growth, decrease in household size, replacement, and the gap bridging activity).

2. The Model and Its Use

In principle the same model and notation has been used as in the former section. The variables and coefficients get an additional subscript u or r referring to urban or rural areas, while the equations (2.2)~(2.6) repeat themselves simply twofold, in addition to (2.1) two equations appear:

$$P_t = P_{ut} + P_{rt} \qquad (3.1)$$

$$P_{ut} = (1 + \alpha_t + \beta_t)\ P_{ut-1}. \qquad (3.2)$$

The latter simply describes the assumptions that urban population grows due to a combination of natural growth(10) and of rural-urban migration, the effect of the latter being described by an independent yearly migration based growth rate β_t. Moreover, we maintain (2.7) for rural areas, but derive $\varepsilon^*_{u_t}$ in the policy form from the condition that the weighted average of $\varepsilon^*_{u_t}$ and $\varepsilon^*_{r_t}$ should remain equal to ε^*_t of HNM 1:

$$(H_{rt}\ \varepsilon^*_{r_t} + H_{ut}\ \varepsilon^*_{u_t})/(H_{u_t} + H_{r_t}) = \varepsilon^*_t = (t - t_0)\ \frac{(\varepsilon^* - \varepsilon_{t_0})}{Z} + \varepsilon_{t_0} \qquad (3.3)$$

(10) Due to a lack of relevant information this figure is taken equal to the national average one.

in which $\varepsilon^*=\varepsilon^*_r=\varepsilon^*_u$ and $Z=\bar{Z}_r=\bar{Z}_u$.

In the analytic form we maintain (2.7) both for rural and urban areas (for the complete set of equations see Appendix 3).

Again we can distingiush a policy form and an analytical form of the model HNM 2, each containing 15 variables in 15 linearly independent equations per year. In the policy form Z_r and Z_u are exogenously given targets and written $\bar{Z}_r$ and $\bar{Z}_u$, and n_{ut} and n_{rt} (urban and rural housing needs, respectively) can be expressed as functions of these targets and of other exogenous or lagged endogenous variables, like in deriving (2.9):

$$\begin{aligned} n_{ut} \approx\ & \underset{(1)}{\varepsilon^*_{ut-1}\alpha_t H_{ut-1}} + \underset{(2)}{\varepsilon^*_{ut-1}\beta_t \frac{P_{ut-1}}{P_{rt-1}} H_{rt-1}} + \underset{(3)}{\varepsilon^*_{ut-1}\gamma_{ut} H_{ut-1}} + \underset{(4)}{\lambda_u \varepsilon_{ut_0} H_{ut_0}} \\ & + \underset{(5)}{\frac{\varepsilon^*_u - \varepsilon_{ut_0}}{\bar{Z}_u} H_{ut-1}} + \underset{(6)}{\varepsilon^*_{ut-1}\{\gamma_{ut}(\alpha_t+\beta_t)H_{ut-1} + \beta_t(H_{ut-1} - \frac{P_{ut-1}}{P_{rt-1}} H_{rt-1})\}} \\ & + \underset{(7)}{\frac{\varepsilon^*_u - \varepsilon_{ut_0}}{\bar{Z}_u} (\alpha_t+\beta_t+\gamma_{ut}+\alpha_t\gamma_{ut}+\beta_t\gamma_{ut})\ H_{ut-1}}\,^{(11)} \end{aligned} \qquad (3.4)$$

in which

$$\gamma_{ut} = \frac{1/\delta_{ut} - 1/\delta_{ut-1}}{1/\delta_{ut-1}};$$

$$\begin{aligned} n_{rt} =\ & \underset{(1)}{\varepsilon^*_{rt-1}\ \alpha_t H_{rt-1}} - \underset{(2)}{\varepsilon^*_{rt-1}\ \beta_t \frac{P_{ut-1}}{P_{rt-1}}\ H_{rt-1}} + \underset{(3)}{\varepsilon^*_{rt-1}\gamma_{rt} H_{rt-1}} + \underset{(4)}{\lambda_r \varepsilon_{rt_0} H_{rt_0}} + \underset{(5)}{\frac{\varepsilon^*_r - \varepsilon_{rt_0}}{\bar{Z}_r} H_{rt-1}} \\ & + \underset{(6)}{\varepsilon^*_{rt-1}\ \gamma_{rt}(\alpha_t - \beta_t \frac{P_{ut-1}}{P_{rt-1}}) H_{rt-1}} + \underset{(7)}{\frac{\varepsilon^*_r - \varepsilon_{rt_0}}{\bar{Z}_r} \left\{(1+\gamma_{rt})\left(\alpha_t - \beta_t\ \frac{P_{ut-1}}{P_{rt-1}}\right) + \gamma_{rt}\right\}} H_{rt-1} \end{aligned} \qquad (3.5)$$

in which

$$\gamma_{rt} = \frac{1/\delta_{rt} - 1/\delta_{rt-1}}{1/\delta_{rt-1}}.$$

N.B. In these formulae $\beta_t \frac{P_{ut-1}}{P_{rt-1}}$ is the migration rate of the rural population. We use in the model β_t related to the urban population, as our data are in terms of this β_t.

