Business Risk, Financial Risk, and Default Risk: Theory and Empirical Results (II)

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VII. Factor Analysis of the Risk-Return Variables

In applying factor analysis to our empirical data of risk and return variables, we have two objectives. The first is to examine if the factor analytic classification of variables is consistent with the theoretical classification of the variables. The second objective is to screen variables to be included in multiple regression equations to reduce the degree of multicollinearity in testing the hypotheses on risk and return relationships.(13) Also, a reduction in the degree of multicollinearity is necessary for a proper interpretation of a discriminant function.

The objective of factor analysis is to divide a given number of variables into groups of variables that are closely correlated with each other. Specifically, factor analysis calculates the standard regression coefficients between observed

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(13) To reduce the problem of multicollinearity, in addition to the factor analysis approach, there are the following methods: simple and partial correlation methods, the stepwise multiple regression method and the ridge regression method. Each method has advantages and disadvantages. For concepts of factor analysis, see Hair and others (1979). For mathematical introduction see Comrey (1973), Harman (1976), and Kim and Mueller (1978). For the computer program see Nie and others, eds., *SPSS*, 2nd ed., 1975, pp. 468-514.

variables and hypothetical unobserved variables. For instance, factor analysis assumes the following relationships between observed variables X_1, X_2,... and unobserved hypothetical factors F_1, F_2,...:

$$\begin{aligned} X_1 &= a_{11}F_1 + a_{12}F_2 + a_{13}F_3 + \cdots \\ &\cdots\cdots\cdots\cdots \\ X_n &= a_{n1}F_1 + a_{n2}F_2 + a_{n3}F_3 + \cdots \end{aligned} \tag{57}$$

where X_1, X_2, ... are the individual variables and F_1, F_2,... are the hypothetical factors. The coefficients a_{11}, a_{12},... are the standardized multiple regression coefficients (or factor loadings or correlation coefficients between the variables and the factors). The optimal number of factors and factor loadings are to be determined such that the factor solution would best fit the observed correlations among the variables.

Assume that as a result of calculation of factor loadings, X_1 and X_3 have higher factor loadings with factor F_1, and X_2 and X_4 have higher factor loadings with factor F_2. Then we infer that variables X_1 and X_3 belong to the same factor group F_1 and variables X_2 and X_4 belong to the same factor group F_2.

As we have seen in the preceding section, we have 9 risk variables and 5 return variables, a total of 14 variables. These variables may be divided into four categories: (1) The return variables related to common stock, $\Delta P/P, D/P, E(R)$. (2) The risk variables related to common stock, $\hat{s}(\Delta P/P)$, $\hat{s}(D/P)$, $\hat{s}(R)$, β. (3) The return variables related to the business and financial conditions of the firm, E/P, Y/V. (4) The risk variables related to the business and financial conditions of the firm, $\hat{s}/Y$, $\hat{s}(E/P)$, α, B/S, B/V. Our objective is to several significant groups of variables that are statistically closely correlated with each other in terms of factor analysis.

The results of calulations of factor loadings are summarized in Tables 4 and 5.[14] Table 4 is the initial unrotated factor loadings matrix. We note that the 14 variables are classified into five factor groups in both tables. In Table 4, the factors are extracted in the order of the percent of variance extracted, and

(14) We have used the following computer program: SPSS, 7.05, Type=PA1, Rotate=Varimax. For applications of factor analysis to financial ratios, see Chen and Shimerda (1981) and their references.

Table 4. Initial Factor Loadings Matrix

	F_1	F_2	F_3	F_4	F_5	h^2 (communality)
1. Earnings/price ratio, E/P	.4978	.7723	.0446	−.0394	−.0123	.8428
2. Business risk, $\hat{s}/Y$	.5664	.0895	.1096	.1672	.7149	.8799
3. Default risk, α	.5832	−.5270	−.2829	−.3876	.0553	.8512
4. Expected return, $E(R)$	.3645	.3313	.7923	−.1471	.0450	.8939
5. Total risk, $\hat{s}(R)$	.6341	−.4364	.3754	.3585	−.3411	.9786
6. Debt/equity ratio, B/S	.8287	−.1783	−.3307	−.3748	−.1074	.9799
7. Financial risk, $\hat{s}(E/P)$	.6945	.0510	−.2774	.3352	.3361	.7871
8. Systematic risk, β	−.2104	−.5329	.4887	−.1391	.2593	.6537
9. Dividend yield, D/P	.1167	.7477	−.3462	−.0082	−.1664	.7202
10. Rate of capital gain, $\Delta P/P$	.35493	.27864	.8263	−.1434	.0568	.9102
11. Risk of dividend yield, $\hat{s}(D/P)$	.2303	.1127	−.3878	.6617	.0194	.6543
12. Risk of capital gain, $\hat{s}(\Delta P/P)$	.6223	−.4375	.3745	.3673	−.3569	.9812
13. Debt/value ratio, B/V	.8144	−.0834	−.3791	−.3591	−.0797	.9492
14. Average return on capital, Y/V	.2072	.7560	.1697	−.0522	−.1898	.6821
Eigenvalue	3.9428	2.9200	2.5538	1.3272	1.0254	11.7534
% of variance	28.2	20.9	18.2	9.5	7.3	
Cumulative %	28.2	49.0	67.3	76.7	84.1	

Note: SPSS, 7.05, Type=PAl(principal-component solution without iteration) is used. *SPSS* (1975), pp. 478–480.

Table 5. Varimax Rotated Factor Loadings Matrix

	F_1	F_2	F_3	F_4	F_5	h^2 (communality)
1. Earnings/price ratio, E/P	.1302	.7565	.4347	−.0377	.2614	.8478
2. Business risk, $\hat{s}/Y$	.1742	−.0555	.3040	−.0202	.8681	.8799
3. Default risk, α	.8604	−.2748	−.0983	.1186	.1084	.8512
4. Expected return, $E(R)$	−.0471	.0938	.9187	.1810	.0787	.8939
5. Total risk, $\hat{s}(R)$	.2203	−.1513	.1785	.9291	.1096	.9786
6. Debt/equity ratio, B/S	.9491	.1535	.0178	.1907	.1375	.9799
7. Financial risk, $\hat{s}(E/P)$	.3341	.2202	−.0976	.2305	.7513	.7871
8. Systematic risk, β	−.1048	−.7576	.2549	.0151	−.0600	.6537
9. Dividend yield, D/P	.0128	.8184	−.0312	−.2205	.0264	.7202
10. Rate of capital gain, $\Delta P/P$	−.0542	.0332	.9274	.2004	.0767	.9102
11. Risk of dividend yield, $\hat{s}(D/P)$	−.1263	.3432	−.4506	.3127	.4687	.6543
12. Risk of capital gain, $\hat{s}(\Delta P/P)$	.2089	−.1480	.1688	.9370	.0964	.9812
13. Debt/value ratio, B/V	.9200	.2357	.0021	.1318	.1729	.9492
14. Average return on capital, Y/V	−.0868	.6948	.4353	−.0427	−.0234	.6820
Eigenvalue	2.7783	2.6772	2.5238	2.0853	1.7048	11.7694
% of variance	19.85	19.12	18.03	14.90	12.18	
Cumulative %	19.85	38.97	57.00	71.90	84.08	

