A Diagrammatic Exposition of the Equilibrium and the Optimum Involving Public Goods

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I. Introduction

Samuelson [5] has shown that Pareto-optimal allocation of resources involving public goods in a decentralized economy is fundamentally incompatible with individual incentives.(1) In such a situation competitive equilibrium allocation is not Pareto-optimal allocation and *vice versa*. The purpose of the present study is to show clearly that the equilibrium involving public goods is not Pareto-optimal in a decentralized economy by the use of simple diagrams which have hitherto not been used in the literature of public finance except a similar diagram used by Malinvaud [3].(2)

The economy I am going to consider in the present study is essentially a

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(1) Since then there has been a number of papers which devised mechanisms to induce Pareto-optimum for the equilibrium involving public goods in the economy with a central government. For a brief review of the literature on those demand revealing mechanisms, see Groves and Ledyard [2].

(2) Malinvaud's paper deals with a planning problem where all the information is centralized by a central institution called "board". The present paper differs from Malinvaud's in that the ecenomy dealt with is a decentralized one. Also, the budget line in my diagram is more flexible and explicit reflextion of the disposable resources or wealth in the economy. More about the difference in the budget lines will be discussed on later pages.

decentralized economy. In this economy, there are two groups of consumers whose preference patterns are homogeneous within the group but different between the two groups. The private and public goods consumed in this economy are produced and supplied from outside the economy. Thus, to be exact, this is a sub-sector of a consumer economy where the allocation of the final goods within this sub-sector is relevant in the context of the present study. No demand revealing mechanism is applied to reach an allocation. Each group of consumers pursue its own interest only. No externality in the consumption of private goods is assumed.

Since this is a decentralized economy, there is no central authority or government that gathers information and controls resource allocation. However, there is a total resource constraint for the entire economy. Each group of consumers divides its spending between private goods and a collective commodity, namely a public good. The mechanism for the distribution of the total resources between the two groups of consumers does not need to be known. Given the initial allocation, where the final allocation would end up and what its properties are, are the problems I will be concerned with in this paper.

In order to help visualize this kind of economy, let us think of the problem of determining the level of fringe benefits in a firm which consists of two groups, namely, a labor union and a management. For the ease of exposition and understanding, the specific problem of the level of fringe benefits between the union and the management will be explained using the diagrams in this paper.[3] But the propositions obtained in regard to the fringe benefits as a public good can be readily generalized to the problem of resource allocation involving any public goods.

In the present study, the workers are treated as an aggregate entity represented by an organization of the workers, say, a labor union. The employment

(3) The present study does not deal with the fringe benefits or the quality of work environment in terms of the labor market structure as analyzed, for example, by Viscusi [6]. Nor does it take into account the heterogeneity of labor as considered by Goldstein and Pauly [1].

level is not affected by the chosen level of the fringe benefits or the job search behavior of the workers. In this sense it is a static model. The management is the other aggregate entity which divides the total receipt of the firm with the union. The question of exactly whom the management consists of is not relevant here. What matters is that such a dichotomy representing two conflicting interests within a firm for the sharing of the receipt of the firm could exist.

The fringe benefits in the context of the present study is a broad term that could include any benefit or improvement of the work environment to the employees whose cost of providing must be borne by either the management or the union or both. It is more appropriate to treat the fringe benefits as a public good rather than a private good. Because although the workers derive most satisfaction from the improved work environment, the management will presumably benefit from the improved productivity of the workers if nothing else. The traditional Edgeworth box diagram is inadequate for the analysis if one of the goods is a public good. Thus in the following analysis I use the simple equilateral triangle diagrams which is reminiscent of the classical Edgeworth box in which the indifference curves and budget lines appear directly. The indifference curves and budget lines in the diagram will be explained first. Then the process of reaching the equilibrium allocation and its deviation from Pareto-optimum will be shown.

II. Budget Line

The equilateral triangle has the following nice property. At any point within the triangle and on the boundary if you draw the line segments that are perpendicular to each side of the triangle, the sum of such line segments is equal to the height of the triangle as illustrated in Figure 1. This property is well known. In the figure, $x+y+z=w$ where w is the height of the equilateral triangle.