These functions are split up into seven or eight components now corresponding to the factors: (1) natural population increase, (2) migration (negative for rural areas), (3) smaller households, (4) replacement, (5) gap bridging, (6)

(11) For the discussion of a small correction term due to the linearization of the terms (5) and (7) see Appendix 4.

and (7) small cross terms which in the numerical exercises are distributed proportionally over the terms (1), (2), (3) and (1), (2), (3), (5), (6), respectively, taking account of the negative migration term in (3.5).

With respect to the analytical form one could construct expressions for Z_u and Z_r analogous to equation (2.10).

3. Empirical Application of HNM 2 to the Republic of Korea, 1972～1976

For the empirical application of HNM 2 the same sources are used as for HNM 1 plus other Korean Statistical Yearbooks. For 1970 the urbanization rate(= share of urban population in total population) is 0.4325, which figure allows to split up P_{t_0}. H_{t_0} is found by using $\delta_{u_0}=5.12$ and $\delta_{r_0}=5.57$.[12] The annual percentage decrease in the average household size is assumed to be identical in urban and rural areas such that the national average remains as in Section II. 3. Moreover $\varepsilon_{r_0}=0.90$ and $\varepsilon_{u_0}=0.56$[12] and we assume $\lambda_u=\lambda_r=0.02$. The critical parameter β_t, which is the difference between the annual and the natural growth rate of the urban population, is taken constant and estimated at 0.057 and 0.050, alternatively.[13] As said before $\alpha_t=0.021-0.001(t-t_0)$ like in Section II. 3.

For the *policy form* we assume again $\bar{Z}_u=\bar{Z}_r=35$ and $\varepsilon^*=1$. Total urban housing needs during 1972～1976 appear to be $1,344\times10^3$ if $\beta=0.050$ and $1,466\times10^3$ if $\beta=0.057$, which equal total needs since rural housing needs are negative, while rural houses cannot be transferred to urban areas.

The building program is *analysed* under the condition that the total program is executed in the cities. Notwithstanding this condition the figures show that the urban housing gap increases, whereas the rural housing gap will be abolished in 1978 or 1990[14] without building any house. The results are indicated in Table 1 of Section IV.

IV. Estimation of Urban Housing Needs Distinguished According to Income Classes: HNM 3

1. Introduction

This section pursues the idea of disaggregation. The model HNM 3 distin-

(12) See Yonsei University [16].

(13) Based on the figures $\beta_{1955\sim60}=0.0250$, $\beta_{1960\sim65}=0.0304$ and $\beta_{1966\sim70}=0.057$ (Yonsei University [16]); Stabilization on this last level implies already in 1976 an urbanization rate of 0.60.

(14) Depending on the alternative values of the migration rate β, and provided that β also keeps its value after the planning period.

guishes also income classes and is applied only to the urban situation since the housing problem appears to exist only or primarily(15) in these areas. Data are needed now with regard to the number of households per income class in the base year, but also a method for projection of these numbers during the period considered. A method for such a projection is developed and described in De Kruijk [6] and more extensively in De Kruijk [7]. The basic idea is to approximate the existing income distribution by a theoretical standard one (e.g. the lognormal distribution), estimating the future values of the parameters of the distribution function and applying the existing deviations from this standard distribution to the projection of the latter. The result of these calculations, which take also account of migration, is indicated by $X^i_{u_t}$, the number of urban households per income class i in year t.

The application of this model HNM 3 requires, furthermore, quite a number of data which are not given in the statistical sources, but which can be determined by reasonable assumptions consistent with the available data used in the former sections.

2. The Model and Its Use

The model is of the same type as HNM 1 and HNM 2, but the variables, equations and H_{u_t} and $\varepsilon_{u_{t_0}}$ are additionally specified for the income classes i ($i = 1, \cdots, 7$). We give the equations only for the urban part, the meaning of the symbols and equations being as before:

$$H^i_{u_t} = X^i_{u_t} \tag{4.1}$$

$$n^i_{u_t} = n^i_{u_{ht}} + n^i_{u_{dt}} + n^i_{u_{rt}} \tag{4.2}$$

$$n^i_{u_{ht}} = \varepsilon^i_{u_{t_0}}(H^i_{u_t} - H^i_{u_{t-1}}) \tag{4.3}$$

$$n^i_{u_{dt}} = \varepsilon^i_{u_{t_0}} \lambda_u H^i_{u_{t_0}} \tag{4.4}$$

$$n^i_{u_{rt}} = \varepsilon^{*i}_{u_t} H^i_{u_t} - \varepsilon^{i*}_{u_{t-1}} H^i_{u_{t-1}} - \varepsilon^i_{u_{t_0}}(H^i_{u_t} - H^i_{u_{t-1}}) \tag{4.5}$$

$$\varepsilon^{*i}_{u_t} = \varepsilon^* - (\varepsilon^* - \varepsilon^{*i}_{u_t}) \frac{(\varepsilon^* - \varepsilon^*_{u_t}) \sum_{i=1}^{7} H^i_{u_t}}{\sum_{i=1}^{7} H^i_{u_t} \{\varepsilon^* - (t - t_0) \frac{(\varepsilon^* - \varepsilon^i_{u_{t_0}})}{Z_u} - \varepsilon^i_{u_{t_0}}\}}. \tag{4.6}$$