Note: SPSS, 7.05, Type PAl (principal-component solution without iteration), Rotate=Varimax is used. *SPSS* (1975), p. 485. Variamx rotation is an orthogonal rotaion that maximizes the variance of the squared loadings in each column.

a careful examination of the factor loadings suggests that most variables are included in factor group 1. Table 5 is the varimax rotated factor loadings matrix. We note that the percent of variance is more evenly redistributed, and the 14 variables are also more evenly distributed among the five factor groups. Table 5 suggests the following factor groups:

Factor group 1: default risk, α
the debt/equity ratio, B/S
the debt/value ratio, B/V (financial leverage)
Factor group 2: the earnings/price ratio, E/P (the cost of equity capital)
the dividend yield, D/P
the income/value ratio, Y/V (the average rate of return on total capital)
systematic risk, β
Factor group 3: the expected return on common stock, $E(R)$
the rate of change in stock price, $\Delta P/P$
Factor group 4: total risk return, $\hat{s}(R)$ (the standard deviation of stock return)
the risk of stock price, $\hat{s}(\Delta P/P)$ (the standard deviation of the rate of change in stock price)
Factor group 5: business risk, $\hat{s}/Y$
financial risk, $\hat{s}(E/P)$
the risk of dividend yield, $\hat{s}(D/P)$ (the standard deviation of dividend yield)

The above results suggest that the factor analysis is indeed able to classify the 14 variables into risk and return types. Factor group 1 includes the risk variables associated with financial leverage. Factor group 2 includes the return variables associated with the earnings of the firm. Factor group 3 includes the return variables associated with common stock. Factor group 4 includes the risk variables associated with the rate of return on common stock. And factor group 5 includes the risk variables associated with the earnings of the firm.

The above factor analytic five group classification of the variables is generally in accordance with the expected four category classification. That is, if we combine group 1 and group 2, the five groups are reduced to four groups. However, there are two variables that are differently classified. First, systematic risk β is a risk associated with common stock, and thus it should be included in group 4, but it is included in group 2. This result is due to the fact that systematic risk is highly correlated with the earnings/price ratio, the income/value ratio and the dividend yield. Second, the risk of dividend yield $s(D/P)$ is also not included in group 4, but in group 5. This result is also due to the fact that the risk of dividend yield is highly correlated with financial risk, though it is not highly correlated with business risk.

VIII. Discriminant Analysis of the Risk Classes of the Firms

1. Discriminant Analysis of Default Risk

In this section, our objective is to calculate discriminant functions with the empirical data so that they could be used to predict the risk class of a firm. The standardized discriminant functions take the following forms:[15]

$$\begin{aligned} Z_1 &= a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \ldots \\ Z_2 &= a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \ldots \qquad (58) \\ &\ldots\ldots\ldots\ldots \end{aligned}$$

where $Z_1, Z_2, \ldots$ are the dependent variables or the discriminant scores, $X_1, X_2, \ldots$ are the independent variables, and $a_{11}, a_{12}, \ldots$ are the discriminant coefficients or discriminant weights. The discriminant coefficients are to be determined such that the between-group variance is maximized relative to the within-group variance.

In order to calculate discriminant functions, we have to divide the sample firms *a priori* into two or more groups on certain criteria. When there is only

(15) For the concepts of discriminant analysis and other multivariate analyses, see Hair and others (1979). Also see Klecka (1980), Tatsuoka (1971), Van de Geer (1971), and Cooley and Lohnes (1971). For the computer program, see *SPSS* (1975). The discriminant coefficients are determined such that the following G ratio is maximized:

$$G = \frac{(\bar{z}_1 - \bar{z}_2)^2}{\sum_i^2 \sum_j^n (z_{ij} - \bar{z}_i)^2}$$

where $\bar{z}_1$ and $\bar{z}_2$ are means of the two groups, and z_{ij} are the z values of jth firm in the ith group. See Hoel (1974, pp. 181-186). Other statistical techniques that can be used to classify groups include probit analysis and logit analysis.

one criterion, we can divide the firms into two groups, and when there are two criteria, we can divide the firms into three groups, and so on. Since the sample size is rather small in our study, we have used only one criterion and divided the firms into two groups. If a firm has a default risk less than or equal to 3.25%, the firm is classified as a low default risk firm, and if a firm has a default risk greater than 3.25%, the firm is classified as a high default risk firm.(16) As a result, we have obtained 18 low default risk firms and 18 high default risk firms.

As the independent variables, we have selected the following three variables, namely, the income/value ratio (Y/V), the debt/value ratio (B/V), and business risk ($\hat{s}/Y$). In Equation (27) we note that these three variables determine the size of t value and thus default risk.

The statistical results of discriminant analysis are summarized in Table 6.(17) The following observations may be made:

(1) The discriminant function is highly significant in differentiating low default risk firms and high default risk firms as indicated by the Wilks' lamda, canonical correlation coefficient, and the chi-square values. The percent of firms correctly classified in each default risk class is 91.67%, and thus only 8.37% of the firms are wrongly classified by the discriminant function.

(2) The centroid of the discriminant score for the high default risk firms is −0.7620, and the centroid for the low default risk firms is 0.7620. Thus, the critical cutting score is equal to zero.

(3) The coefficients of the standardized discriminant function represent the relative importance of each independent variable in differentiating the default risk classes of the firms. The absolute values of the coefficients suggest that the order of importance is the debt/value ratio (B/V), business risk ($\hat{s}/Y$) and

(16) The median for default risk in Table 1 is 2.90. It is obtained by SPSS which uses the median formula for a frequency distribution (*SPSS*, p. 183). The median 3.25 is obtained by Minitab which gives the median for an ordered set.

(17) Table 6 is obtained by using direct and stepwise methods (Rao and Wilks), SPSS, 7.05.

Table 6. Discriminant Analysis of the Default Risk Classes of Firms

A. Discriminant Functions

	(1) Standardized discriminant function	(2) Unstandardized discriminant function	(3) F-ratio
(1) $\hat{s}/Y$ (Business risk)	−0.4216	−0.0226	7.78
(2) B/V (Debt/value ratio)	−0.7813	−0.0473	26.82
(3) Y/V (Income/value ratio)	0.2898	0.0391	3.95
(4) constant		1.1023	

Note: (1) Centroids of groups in reduced space: Group 1=0.0762, Group 2=−0.7620, (2) Eigenvalue=1.4826, (3) Canonical correlation=0.7728, (4) Wilks' lamda=0.4028, (5) Chi-square=29.55, P<0.000, d.f.=3.