The total receipt of the firm net of all the operating costs other than

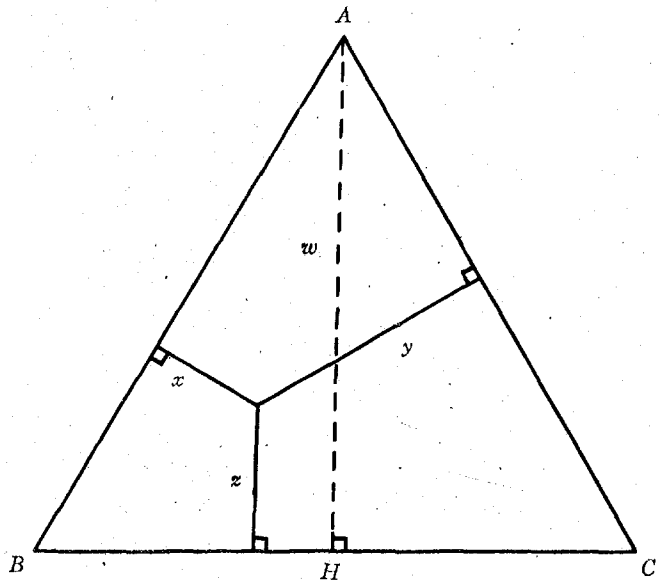


Fig. 1

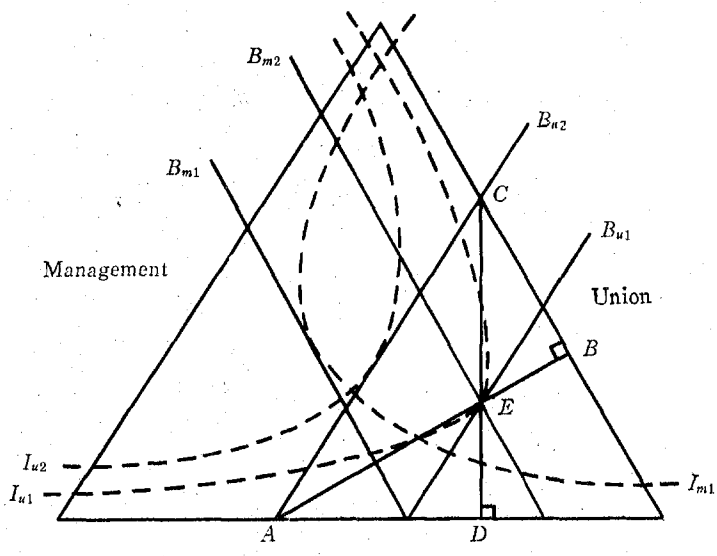


Fig. 2

labor cost can be represented by w in the figure. w is initially divided between the management and the union whose shares can be represented by x and y respectively. Any fringe benefits which can be represented by z, are provided at

the sacrifice of the share of either the management or the union or both. The share of the union will be distributed among the workers as the wage. How it will be distributed is not relevant in the present study. The union considered here is a closed shop in the sense that all the workers employed belong to the union. The union is in fact assumed to represent the preference of a typical worker and hereafter the union will be treated as if it were an individual worker.

The utility of the union depends on the level of the wage and fringe benefits. The indifference curves of the union are represented as I_{u1} and I_{u2} in Figure 2 where $I_{u2} > I_{u1}$. The budget lines of the union are represented as B_{u1} and B_{u2} in the figure where $B_{u2} > B_{u1}$. These budget lines are parallel to the side of the triangle marked as management.[4] At point A, there are no fringe benefits and the amount of the wage is AB. At point C, the level of the fringe benefits is CD and there is no wage. Normally the union will have both the wage and some fringe benefits. For example, at point E, the level of the fringe benefits is ED and the wage is EB. For the maximum utility, the union will want to reach a point such as E where the indifference curve is tangent to the budget line. The utility of the management depends on its share of profits and also on the level of the fringe benefits. The indifference curves and budget lines of the management can be represented similarly as shown in the figure by I_{m1} and B_{m1} respectively.

A special feature of the budget line involving a public good is that as soon as one party provides the public good, in this case the fringe benefits, this provision has an immediate effect of expanding the budget line of the other party since the consumption of the fringe benefits by the other party cannot be prevented due to the non-exclusion property of the public good. In other words, the other party becomes the free rider. Thus arises the necessity for

(4) In Malinvaud's diagram, the budget line (common for both groups) is represented as a line such as AH in Figure 1. And this common budget line pivots around the midpoint H. This, however, does not pose any contradiction to the individualized budget lines of mine. Because what Malinvaud called a budget line is actually a price (or taxes on public goods) indicator line in his centralized planning economy.

the bargaining and negotiation between the union and management for the provision of the fringe benefits. In Figure 2, suppose the original budget lines are B_{m1} and B_{u1} with no fringe benefits. As soon as the union provides ED of fringe benefits at the cost of some wage, the budget line of the management automatically expands to B_{m2}.