This formula is derived by first defining preliminarily the $\varepsilon^{*i}_{u_t}$'s like in (2.7) and correcting them by multiplying their differences with ε^* by a factor a_t, so as to make their weighted average equal to $\varepsilon^*_{u_t}$ in HNM 2. The formula applies

(15) Even if the total needs in the rural areas are negative, certain income groups may display positive needs, however.

only in the policy form, in which $X^i_{u_t}$ and $\bar{Z}_u(=\bar{Z}^i_u)$ are exogenous.

In the analytical form $n^i_{u_t}$ would become exogenous ($\bar{n}^i_{u_t}$) and (4. 6) could be replaced by equations relating $Z^i_{u_t}$(now endogenous and potentially different per income class, while the $X^i_{u_t}$ have remained exogenous) to $\varepsilon^i_{u_t}$ similar to (A. 15′). As the Z^i_u depends critically on the $\bar{n}^i_{u_t}$, for which on the basis of the available data no reasonable assumption with respect to its distribution over income classes can be made, we calculate in this paper only an aggregate $Z_{u_1 \cdots T}$ from an aggregate $\varepsilon_{u_{t_0}+T}$, which amounts to the same calculation as in HNM 2.

3. Empirical Application of HNM 3 to the Republic of Korea, 1972～1976

As for the data, α_i and β_i and λ_u are assumed to be the same for all income classes having the values as in the former sections. Taking 10 p.c. for the urban growth of income (expenditure) per household over 1972～76 and assuming a constant relative income distribution (which has been the case during 1964～70, Chenery *et al.* [5]) suffice to calculate the $X^i_{u_t}$. Finally, for the seven income classes i different values for $\varepsilon^i_{u_0}$ are assumed, viz. 0.42/0.50/0.55/0.60/0.65/0.70/1, consistent with the average $\varepsilon_{u_0}=0.56$. In the calculations of total needs it could be assumed that negative needs for one income class cannot be used to supply to (positive) needs of other income classes, the sum of the latter needs therefore representing total needs.(16) The results of the calculations are summarized in Table 1 and Table 2 of Section V.

4. Some Conclusions from the Calculations with the Three Models

These figures give the following indications:

(1) Disaggregation into urban and rural areas and into income classes throws considerable light on where and in which income classes housing needs are most prominent.

(2) The housing problem exists mainly in the urban area.

(3) The relative importance of the needs due to migration in the city, being near to 40 p.c., is by far the largest compared to other factors in the urban area. Moreover, in the increased gap bridging activity in HNM 2 as compared to HNM 1 migration exerts also some influence.

(4) The sensitivity of the total needs for β is high above a certain critical value of β, due to the impossibility to transfer negative rural needs to urban areas.

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(16) With the given data for the urban area this assumption need not be used in the calculations.

Table 1. Housing needs ($\times 10^3$) induced by various factors during the plan period 1972~76 and the evaluating parameters of the house building program during 1972~76 according to alternative values of the rural-urban migration rate (β), Republic of Korea

type of model	area	policy model: housing needs due to: increase of the number of households due to: natural population increase (1)	migration (2)	smaller households (3)	replacements (4)	gap bridging (5)	total housing needs (6)	number of years in which the housing gap is abolished ($z_{1..T}$) (7)	analytical model: housing units/households ratio (ε_t) ε_{1970} (8)	ε_{1976} (9)	relative change of ε during 1971~76 ($\Delta\varepsilon_T$) (10)
HNM 1	whole country	421	—	195	444	230	1290	negative	0.75	0.68	−0.09
HNM 2 (β= 0.05)	urban	183	517	118	151	375	1344	negative	0.56	0.52	−0.07
	rural	238	−705	77	293	43	negative (−54)	20	0.90	0.93	0.03
HNM 2 (β= 0.057)	urban	187	608	120	151	400	1466	negative	0.56	0.50	−0.11
	rural	234	−818	75	293	40	negative (−176)	8	0.90	0.98	0.09
HNM 3 (β= 0.05)	urban	183	517	118	151	375	1344	negative	0.56	0.52	−0.07
HNM 3 (β= 0.057)	urban	187	608	120	151	400	1466	negative	0.56	0.50	−0.11

(5) The decline of the urban housing units/households ratio by 9~12 per cent during a six year period implies a dim outlook for the urban housing situation.

(6) Additional refinements -e.g. into counties, cities or quarters -will lead undoubtedly to a further increase of total housing needs if one of these entities appears to end up with a surplus of houses, as far as such a surplus cannot be used for relieving deficits in other entities.