B. Prediction Results

	Group 1	Group 2	Total
Group 1($\alpha \leq 3.25\%$)	18 (100%)	0 (0.0%)	18
Group 2($\alpha > 3.25\%$)	3 (16.7%)	15 (83.3%)	18
	21	15	36

Note: The percent of firms correctly classified=(18+13)/36=0.9167.

$$t=\frac{p-0.5}{\sqrt{\frac{0.5(1-0.5)}{n}}}=\frac{(0.9167-0.5)}{\sqrt{\frac{0.5(1-0.5)}{36}}}=5.00$$

where p=the percent of cases correctly classified,
n=the sample size or the number of cases.

the income/value ratio (Y/V). As the F-ratios suggest, all the above variables are highly significant.

(4) The signs of the coefficients suggest that the greater the business risk and the debt/value ratio are, the greater is the chance that the firm will be classified as a high default risk firm. However, the greater the income/value ratio, the smaller is the chance that the firm will have a high default risk. These results are consistent with Equation (27).

(5) Multicollinearity among the independent variables would make discriminant functions highly unstable and lead to a misleading interpretation. However, an examination of the simple correlation coefficients (in Tables 2 and 3) of the independent variables of the discriminant function and also their factor groups does not suggest any significant multicollinearity among the independent vari-

ables.[18] Thus we may state that the discriminant function in Table 6 should be quite reliable.

2. Discriminant Analysis of Systematic Risk

A similar methodology is used to calculate a discriminant function with respect to systematic risk β. If a firm has a systematic risk less than or equal to 1.0, the firm is classified as a low systematic risk firm, and if a firm has a β greater than 1.0, the firm is classified as a high systematic risk firm. As a result, we have obtained 18 low systematic firms and 18 high systematic firms.

As to the determinants of systematic risk, as Equation (23) states, it is determined by the covariance of the return on a given stock and the return on the market portfolio, and the variance of the return on the market portfolio. However, we are uncertain *a priori* what variables are statistically significant in influencing the covariance. Thus, out of the 14 risk and return variables, excluding systematic risk β, we have included 13 remaining variables in the stepwise multiple discriminant procedure so that the stepwise method could select an optimal set of independent variables.[19]

The results of the stepwise multiple discriminant analysis are summarized in Table 7. The following points may be noted:

(18) The simple correlation coefficients are: 0.269 between $\hat{s}/Y$ and B/V, 0.085 between $\hat{s}/Y$ and Y/V, and 0.064 between B/V and Y/V. None of the above correlation coefficients is significant at the 5% level. Also, in the factor classification, each variable belongs to a different factor group.

Altman (1968) included four financial ratios in his discriminant model to predict corporate bankruptcies: the net working capital/total assets, retained earnings/total assets, earnings before interest and taxes/total assets, market value of equity/book value of total debt. In a recent new model, the Zeta model, Altman, Haldeman, and Narayana (1977) included seven financial ratios: EBIT/total assets, normalized standard error of estimate of EBIT, logarithm of EBIT/total interest payments, retained earnings/total assets, current asset/current liability, five year average of market value of equity/total capital, total assets. They tested both quadratic and linear models. They found the linear model performed better.

(19) Out of the 14 risk-return variables, systematic risk β is excluded from the independent variables since it is used as the basis for the classification of the firms into two groups.

In SPSS, the Rao and Wilks methods, among others, provide the stepwise discriminant results. The stepwise discriminant procedure selects an optimal (not a maximal) set of variables out of given total independent variables such that an additional variable does not contribute to the degree of discriminating power of the function as indicated by the smallest Wilks' lamda, the largest Rao's V or the largest overall multivariate F-ratio. See *SPSS*, pp. 447-448

Table 7. Discriminant Analysis of the Systematic Risk Classes of Firms

A. Discriminant Functions

	(1) Standardized discriminant function	(2) Unstandardized discriminant function	(3) F-ratio
(1) E/P (Earnings/price ratio)	0.8363	0.1723	5.12
(2) D/P (Dividend yield)	−0.7187	−0.4051	9.30
(3) $\hat{s}(E/P)$ (Financial risk)	−0.4435	−0.1704	4.04
(4) $\hat{s}(D/P)$ (Dividend risk)	−0.2343	−0.7159	1.17
(5) Y/V (Income/value ratio)	−0.9111	−0.1228	7.35
(6) α (Default risk)	−0.6042	−0.0190	8.56
(7) Constant		2.6848	

Note: (1) Cetroids of groups in reduced space: Group 1=−0.7087, Group 2=0.7087, (2) Eigen-value=1.0689, (3) Canonical correlation=0.7188, (4) Wilks' lamda=0.4834, (5) Chi-square=22.54, P<0.001, d.f.=6.

B. Prediction Results

	Group 1	Group 2	Total
Group 1 ($\beta \leq 1.0$)	16 (88.9%)	2 (11.1%)	18
Group 2 ($\beta > 1.0$)	3 (16.7%)	15 (83.3%)	18
	19	17	36

Note: The percent of firms correctly classified=(16+15)/36=0.8611.

$$t=\frac{0.8611}{\sqrt{\frac{0.5(1-0.5)}{36}}}=4.33.$$

(1) The discriminant function is highly significant in differentiating low systematic risk firms and high systematic risk firms as indicated by the Wilks' lamda, canonical correlation coefficients, and the chi-square values. The percent of firms correctly classified in each systematic risk class is 86.11 %, and thus 13.89% of the firms are wrongly classified by the discriminant function. However, judging from the hit-ratios, the systemaintic risk discriminant function is less satisfactory than the default risk discriminant function in its performance.

(2) The centroid of the discriminant score for the low systematic risk firms is -0.7087, and the centroid for the high systematic risk firms is 0.7087. Thus the critical cutting score is equal to zero.

(3) The absolute values of the coefficients of the standardized discriminant function suggest that the order of the relative significance of the independent variable is: the income/value ratio (Y/V), the earnings/price ratio (E/P), the dividend yield (D/P), default risk (α), financial risk $(\hat{s}(E/P))$, and dividend risk $(\hat{s}(D/P))$. The F-ratios suggest that all the independent variables are highly significant except the dividend risk.

(4) We note that only the earnings/price ratio has a positive sign, but all other variables have negative signs. Particularly, it should be noted that default risk has a negative sign, too. It suggests that if a firm has a greater default risk, the firm tends to have a lower systematic risk.