III. Rational Set

Let us assume that there exists a minimum subsistence level of wages below which the share of wages must not decrease in exchange for the fringe benefits. Thus the indifference curves of the union will always lie to the left of the minimum subsistence level. Similarly let us assume further that there exists a minimum acceptable profit level of the management which reflects the opportunity cost of the management. The indifference curves of the management will always lie to the right of the minimum acceptable profit level. These restrictions are introduced in order to avoid some unrealistic situations where the union and the management have almost none in comparison to the

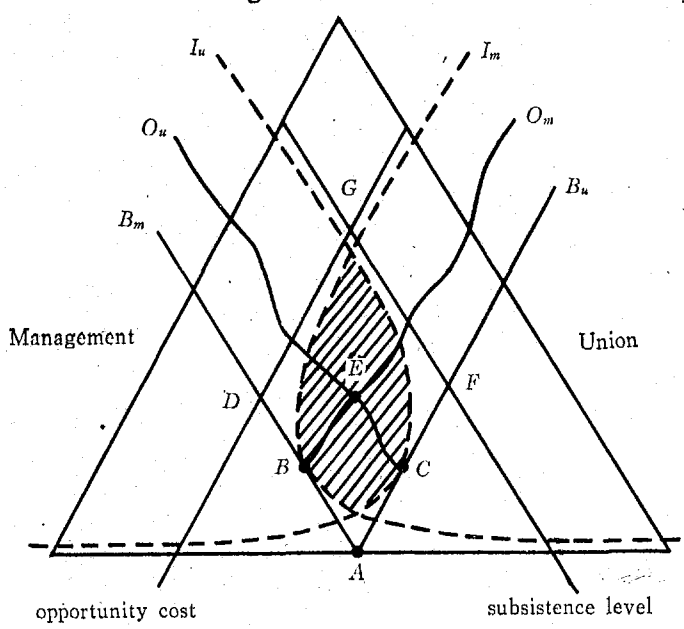
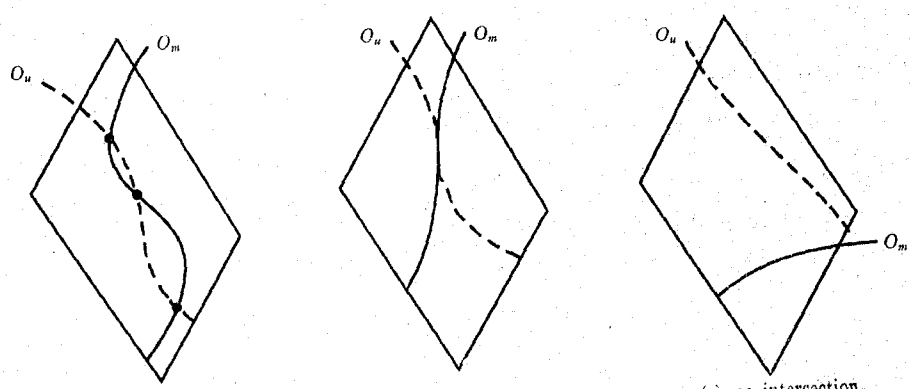