V. Projection of the Increase in Effective Demand for Housing in the Republic of Korea, 1972~1976

After having estimated the needs for new housing units during the plan period, in this section the increase in effective demand for additional urban housing units is estimated based on the additional purchasing power of the households in the different income classes under the assumption of no relative

Table 2. Projected income (expenditure) distribution and the resulting increase in effective demand for housing in urban areas, Republic of Korea, 1972～1979, compared to the needs for housing

monthly consumption expenditure range in 10^3 won	1971 actual number of households[(1)] $\times 10^3$ (1)	1971 theoretical number of households[(2)] $\times 10^3$ (2)	1971 rent per household in won (3)	1971 assumed houses/households ratio ($\varepsilon'_{u_{t0}}$) (4)	1971 normative houses/households ratio ($\varepsilon^*_{u_t}$) $\beta=0.05$ (4A)	1971 normative houses/households ratio ($\varepsilon^*_{u_t}$) $\beta=0.057$ (4B)	1971 assumed number of houses effectively demanded[(3)] $\times 10^3$ (5)	1971 rent per house in won (6)=(3)/(4)	1976 normative houses/households ratio ($\varepsilon^*_{u_t}$) $\beta=0.05$ (7)	1976 normative houses/households ratio ($\varepsilon^*_{u_t}$) $\beta=0.057$ (8)
0～20	637	666	2,850	0.42	0.468	0.470	267	6,790	0.591	0.601
20～28	799	727	3,900	0.50	0.541	0.543	400	7,800	0.648	0.657
28～36	579	579	53,400	0.55	0.587	0.589	316	9,710	0.683	0.691
36～44	368	379	6,180	0.60	0.633	0.634	221	10,300	0.719	0.725
44～52	191	229	7,550	0.65	0.679	0.680	123	11,620	0.754	0.7595
52～60	104	133	8,490	0.70	0.725	0.726	73	12,130	0.788	0.793
60～	217	182	14,230	1.00	1.000	1.000	217	14,230	1.000	1.000
total/average	2,895	2,895	5,430	0.56	0.596	0.5975	1,617	9,700	0.709	0.716

monthly consumption expenditure range in 10 won	1976 final projection of the number of households $\times 10^3$ $\beta=0.05$ (9)	1976 final projection of the number of households $\times 10^3$ $\beta=0.057$ (10)	1976 additional number of households $\times 10^3$ $\beta=0.05$ (11)	1976 additional number of households $\times 10^3$ $\beta=0.057$ (12)	1972～70 additional number of houses effectively demanded $\times 10^3$ $\beta=0.05$ (13)=(11)×(3)/(6)	1972～70 additional number of houses effectively demanded $\times 10^3$ $\beta=0.057$ (14)=(12)×(3)/(6)	1972～70 housing needs (exclusive of replacement needs) $\beta=0.05$ (15)	1972～70 housing needs (exclusive of replacement needs) $\beta=0.057$ (16)	1972～70 number of houses effectively demanded related to housing needs (exclusive of replacement needs) $\beta=0.05$ (17)=(13)/(15)	1972～70 number of houses effectively demanded related to housing needs (exclusive of replacement needs) $\beta=0.057$ (18)=(14)/(16)
0～20	665	690	30	52	13	52	96	114	0.14	0.19
20～28	1,039	1,077	242	275	121	275	243	274	0.50	0.50
28～36	842	873	265	292	146	292	237	261	0.62	0.62
36～44	588	609	221	240	133	240	190	208	0.70	0.69
44～52	337	349	147	158	96	158	125	135	0.77	0.76
52～60	201	209	97	104	68	104	83	90	0.82	0.81
60～	435	451	219	233	219	233	219	233	1.00	1.00
total/average	4,107	4,258	1,221	1,354	796	873	1,193	1,315	0.67	0.66

Sources: Republic of Korea [12], Yonsei University [16]

(1) The figures given here are averages of the slightly deviating figures for $\beta=0.05$ and $\beta=0.057$.

(2) The parameters of the lognormal distribution are for 1971: θ (mean of logs)$=10.2625$, λ (standard deviation of logs)$=0.4851$ and for 1976: $\theta=10.3625$, $\lambda=0.4851$.

(3) Apart from the somewhat arbitrary assumption about the figures in column (4) the total number is assumed to be 1,617 on the basis on the actual figure for 1970 and the assumption of an equal time distribution of the planned building activity over time.

change in prices for housing units. This estimate will be compared to that of the needs and to the building program.

In the former section already the households per income class have been projected (X'_{w_t}) by making use of the estimated changes of the parameters of a theoretical distribution, i.e. the lognormal distribution. The initial—1971—parameters of this distribution are estimated by the numerical optimization method according to Box [2] which is called the complex method.(17) The goodness-of-fit criterion proposed by Gastwirth and Smith [10] is satisfied for the density function fitted here.