(5) Finally, we have to check multicollinearity of the discriminant function. A review of the simple correlation coefficients (Tables 1 and 2) and the factor loadings suggests that there are significant correlations among the independent variables. For instance, the income/value ratio, the earnings/price ratio, and the dividend yield belong to the same factor group 2, and financial risk and the risk of dividend yield belong to the same factor group 5. These results suggest that the stepwise multiple discriminant procedure does not guarantee the elimination of multicollinearity.

When there exists multicollinearity, the results of discriminant analysis can be unstable and lead to a misleading interpretation. For these reasons, we have calculated alternative discriminant functions. This time, we have selected only one variable from each factor group. The variables selected as representing each factor group are: the earnings/price ratio, business risk, the expected return on common stock, its total risk, and default risk. The results are summarized in Table 8.

Discriminant functions (1) and (2) in Table 8 are obtained when all the independent variables are simultaneously included in the equations (the direct method). We note that the earnings/price ratio and default risk have the same negative signs as in Table 7. Discriminant functions (3) and (4) are obtained by the stepwise procedure (the Rao method). We note that the signs of the

Table 8. Discriminant Analysis of the Systematic Risk Classes of Firms

A. Discriminant Functions

	(1) Standard-ized function	(2) Unstan-dardized function	(3) Standard-ized function	(4) Unstan-dardized function	(5) F-ratio
(1) E/P (Earnings/price ratio)	−0.8235	−0.1696	0.7364	0.1517	3.95
(2) $\hat{s}/Y$ (Business risk)	−0.0476	−0.0026			
(3) $E(R)$ (Expected return)	0.3656	0.0178			
(4) $\hat{s}(R)$ (Total risk)	−0.1273	−0.0100			
(5) α (Default risk)	−0.6905	−0.0218	0.7926	0.0250	4.57
(6) Constant		2.3317		−2.2736	

Note: (1) Centroids of groups in reduced space: Group 1 = −0.4418, Group 2 = 0.4448,
(2) Eigenvalue = 0.2556,
(3) Canonical correlation = 0.4512,
(4) Wilks' lamda = 0.7965,
(5) Chi-square = 7.169, P<0.208, d.f. = 5.

(1) Centroids of groups in reduced space: Group 1 = 0.4228, Group 2 = −0.4228,
(2) Eigenvalue = 0.4289,
(3) Canonical correlation = 0.4289,
(4) Wilks' lamda = 0.8161,
(5) Chi-square = 6.71, P<0.035, d.f. = 2.

B. Prediction Results (applies to both functions)

	Group 1	Group 2	Total
(1) Group 1 ($\beta \leq 1.0$)	13 (72.2%)	5 (27.8%)	18
(2) Group 2 ($\beta > 1.0$)	3 (16.7%)	15 (83.3%)	18
	16	20	36

Note: The percent of firms correctly classified = (13+15)/36 = 0.7778.

$$t = \frac{0.7778 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{36}}} = 3.334.$$

discriminant coefficients of the earnings/price ratio and default risk are reversed from negative to positive signs. However, this does not imply that the effects of these variables have changed. When we observe the centroids of the two groups for discriminant functions (1) and (2), the centroids are −0.4448 and 0.4448 for group 1 and group 2, respectively. While for discriminant functions (3) and (4), the centroids are 0.4228 and −0.4228 for group 1 and group 2, respectively. Thus, the reversed signs are due to reversed signs of the centroids and not due to the changes in the effects of the independent variables.

However, comparing discriminant function (1) of Table 7 and discriminant function (1) of Table 8, there are only two common independent variables, namely, the earnings/price ratio (E/P) and default risk (α). Default risk has negative signs in both functions, and thus its effects are consistent. However, the earnings/price ratio has a positive sign in Table 7, while it has a negative sign in Table 8. These inconsistent results are apparently due to the fact that the earnings/price ratio is highly correlated with other independent variables such as the dividend yield, financial risk and the income/value ratio. Thus, the most reliable discriminant function must be said to be function (3) or (4) in Table 8.

In effect, discriminant function (3) or (4) in Table 8 suggests the following: (1) If a firm has a higher earnings/price ratio, the firm tends to have a lower systematic risk. (2) If a firm has a higher default risk, the firm tends to have a lower systematic risk.[20]

IX. Regression Analysis of Risk and Return

In Section VI, we have listed four models regarding the relationships between risks and returns. In this Section, we will examine those hypotheses using multiple regression analysis of the empirical data. The four models may be tested in the following forms:

$$E/P = F_1(B/S,\ \hat{s}/Y, \ldots, e_1) \qquad (59)$$

$$V/Y = F_2(B/V,\ \hat{s}/Y, \ldots, e_2) \qquad (60)$$

$$E(R) = F_3(\hat{s}(R),\ \ldots, e_3) \qquad (61)$$

$$E(R) = F_4(\beta,\ \ldots, e_4) \qquad (62)$$

where E/P=the cost of equity capital, B/S=the debt/equity ratio, $\hat{s}/Y$=business risk, V/Y=the average cost of capital (pre-tax), B/V=the debt/value

(20) This result is consistent with Brigham and Crum's observations (1977). See footnote (2) of this paper. A possible explanation for this phenomenon is that when the firm's default risk is large, dividend yield is already close to zero, and the stock price is close to a bottom line and tends to stay constant. As a result the expected return is close to zero, and the slope of the characteristic line, i.e., the β value will be small.

ratio as a measure of financial leverage, $E(R)$=the expected return on common stock, $\hat{s}(R)$=total risk or the standard deviation of the rate of return on common stock, and β=systematic risk of the rate of return on common stock. All the above variables are measured in percents except for systematic risk β which is measured in whole numbers.

Since the expected relationships were already explained in Section VI, now we will discuss the empirical results. A large number of regression equations were calculated by ordinary least squares method with the empirical data for the 36 corporations. Some of the relevant regression equations are listed in Tables 9 and 10. We may summarize the following observations:

1. The Cost of Capital Hypotheses

According to the Modigliani-Miller model (1958, 1963) and the Miller model (1977), Equations (51) and (53), the cost of equity capital should be linearly and positively related to the debt/equity ratio. In regression Equation (1) of Table 9, indeed the debt/equity ratio (B/S) has a positive sign and is significant at the 5% level.[21] This result is consistent with both the Modigliani-Miller model and the Miller model. However, it should be noted that the correlation coefficient is extremely low.

Furthermore, as Modigliani and Miller argue, in order to test the net effect of financial leverage, we have to hold business risk constant. For this reason, in many previous empirical studies, they selected firms of the same industry assuming that such firms should have approximately the same business risk. However, firms of the same industry tend to have just as much significant differences in the degrees of business risk as the firms of different industries.[22] Thus, whether the firms are of the same industry or not, the business risk variable should be included in a regression equation to measure the net effect of financial leverage on the cost of capital.

(21) When the simple correlation coefficient is tested at a two tail 5% level it is not significant, but when the regression coefficient is tested at a one tail 5% level, it is significant.