Fig. 3



(a) multiple (b) coinciding (c) no intersection

Fig. 4

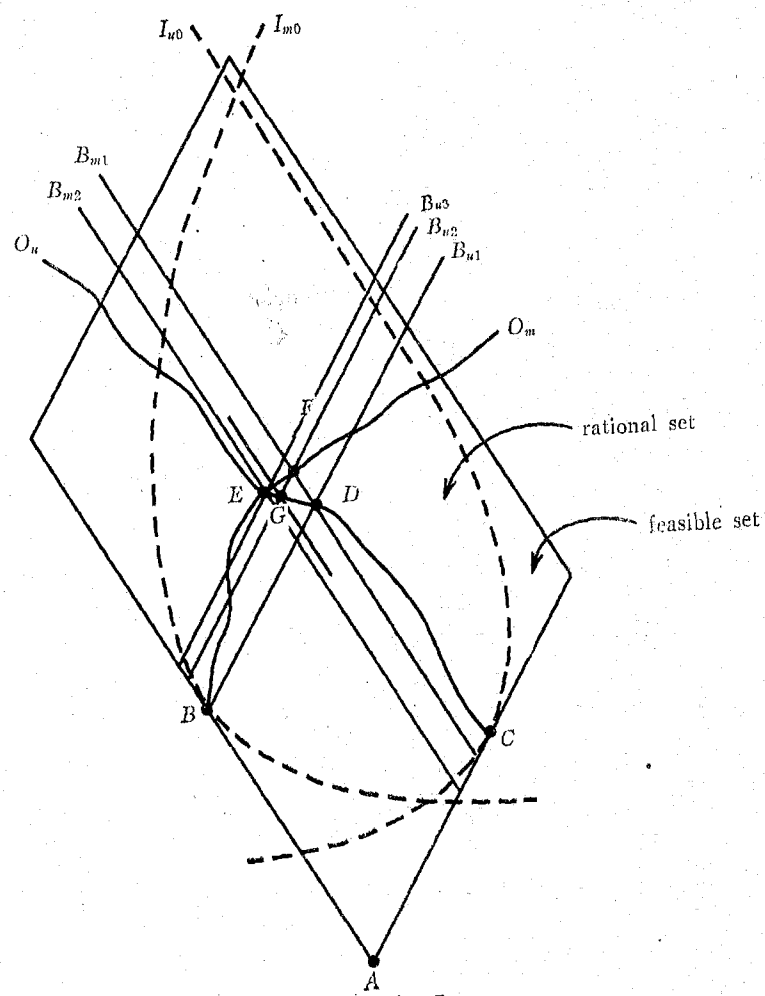


Fig. 5

ridiculously high level of fringe benefits. The feasible set of bargaining is, then, reduced to the rhomboid $ADGF$ in Figure 3 where the initial allocation of the shares of the union and the management is at A with no provision of the fringe benefits.

At the initial allocation A, neither the union nor the management is maximizing its utility. The utility maximizing point given the budget line is where the indifference curve is tangent to the budget line. Allocation B is the utility maximizing point for the management given the initial allocation A, the budget line B_m and the indifference curve I_m. Allocation C is the utility maximizing point for the union given the initial allocation A, the budget line B_u and the indifference curve I_u in the figure. A utility maximizing entity would want to move the allocation to such a point. Let us call the locus of all the utility maximizing points as an offer curve. Then, the curves denoted by O_m and O_u represent the offer curves of the management and the union respectively.

The shaded area surrounded by the indifference curves I_m and I_u represents the set of Pareto-superior allocations within the rhomboid-shaped feasible set. Let us call this shaded area as rational set. The interior of this set can be reached only if both parties forego some of their share of the profits for the provision of the fringe benefits. Under the usual assumption of no bliss point and the strong convexity of the preferences, there exists a unique intersection of the offer curves in the interior of the rational set such as allocation E in Figure 3. However, the intersection may not always be unique or exist under the same assumption. These cases are depicted in Figure 4.

IV. Equilibrium and Pareto-optimum

For the simplicity of the exposition, let me consider only the case where there is a unique intersection of the offer curves. The other cases will also yield comparable results. Suppose that the management wants to move to the allocation B from A in Figure 5. Once the allocation is at B, the union can

now move along its new budget line B_{u1} to maximize its utility at D on its offer curve. The management will not oppose this movement because its utility at D is higher than its utility at B. But at D the management wants to move along its new budget line B_{m1} to allocation F on its offer curve. At F the union wants to move along its new budget line B_{u2} to allocation G on its offer curve. At G, the utility of the management is lower than its utility at F. The management cannot oppose this movement as long as the union is moving along its budget line. However, at G, the management will want to move along its new budget line to an allocation on its offer curve. This process will continue until allocation E which is the intersection point of the two offer curves, is obtained. At E, both the management and the union are maximizing their utility given their respective budget lines because allocation E is on their offer curves. This allocation E is the competitive equilibrium.

However, E is not a Pareto-optimal allocation. Portions of Figure 5 is depicted in an expanded version in Figure 6. The shaded region surrounded by two indifference curves I_{u1} and I_{m1} which are tangent to their respective budget line at E in Figure 6, represents Pareto-superior allocations compared to E. Thus the competitive equilibrium E is not a Pareto-optimal allocation. Proposition 1 is hereby obtained.