From this projection and from the data on rent paid—including estimated rent for owner occupied dwellings—per income class (Bureau of Statistics [3]) the additional amounts for rent "available" per income class over the planning period can be estimated. Applying to these amounts the average rent per housing unit (calculated from rents per household and the ε'_{w_0} of the former section, 1971 prices, and being different as between income classes) in each income class separately we get the increase in effective demand for housing units per income class as presented in Table 2.

The figures show that the additional effective demand expressed in numbers of housing units is ca. 800, 000 in urban areas, which is ca. two thirds of the urban needs (exclusive of replacement) during the plan period. The building program implies a net urban increase of about 515, 000 housing units (new houses less replacements)(18) which falls behind the effective demand. Since one might expect that market forces will do their job in South Korea, prices will increase and the number of additional housing units effectively demanded will decrease to this 515, 000, which is less than two fifths of total urban needs. The comparison of needs and additional effective demand for the income groups distinguished displays a far greater discrepancy for the lowest income groups than for the higher ones, due to the relatively larger increase of the norm $\varepsilon^{*}{}'_{w_t}$ for the lower income groups (which remains, however, at the planning period substantially lower than the norm of the highest income groups, see Table 2).

(17) Hans de Kruijk would like to thank Drs. H.K. van Dijk and Prof. Dr. T. Kloek for drawing his attention to this method and for their readiness to put their computer program at his disposal.

(18) Assumedly executed fully in urban areas and including the 151, 000 housing units for replacement.

Though the calculations project a dim housing future in the Republic of Korea it might still be better than in many other developing countries.

Moreover, extra efforts in the housing area in view of the shortfall of a program as compared to the increase in effective demand and, even more, to the needs should be weighed against the requirements in other areas critical for development in an overall assessment.

VI. Summary and Conclusions

The purpose of this paper has been to present a series of very simple models, which could be used, on the basis of general data available for quite a few developing countries, to get a rough quantitative impression of the (basic) needs for housing units arising during a planning period or to provide a rough quantitative indication of some implications of a proposed housing units building program for a certain planning period. The three models proceed from one working with national aggregates (HNM 1) via one which distinguishes urban and rural areas (HNM 2) to one which disaggregates these categories even further over income classes of the population (HNM 3). They employ a postulated normative minimally acceptable housing units/households ratio ε^*. In the policy form of the models the number of years $\bar{Z}$ is chosen during which the gap between the existing housing units/households ratio and ε^* is to be bridged according to an assumed time path and the implied needs during a planning period are calculated. In the analytical form the number of years $Z_{1\cdot\cdot T}$ which it will take to bridge this gap is calculated as resulting from the continuation of the gradual gap bridging during a planning period which is implied by a chosen program of building housing units.

In this way not only the total needs during a planning period are calculated, but also the relative importance of the growth of the population, of the decrease in average household size, of the wear and tear of existing housing units, of the gap bridging policy choices, of rural-urban migration and of the development of the incomes of different income classes as projected by a method of Hans de Kruijk. These can be compared with the projected demand for housing units to be built during the planning period employing again the above mentioned income distribution projection method.

Exercises with these models are made for data of the Republic of Korea referring to its Third Five Year Plan Period 1972~1976. Some main conclusions of these exercises are as follows:

(1) The housing problem exists mainly in the urban areas and the needs due to migration are the largest factor there, while the outmigration from rural areas largely solves the housing problem there (in the aggregate). This shows at the same time the importance of rural-urban disaggregation of the models, which reveals clearly larger needs than estimated with HNM 1.

(2) Population increase and wear and tear are the second and third important factors for determining housing needs.

(3) All needs estimates are roughly 2 times higher than the proposed housing program, which falls even short of the demand for new housing units in urban areas. This program implies a deterioration in the overall as well as urban housing units/households ratio, even while it is relatively more extensive than in quite a few, if not most, developing countries.

These conclusions depend of course on the data given and specific assumptions made for the Republic of Korea.

(4) The ratio of the increase in effective demand and the needs per income class is much lower in the lower income classes than on the average, which suggests the need for special efforts to make up for the deficiency of market forces. This need should be weighed against the same in other areas critical for development including income distribution considerations.

Appendix 1: Six Cases of the Signs of $(\varepsilon^*-\varepsilon_0)$[19] and of $Z_{1\cdots T}$ in (2. 10)

One can distinguish six cases for the values of ε^* in relation to ε_0 and the sign of $Z_{1..T}$ or $(\varepsilon_T-\varepsilon_0)$:

(1) $\varepsilon^*>\varepsilon_0$ and $Z_{1..T}>0$ $(\varepsilon_T>\varepsilon_0)$: the housing shortage decreases and will be abolished in $Z_{1..T}$ years[20] from year t_0.

(2) $\varepsilon^*>\varepsilon_0$ and $Z_{1..T}<0$ $(\varepsilon_T<\varepsilon_0)$: the housing shortage increases over time.