(22) Wippern (1966) and Gonedes (1969) found that firms of the same industry have significantly different degrees of business risk as much as among the firms of the different industries.

Table 9. Regression Results for the Cost of Capital

	(1) Intercept	(2) B/S (42.45)	(3) $\hat{s}/Y$ (26.65)	(4) $(B/S)^2$ (3082.8)	R^2	$\bar{R}^2$	SEE	F
(1) E/P (12.15)	10.51 (8.62)	0.0388 (1.77)*			0.084	0.057	4.71	3.12
(2)	9.079 (6.03)	0.0303 (1.36)	0.067 (1.56)		0.147	0.095	4.62	2.84
(3)	9.979 (7.28)		0.0816 (1.93)*		0.147	0.095	4.62	2.84
(4)	7.932 (4.93)	0.1998 (2.72)*		−0.0014 (−2.29)*	0.209	0.161	4.45	4.36
(5)	7.192 (4.16)	0.1754 (2.31)*	0.0485 (1.14)	−0.0012 (−1.99)*	0.240	0.169	4.43	3.37
(6)	9.183 (6.08)	0.0295 (1.33)	0.0968 (1.84)*	−0.0302 g (−0.98)	0.172	0.094	4.62	2.21
	Intercept	B/V (25.80)	$\hat{s}/Y$ (26.65)	$(B/V)^2$ (930.86)	R^2	$\bar{R}^2$	SEE	F
(7) Y/V (18.41)	17.67 (7.54)	0.0285 (0.37)			0.004	−0.025	7.51	0.14
(8)	17.51 (1.96)		0.0336 (0.49)		0.007	−0.022	7.50	0.25
(9)	17.13 (6.28)	0.0197 (0.24)	0.0289 (0.40)		0.009	−0.051	7.61	0.15
(10)	14.44 (4.07)	0.3779 (1.20)		−0.0062 (−1.21)	0.046	−0.012	7.46	0.80
(11)	13.93 (3.65)	0.3680 (1.21)	0.0278 (0.39)	−0.0062 (−1.19)	0.051	−0.038	7.56	0.57
(12)	17.21 (6.22)	0.0188 (0.23)	0.0524 (0.60)	−0.0241 g (−0.47)	0.016	−0.077	7.70	0.17

Note: The numbers in parentheses under the dependent and independent variables are their mean values. All values are in %. The numbers in parentheses below the regression coefficients are their t-ratios. * Significant either at the 5% level or 1% level. g = the growth rate of earnings (EBIT) (%).

For this reason, we have included both the debt/equity ratio and business risk in Regression Equation (2) of Table 9. The result is that none of the two independent variables are significant at the 5% level, though the simple correlation coefficient between the two variables is not significant. We note that business risk is significant in Regression Equation (3) where business risk is the only independent variable.

To examine a possible non-linear relationship, Regression Equations (4) and (5) are calculated. We note that business risk is not significant, but the two debt/equity ratio variables are all significant at the 5% level. In Regression Equation (6), we have included the growth rate of earnings (EBIT), but it is not significant. Since there is a high correlation between the growth rate of EBIT and business risk, we have tested regression equations with the growth rate of EBIT but without business risk, but the growth rate was still not significant.

Similarly, to test the statistical relationship between the average cost of capital (Y/V) and financial leverage or the debt/value ratio (B/V), a large number of regression equations were calculated. Some of the results are shown in Regression Equations (7)∼(12). We note that the debt/equity ratio is not significant in both equations. From the above regression results, we may make the following interpretations:

First, as to the cost of equity capital, either the debt/equity ratio or business risk is significant in explaining the variations in the cost of equity capital. These results are consistent with both the Modigliani-Miller hypothesis and the Miller hypothesis, i.e., Equations (51) and (53). However, it should be noted that the coefficient of determination is extremely low.

Second, the statistical insignificance of the regression equations, (7)∼(12), for the average cost of capital does not support the Modigliani-Miller hypothesis or Equation (52), but supports the Miller market equilibrium hypothesis or Equation (54) in which financial leverage is independent of the average cost of capital.

Third, for the cost of equity capital, the nonlinear, quadratic regression equations are a little better than the linear regression equations in $\bar{R}^2$, *SEE* and F values. These results suggest that the cost of debt capital may not stay constant, but may increase with financial leverage.

In the linear Modigliani-Miller model and the Miller model, the cost of debt capital, i.e., the rate of interest is assumed constant, independent of financial leverage. However, we may assume that the cost of equity capital rises as the amount of debt capital increases. So, if we assume that the cost of debt capital

(i) is an increasing linear function of the debt/equity ratio, we may write[23]

$$i=c+d(B/S) \qquad (63)$$

where i=the market rate of interest, c=the leverage risk free rate of interest, d=the leverage risk premium rate of interest, and B/S=the debt/equity ratio. Substituting Equation (63) in Equations (41)～(54), we obtain the nonlinear Modigliani-Miller model and Miller's market equilibrium model.

Nonlinear Modigliani-Miller model:

$$E/P=\rho+(\rho-c)(1-t_c)(B/S)-d(1-t_c)(B/S)^2 \qquad (64)$$

$$Y(1-t_c)/V=\rho-\rho t_c(B/V) \qquad (65)$$

Nonlinear Miller model:

$$E/P=\rho+(\rho-c)(B/S)-d(B/S)^2 \qquad (66)$$

$$Y(1-t)/V=\rho \qquad (67)$$

In effect, the regression results for the 36 corporations tend to support the nonlinear version of the Miller market equilibrium model.

The nonlinear cost of equity capital model is depicted in Figure 4. When the average cost of capital k_a is constant, the optimal capital structure is indeterminate. However, if the objective of the firm is to maximize the cost of equity capital, the optimal capital structure is determined at $(B/S)^*$, where the cost of equity capital $K_e=E/P$ is a maximum.[24]

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(23) The possibility of a non-linear interest rate function is explained in Modigliani-Miller (1958, pp. 274-276 and Figure 2). A general interest rate function is given as $i=f(B/S)$.
Another form of interest rate function is given by Haley and Schall (1979, p. 38): $i=a+(k_a-a)(B/V)^2$, where a=the leverage free rate of interest, k_a=the average cost of capital, and B/V=the debt/value ratio. In Haley and Schall, if $B/V=0$, $i=a$; if $B/V=1$, $i=k_a$.