Proposition 1: A competitive equilibrium allocation involving public goods in a decentralized economy is individually rational but not Pareto-optimal.

Let us call the shaded region in the figure as a bargainable set. Pareto-optimal allocations would occur at the tangency points of the two indifference curves. The locus of such points of tangency is denoted by *P.O.* in the figure The allocations on Pareto-optimal locus *P.O.* are not on the offer curves. To be individually rational, an allocation must be utility maximizing allocation given the budget line. In other words, it must be on the offer curve. Thus Pareto-optimal allocations are not individually rational allocations. Proposition 2 is hereby obtained.

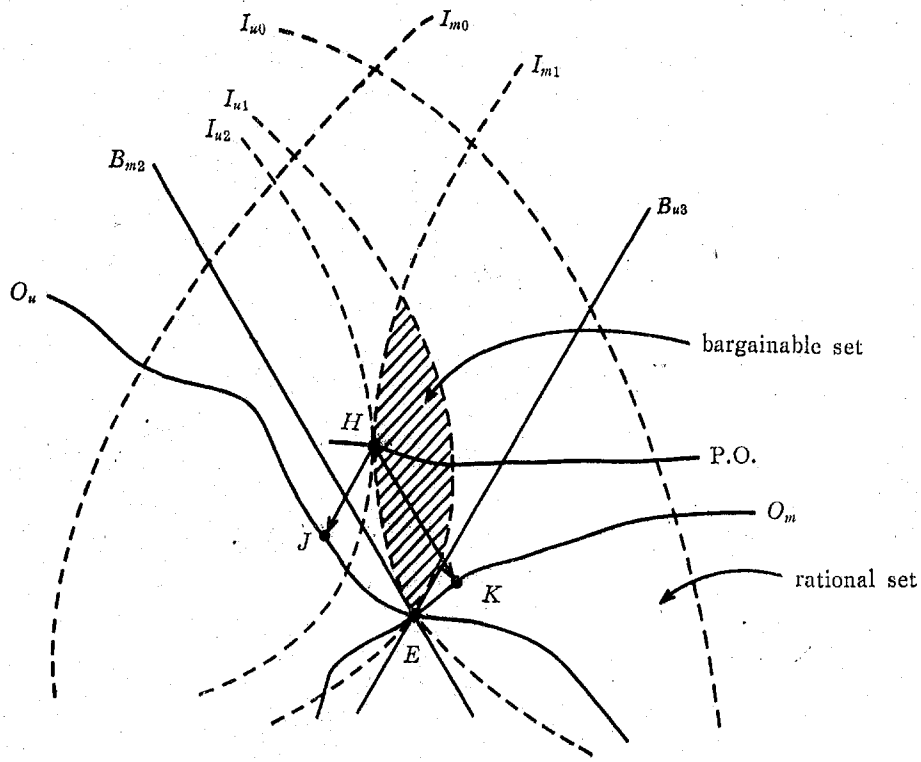


Fig. 6

Proposition 2: Pareto-optimal allocations within the bargainable set involving public goods in a decentralized economy are not individually rational.

Suppose that a very strong union bargained for its utility maximizing allocation within the bargainable set such as point H. However, once the allocation is at H, the union wants to move to an individually rational allocation on its offer curve such as J. At H, there is also pressure for the management to move to its individually rational allocation such as K on its offer curve. Once the allocation is moved to a point such as either J or K, the whole process (as has been shown in Figure 5) of the utility maximization of both groups would continue downward until it reaches the original equilibrium allocation E. The allocations such as H, J, and K are unstable. Thus a Pareto-optimal

allocation cannot be a competitive equilibrium.(5) Propositions 3 and 4 are hereby obtained.

Proposition 3: A competitive equilibrium allocation involving public goods in a decentralized economy is stable.

Proposition 4: A Pareto-optimal allocation involving public goods in a decentralized economy is not a competitive equilibrium.

References

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(5) The point where the two indifference curves coincide is both Pareto-optimum and the competitive equilibrium in Malinvaud's diagram. This is because in his diagram, there is a common budget line which can be tangent to both indifference curves at such a point. But in my diagram, such a point is Pareto-optimal but not a competitive equilibrium because of the difference in the budget lines as explained earlier.