(3) $\varepsilon^*=\varepsilon_0$ and $Z_{1..T}=0$ $(\varepsilon_T>\varepsilon_0)$: the numbers of housing units and of households are balanced in year t_0, but an increasing surplus starts to arise.

(4) $\varepsilon^*=\varepsilon_0$ and $Z_{1..T}=0$ $(\varepsilon_T<\varepsilon_0)$: the same as (3), but now an increasing shortage starts to arise.

(19) ε_T and ε_0 are abbreviations of ε_{t_0+T} and ε_{t_0}, respectively.

(20) Provided that ε_t keeps its change also for later years.

(5) $\varepsilon^* < \varepsilon_0$ and $Z_{1 \cdots T} > 0$ ($\varepsilon_T < \varepsilon_0$) : a housing surplus exists, but in $Z_{1 \cdots T}$ years[(20] from t_0 the number of houses and households will be balanced.

(6) $\varepsilon^* < \varepsilon_0$ and $Z_{1 \cdots T} < 0$ ($\varepsilon_T > \varepsilon_0$): the existing housing surplus increases over time.

In all cases where $\varepsilon_T = \varepsilon_0$ (then $Z_{1 \cdots T} = \infty$ or undetermined) the existing shortage, surplus or balance in housing will remain.

Appendix 2: Data Base for the Estimates of α_t and of δ_t

The Third Five Year Plan of Korea (Government of the Republic of Korea [12]) estimates the annual growth rate of the population at 1.5 per cent and a decrease of the average number of persons per household of 0.20 during the plan period. Combining the figures of Table A.1 and the estimates of the plan, we assume the 1.5 per cent for the annual growth rate of the population to apply to 1976 and interpolate linearly between 1970 and 1976, while we distribute the 0.20 decrease in the average number of households in 1972~1976 evenly over the years.

Table A.1. Population and the number of persons per households of the Republic of Korea, 1960~1970

t	$P_t \times 10^3$	$\frac{P_t - P_{t-1}}{P_{t-1}} \times 100$	$\delta_t = \frac{P_t}{H_t}$
1960	—	—	5.71
1961	25,402	2.86	—
1962	26,125	2.85	—
1963	26,868	2.84	—
1964	27,631	2.84	—
1965	28,377	2.70	5.66
1966	29,086	2.50	5.62
1967	29,784	2.40	—
1968	30,470	2.30	—
1969	31,139	2.20	—
1970	31,793	2.10	5.37

Sources: first column: ECAFE, [9], third column: Republic of Korea [12]

Note: The population figure of the Korean Statistical Yearbook for 1970 excludes foreigners and is related to October. Therefore we use the ECAFE-figures.

Appendix 3: Housing Need Model 2

$$P_t = (1+\alpha_t)\, P_{t-1} \qquad \text{(A.1)}$$

$$P_{rt} = P_t - P_{ut} \qquad \text{(A.2)}$$

$$P_{ut}=(1+\alpha_t+\beta_t)\ P_{ut-1} \quad \text{(A.3)}$$

$$H_{rt}=\frac{1}{\delta_{rt}}\ P_{rt} \quad \text{(A.4)}$$

$$H_{ut}=\frac{1}{\delta_{ut}}\ P_{ut} \quad \text{(A.5)}$$

$$n_{rt}=n_{r_{ht}}+n_{r_{dt}}+n_{r_{gt}} \quad \text{(A.6)}$$

$$n_{ut}=n_{u_{ht}}+n_{u_{dt}}+n_{u_{gt}} \quad \text{(A.7)}$$

$$n_{r_{ht}}=\varepsilon_{rt_0}\ (H_{rt}-H_{rt-1}) \quad \text{(A.8)}$$

$$n_{u_{ht}}=\varepsilon_{ut_0}\ (H_{ut}-H_{ut-1}) \quad \text{(A.9)}$$

$$n_{r_{dt}}=\varepsilon_{rt_0}\ \lambda_r\ H_{rt_0} \quad \text{(A.10)}$$

$$n_{u_{dt}}=\varepsilon_{ut_0}\ \lambda_u\ H_{ut_0} \quad \text{(A.11)}$$

$$n_{r_{gt}}=\varepsilon^*_{r_t}\ H_{rt}-\varepsilon^*_{r_{t-1}}H_{rt-1}-\varepsilon_{rt_0}\ (H_{rt}-H_{rt-1}) \quad \text{(A.12)}$$

$$n_{u_{gt}}=\varepsilon^*_{u_t}\ H_{ut}-\varepsilon^*_{u_{t-1}}\ H_{ut-1}-\varepsilon_{ut_0}\ (H_{ut}-H_{ut-1}) \quad \text{(A.13)}$$

$$\varepsilon^*_{r_t}=(t-t_0)\frac{(\varepsilon^*_r-\varepsilon_{rt_0})}{Z_r}+\varepsilon_{rt_0} \quad \text{(A.14)}$$

$$\varepsilon^*_{u_t}=\frac{\{(t-t^0)\ \frac{\varepsilon^*-\varepsilon_{t_0}}{Z}+\varepsilon_{t_0}\}\ (H_{ut}+H_{rt})-H_{rt}\ \varepsilon^*_{r_t}}{H_{ut}} \quad \text{(A.15)}$$