(24) Figure 17-2 on page 359 of Brealey and Myers (1981) is exactly consistent with our regression results. Also see Figure 11-3 on page 431 of Mao (1969). Other textbooks show convex curves for the cost of equity capital in the presence of the bankruptcy and agency costs. Brigham and Weston (1981, pp. 608, 612), Van Horne(1980, p. 276), Copeland and Weston (1979, p. 306).
For alternative empirical results see Modigliani-Miller (1958, 1966), Barges (1963), Weston (1963), Melnyk (1977).
The regression results in Table 9 are subject to the following problems: (1) The random variable P is in the denominator of both the dependent variable(E/P) and the independent variable (B/S, where $S=PN$). This causes an upward bias in the regression coefficient. For this reason, Barges (1963) used the book value of financial leverage instead of the market value. (2) The numerator of the dependent variable E, the earnings per share, will depend upon the value of equity capital as well as the value of the debt capital. On the

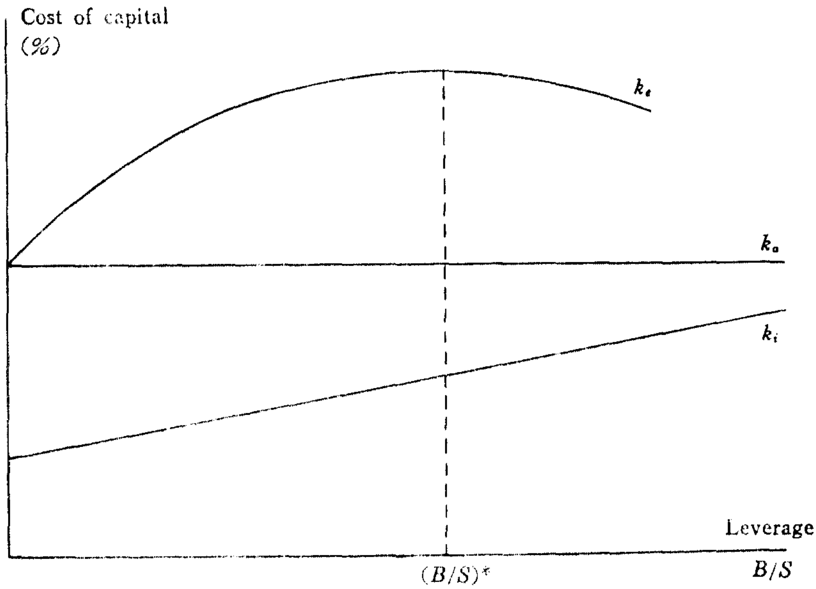

Fig. 4. Nonlinear Cost of Equity Capital

2. The Capital Market Hypotheses

As stated before, the current theories on the capital markets consist of two hypotheses: the capital market line hypothesis (the capital market theory), Equation (55), and the security market line hypothesis (the capital asset pricing model), Equation (56). These two equations are expressed in functional forms as Equations (61) and (62). With the 36 corporate data, we have calculated a large number of regression equations by the ordinary least squares methed. Some of the regression results are summarized in Table 10. We may make the

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other hand, the value of equity capital will depend upon the earnings per share. This will cause a simultaneous equation bias, an upward bias both in the intercept and the regression coefficient. Hu (1973, p. 125). To remove such a bias, instead of the ordinary least squares method, the two-stage least squares method may be used as was done so by Modigliani-Miller (1966).

Also, we have included the growth rate of EBIT as an additional independent variable, but it was not significant. The growth rate g was highly significant in Weston (1963).

The modern theory of finance appears to be on the agreement that the goal of a financial manager is to maximize the value of the firm. However, we could argue that the goal of management is to maximize the value of the firm, while the goal of stock holders is to maximize the rate of return on common stock, i.e., the cost of equity capital. If the power of the stock holders is dominant over the power of management, the goal of the firm will be to maximize the rate of return on common stock. For discussion of conflicting goals of management and stock holders, see Donaldson (1963) and Lewellen (1969).

Table 10. Regression Results for the Expected Return on Common Stocks

	(1) Intercept	(2) $\hat{s}(R)$ (20.37)	(3) $\hat{s}(R)^2$ (419.94)	(4) $\hat{s}/Y$ (26.65)	(5) α (17.24)	R^2	$\bar{R}^2$	SEE	F
(1) $E(R)$ (21.12)	11.76 (1.85)	0.4596 (1.73)*				0.081	0.054	19.90	2.99
(2)	14.01 (1.19)	0.2404 (0.24)	0.0039 (0.23)			0.082	0.063	19.80	1.79
(3)	9.188 (0.77)	0.0746 (0.08)	0.0053 (0.32)	0.2780 (1.51)		0.143	0.063	19.80	1.79
(4)	12.35 (2.12)			0.3291 (1.83)*		0.141	0.089	19.60	2.70
(5)	5.22 (0.71)	0.488 (1.78)*		0.3279 (1.80)*	−0.1612 (−1.46)	0.194	0.119	19.30	2.57
	Intercept	β (1.04)	β^2 (1.12)	$\hat{s}/Y$ (26.65)	α (17.24)	R^2	$\bar{R}^2$	SEE	F
(6) $E(R)$ (21.12)	6.857 (0.37)	13.71 (0.79)				0.018	−0.011	20.60	0.62
(7)	121.00 (1.58)	−198.80 (−1.43)	95.32 (1.54)			0.083	0.028	20.20	1.50
(8)	125.60 (1.73)	−228.50 (−1.72)*	109.90 (1.86)*	0.3793 (2.17)*		0.201	0.126	19.20	2.67
(9)	−4.822 (−0.26)	16.17 (0.96)		0.3424 (1.90)*		0.115	0.061	19.90	2.13
(10)	−5.717 (−0.30)	17.62 (1.04)		0.3928 (2.11)*	−0.1138 (−1.04)	0.143	0.063	19.80	1.78
(11)	21.95 (5.56)				−0.0482 (−0.44)	0.006	−0.024	20.70	0.19
(12)	−1.456 (−0.08)	13.50 (0.80)		$0.5060\hat{s}(e)$ (1.95)*		0.119	0.066	19.80	2.23
(13)	96.65 (1.28)	−167.6 (1.23)	81.27 (1.34)	$0.4608\hat{s}(e)$ (1.78)*		0.166	0.088	19.60	2.12
(14)	83.84 (1.05)	−152.5 (−1.09)	74.40 (1.19)	$1.018\hat{s}(e)$ (0.98)	$-0.0103\hat{s}(e)^2$ (−0.56)	0.174	0.068	19.80	1.64

Note: The numbers below the dependent and independent variables are their mean values. The numbers below the regression coefficients are their t-ratios. * Significant either at the 5% level or 1% level.

following observations:

First, according to the capital market line hypothesis, the expected return $E(R_i)$ should be linearly and positively correlated with its total risk or the standard deviation of the rate of return on common stock $\hat{s}(R_i)$. In Regression Equation (1) of Table 10, indeed total risk $\hat{s}(R)$ has a positive sign and it is significant at the 5% level. This result is consistent with the capital market line hypothesis that the expected return is a linear positive function of its total risk. However, we note that the coefficient of determination is extremely low.