$(\varepsilon^*=\varepsilon^*_r=\varepsilon^*_u;\ Z=Z_r=Z_u)$

in the policy form, and

$$Z_{rt}=(\varepsilon^*_r-\varepsilon_{rt_0})/(\varepsilon_{rt}-\varepsilon_{rt-1}) \quad \text{(A.14')}$$

$$Z_{ut}=(\varepsilon^*_u-\varepsilon_{ut_0})/(\varepsilon_{ut}-\varepsilon_{ut-1}) \quad \text{(A.15')}$$

$(\varepsilon^*_u=\varepsilon^*_r=\varepsilon^*)$

in the analytical form.

β_t: yearly urban population growth rate due to rural-urban migration.

u,r: indexes for urban and rural variables, respectively.

For further explanation, see Sections II. 1 and III. 2.

Appendix 4: Discussion of the Time Path Assumption for ε^*_t, of Its Implications and of Some Alternatives

The choice of the time path for attaining the norm for the housing units /households ratio made in this paper is a simple but by no means the only possible or obvious one. Moreover, the total needs calculated by the models in this paper is not insensitive to this choice. Finally, the disaggregation in HNM 2 and HNM 3 requires some further choices in choosing the respective time paths, which have been mentioned only implicitly in the text. Therefore we make a few more remarks on the choice of the time path(s),in order to allow the reader to consider alternatives to our choices made.

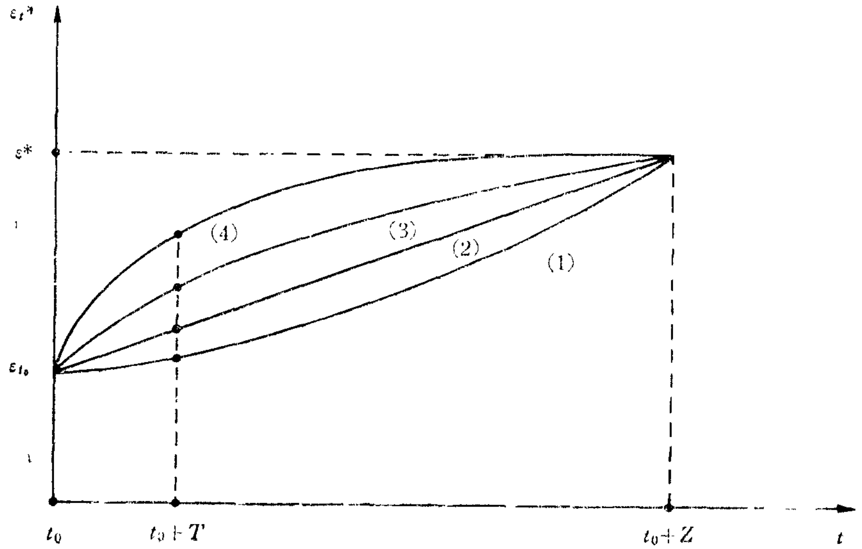
After the acceptance of the logic of choosing a norm ε^* and a fixed time horizon t_0+Z when the norm ε^*_t should be satisfied, the choice of an arithmetical series (equal yearly increases) for the yearly norms ε^*_t represents a gradualist approach. We mention here three alternative choices, one even more gradualist (a geometrical series; equal yearly growth rates), and two less gradualist (a logarithmic and an inversely quadratic series). Their degree of gradualism can be characterised by the value of $\varepsilon^*_{t_0+T}(f)$, f defining the time path of ε^*_t. These values are most easily compared graphically. We give the four choices mentioned here with the corresponding formulae (2.7) and their continuous graphs.

(1) geometric series: $\varepsilon^*_t=\varepsilon_{t_0}\left(\frac{\varepsilon^*}{\varepsilon_{t_0}}\right)^{\frac{t-t_0}{Z}}$.

(2) arithmetical series: $\varepsilon^*_t=(t-t_0)\frac{(\varepsilon^*-\varepsilon_{t_0})}{Z}+\varepsilon_{t_0}$.

(3) logarithmic series: $\varepsilon^*_t=\varepsilon_{t_0}\ \ln\left\{\left(\frac{e^{\varepsilon^*/\varepsilon_{t_0}}-e}{Z}\right)(t-t_0)+e\right\}$.

(4) inversely quadratic series: $\varepsilon^*_t=\frac{(\varepsilon^*-\varepsilon_{t_0})}{\sqrt{Z}}\sqrt{t-t_0}+\varepsilon_{t_0}$.



From the formulae it will be clear that the use of (1), (3) or (4) complicates the model considerably and makes numerical approximations by computer-calculations required for its solution.