To examine a possible nonlinear relationship, Regression Equations (2) and (3) are calculated. None of the variables are significant at the 5% level. In Regression Equation (4), business risk is the only independent variable, and it is significant. When total risk, business risk and default risk are included in Regression Equation (5), total risk and business risk are significant, but default risk is not.

Second, to test the security market line hypothesis, the expected return is regressed on systematic risk β. Some of the regression equations are listed in Table 10. In Regression Equation (6), systematic risk β is not significant at the 5% level. This result does not support the security market line hypothesis.

In order to examine the possibility of a nonlinear relationship, Regression Equations (7) and (8) are calculated.[25] In Equation (7), systematic risk β and β^2 are included, and both variables are not significant. But when β, β^2, and business risk $\hat{s}/Y$ are included in Equation (8), all the three variables are significant. When systematic risk and business risk are included in Regression Equation (9), only business risk is significant. When systematic risk and default risk are included in Equation (10), only business risk $\hat{s}/Y$ is signifi-

(25) If the market is not efficient, we should have a nonlinear equation in systematic risk β. To show this, rewriting Equation (55),

$$E(R_i)=R_F+\frac{E(R_m)-R_F}{\sigma_m}\sigma_i \qquad \text{(a)}$$

where $\sigma_i^2=\beta_i^2\sigma_m^2+\sigma_e^2$. (b)

If the market is efficient,

$$\sigma_e^2=0, \qquad \text{(c)}$$

$$\sigma_i=\beta_i\sigma_m. \qquad \text{(d)}$$

Substituting (d) in (a), we have the security market line:

$$E(R_i)=R_F+[E(R_m)-R_F]\beta_i \qquad \text{(e)}$$

However, if the market is not efficient, $\sigma_e^2\neq 0$. Substituting (b) in (a), we have a nonlinear equation in β:

$$E(R_i)=R_F+\left[\frac{E(R_m)-R_F}{\sigma_m}\right]\frac{[\beta_i^2\sigma_m^2+\sigma_e^2]}{\sigma_i}$$

$$=R_F+[E(R_m)-R_F]\left[\frac{\beta_i^2\,\sigma_m}{\sigma_i}\right]+[E(R_m)-R_F]\frac{\sigma_e^2}{\sigma_m\,\sigma_i} \qquad \text{(f)}$$

where $\sigma_m=\hat{s}(R_m)$, $\sigma_i=\hat{s}(R_i)$, and $\sigma_e=\hat{s}(e_i)$.

cant.[26] In Equation (11), default risk is the only independent variable, but it is not significant. When both systematic risk and unsystematic risk are included in Regression Equations (12) and (13), only unsystematic risk is significant and systematic risk is not.

In effect, the above regression results do not support the security market line hypothesis (CAPM), but they tend to support the capital market line hypothesis. However, we cannot refute a theory with empirical results alone as long as such empirical results are subject to certain statistical problems. First, not only the sample size is rather small, but it may be subject to sampling errors and measurement errors. For instance, if the number of observation years is different to calculate the expected return, total risk and systematic risk, the statistical results may be different. And *a priori* we cannot tell exactly what is the optimal number of observation years.

Second, another problem lies in the fact that *ex post* average returns are used as the proxies for the *ex ante* expected returns on the market portfolio and individual securities. Thus, if the sample statistical data are taken during the prosperity years of the stock market, the average return on the market portfolio $\bar{R}_m$ will be greater than the risk free rate R_F, and the security market line may be positively sloped, and it may be consistent with the capital asset pricing model. However, if the statistical data are taken during the recession

(26) During the period of 1975～78, the Standard and Poor's 500 composite stock prices are 86.16, 102.01, 98.20, and 96.02. Thus the rates of change in the SP stock prices are 18.40%, −3.73%, and −2.22% during 1976～78. The dividend yields are 3.77%, 4.62%, and 5.28% during 1976～78. The average rate of change in the stock prices is 4.15%, and the average dividend yield is 4.56% during 1976～78. If the dividend yields are not included, the variance of the stock return is 152.86%. If the dividend yields are included, the variance of the stock return is 205.94%. We have calculated unsystematic risk by using Equation (32):

$$V(e_i)=V(R_i)-\beta_i^2 V(R_m)$$

The above method produced negative values of unsystematic risk for some corporations. Thus we have taken absolute values of the variance to calculate the standard deviation of the stock returns. Regression Equations (12)～(14) in Table 10 are obtained by using $V(R_m)=152.86$. When $V(R_m)=205.94$, both systematic risk and unsystematic risk were not significant. The square root of $V(e_i)$ is defined as $\hat{s}(e_i)$, i.e., the standard deviation of individual stock returns.