A somewhat different approach to choosing a time path would be to formulate it not for ε^*_t but for n_t, by not distributing bridging the gap $(\varepsilon^*-\varepsilon_{t_0})$ over Z years, but by doing so far the total building program that follows from the calculation of the needs over Z years:

$$n_{t_0+1,\cdots,t_0+Z}=\varepsilon^* H_{t_0+Z}-\varepsilon^* H_{t_0}(1-\lambda Z) \text{ (assuming (2.5) valid over } Z \text{ years).}$$

While such an approach recommends itself because it formulates the time path in terms of concrete activities, its drawback would be the necessity of making somewhat speculative assumptions about the development of H_t over a long period of time for determining the needs during the medium term plan period. The approach followed in this paper avoids such an intergenerational dependence.

It could be noticed that the determination of ε^*_{rt} and ε^*_{ut} carries an element of arbitrariness. While the definitions used ascertain consistency between HNM 2 and HNM 1 with regard to ε^*_{rt}, ε^*_{ut} on the one hand and ε^*_t on the other hand, the gradual arithmetic time path of ε^*_{rt} renders the time path of ε^*_{ut} more than arithmetic.

Actually it follows from (A. 15) that ε^*_{ut} is for $t=t_0+1,\cdots,t_0+T-1$ above its arithmetic path, since the second term in

$$\varepsilon^*_{ut}=\left\{(t-t_0)\frac{(\varepsilon^*_u-\varepsilon_{ut_0})}{Z_u}+\varepsilon_{ut_0}\right\}+\left\{\frac{(t-t_0)}{Z}-1\right\}(\varepsilon_{t_0}-\varepsilon_{rt_0})\left(\frac{H_{rt_0}}{H_{ut_0}}-\frac{H_{rt}}{H_{ut}}\right)$$

is positive as H_{rt}/H_{ut} decreases over time. Therefore

$$\varepsilon^*_{ut}-\varepsilon^*_{ut-1}=\frac{(\varepsilon^*_u-\varepsilon_{ut_0})}{Z_u}+(\varepsilon_{t_0}-\varepsilon_{rt_0})\left[\left\{\frac{(t-t_0)}{Z}-1\right\}\left(\frac{H_{rt-1}}{H_{ut-1}}-\frac{H_{rt}}{H_{ut}}\right)+\frac{1}{Z_u}\left(\frac{H_{rt_0}}{H_{ut_0}}-\frac{H_{rt-1}}{H_{ut-1}}\right)\right] \quad \text{(A. 16)}$$

in which

$$H_{ut}=(1+\alpha_t+\beta_t)(1+\gamma_{ut})H_{ut-1}, \quad H_{rt}=(1+\alpha_t-\beta_t\frac{P_{ut-1}}{P_{rt-1}}(1+\gamma_{rt})H_{rt-1}.$$

This second term of (A. 16) multiplied by $(1+\alpha_t+\beta_t)(1+\gamma_{ut})H_{ut-1}$, constitutes the small correction term not written in (3. 4), but hinted at in the note to (3. 4).

Moreover, at the values of the migration rate used on the basis of available data $\varepsilon_{rt}>\varepsilon^*_{rt}$ for $t<t_0$ even without any building carried out in the rural areas.

One might argue that there should be in this case rather consistency between ε_{r_t} and $\varepsilon^*_{u_t}$ on the one hand, and ε^*_t on the other hand, implying a lower $\varepsilon^*_{u_t}$ than follows from (A. 15), as long as $\varepsilon_{r_t} > \varepsilon^*_{r_t}$. Then the difference between $\sum_{t=1972}^{1976} n_t$ in HNM 1 and $\sum_{t=1972}^{1976} n_{u_t}$ in HNM 2 would become less or zero. However, the argument might loose its force when ε_{r_t} is greater than 1. The authors leave this open for discussion. A somewhat similar element of arbitrariness is involved in the definition of $\varepsilon^{*\prime}_{u_t}$ in HNM 3, which also ascertains consistency with $\varepsilon^*_{u_t}$ in HNM 2, in a relatively simple way. The same kind of defining procedure could also be used for defining $\varepsilon^*_{u_t}$ and $\varepsilon^*_{r_t}$ in HNM 2, consistently with ε^*_t in HNM 1.

Finally it may be useful to notice that, although the models are written in yearly equations, both their policy form and their analytical form are intended to be used for the estimation of total needs or of the implications of a housing building program for a planning period as a whole. Moreover, as the perspective is from a base year to onwards(21) the models can best be used for a period following immediately this base year. If already some years have elapsed since the base year t_0 and the ε_t would deviate clearly from the ε^*_t, one should either calculate separately how to distribute over future years the backlog or surplus of housing units as defined in comparison with the outcome of the model, or forget the years since the base year t_0 and take the year concerned as the new base year, and apply the model from then onwards.

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(21) Therefore in (2.6) ε^*_{t-1} and not ε_{t-1} appears, and n_{g_t} in the model is independent of the particular value of ε_{t-1}, but interconnected with all n_{g_t} for other years before t_0+Z, because of the time path assumption for ε^*_t.

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