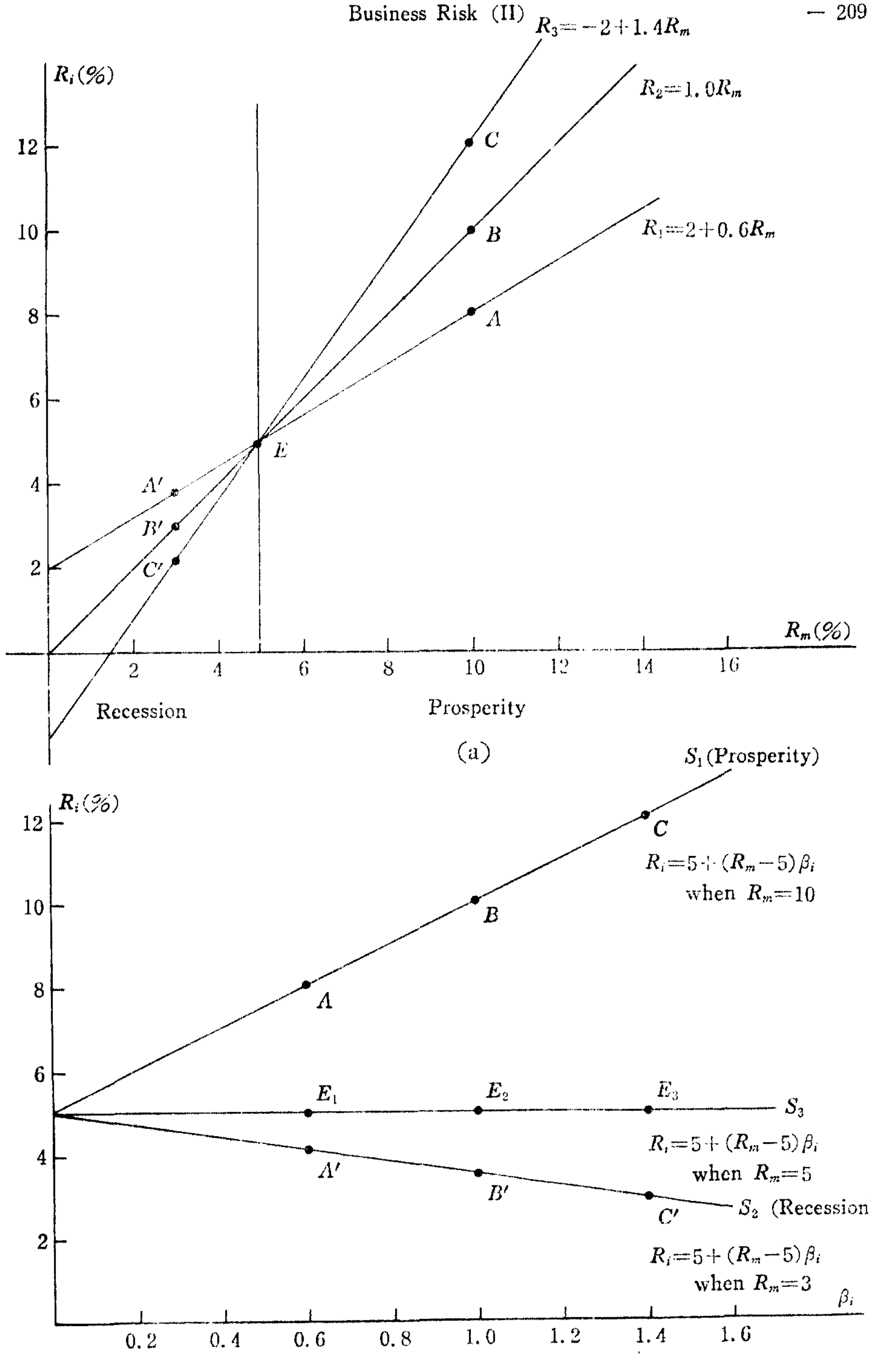

Fig. 5. Characteristic Line and Security Market Line (Cross-Section Data)

years of the stock market, when $\bar{R}_m < R_F$, then the security market line will be negatively sloped. If the statistical data are taken during the stagnant period of the stock market, when $\bar{R}_m = R_F$, the security market line may be horizontal and systematic risk β may be insignificant.

This point is illustrated in Figure 5. In panel (a), systematic risks, i.e., β_1, β_2, and β_3 are obtained from the three characteristic lines using the return data of both prosperity and recession periods of the stock market. These systematic risk values are then used to draw the security market line. However, panel (b) illustrates that the slope of the security market line would depend upon the stock market conditions. At the time of prosperous stock market, when $\bar{R}_m > R_F$, the security market line will be positively sloped. At the time of recession in the stock market, when $\bar{R}_m < R_F$, the security market line will be negatively sloped. And when the stock market is stagnant, when $\bar{R}_m = R_F$, the security market line will be horizontal and the *ex post* rate of return on stock will be independent of systematic risk.

The above considerations suggest that even if the theoretical capital asset pricing model could be valid, an empirical *ex post* security market line could be inconsistent with the theoretical security market line. Thus, the practical usefulness of the capital asset pricing model may have only a limited value, and may need a careful caution in its applicationin and interpretation.[27]

(27) For other empirical studies on the capital asset pricing model, see Sharpe (1965), Jensen (1969), Douglas (1968), Miller and Scholes (1972), Black, Jensen, and Scholes (1971), Friend and Blume (1971), Blume and Friend (1973), and Fama and MacBeth (1973). The above empirical studies do not support the CAPM except the last Fama and MacBeth. The above studies are briefly summarized in Tinic and West (1979, pp. 308–319). Also Modigliani and Pogue (1974) found that both systematic risk and unsystematic risk are significant. Reinganum (1981) found that portfolios of the same systematic risk yielded different returns. That is, the portfolio containing the smallest firms realized average rates of return more than 20 % per year higher than those of the portfolio containing the largest firms.

For the empirical results for the risk-premium curves (the capital market line), see Soldofsky and Max (1978).

For a critique of past and present empirical tests of the CAPM, see Roll (1977, 1978). He argues that the true market portfolio should include all assets such as stocks, bonds, options, human capital, houses, land, etc., and thus the CAPM is untestable unless the exact

3. Default Risk and Systematic Risk

In Section VIII, we have examined determinants of risk levels of the firms using discriminant analysis. In this Section, we examine the statistical significance of the determinants using regression analysis. Some of the regression results are summarized in Table 11.

In Regression Equation (1), the dependent variable is the nominal value of default risk. When the three independent variables are included, the debt/value ratio (B/V) and the income/value ratio (Y/V) are significant, but business risk ($\$/Y$) is not significant at the 5% level. The linear multiple regression equation explains about 66.8% of total variation in default risk of the firms.

In Regression Equation (2), the dependent variable is the categorical value of default risk as in Table 6. That is, if the firm's default risk is less than or equal to 3.25% (the median), the default risk level is assigned the value of 1, and if the firm's default risk is greater than 3.25%, the firm's risk level is assigned the value of 2. In Regression Equation (3), all the three independent variables are significant and have expected signs. That is, if business risk and the debt/value ratio is low, the firm tends to have a high default risk level. These results are consistent with those of discriminant analysis in Table 7.

A similar method was applied to regression analysis of systematic risk of the rate of return on common stocks of the firms. In Regression Equations (3)~(6), β_i is the nominal value of systematic risk, and β_c is the categorical value of systematic risk. That is, if systematic risk is less than or equal to 1.0 (systematic risk of the market portfolio), the systematic risk level of the firm

composition of the true market portfolio is known and used in the test. Thus even if abnormal returns exist using the market portfolio of stocks, we cannot necessarily conclude that the capital markets are inefficient, and thus the CAPM cannot be used to measure the abnormal returns. Mayers and Rice (1979) argue that until a better model is found the proxy market portfolio may be used to test the CAPM, and the security market line may be used to evaluate portfolio performance.

As an alternative to the mean-variance (MV) CAPM, there is the linear risk tolerance (LRT) CAPM. See Roll (1973), Grauer (1978, 1981), Hakansson (1970, 1971), and Rubinstein (1974, 1976).

Table 11. Regression Results for Default Risk and Systematic Risk

A. Default Risk

	(1) Intercept	(2) $\hat{s}/Y$	(3) B/V	(4) Y/V	R^2	$\bar{R}^2$	SEE	F
(1) α_i (17.24)	9.806 (0.96)	0.1688 (0.94)	1.355 (6.66)*	−1.740 (−3.97)*	0.668	0.636	19.1	21.44
(2) α_c (1.2)	1.068 (5.97)	0.0088 (2.79)*	0.0185 (5.18)*	−0.0153 (−1.99)*	0.597	0.559	0.337	15.86

B. Systematic Risk

	(1) Intercept	(2) E/P	(3) $\hat{s}/Y$	(4) $E(R)$	(5) $\hat{s}(R)$	(6) α	R^2	$\bar{R}^2$	SEE	F
(3) β_i (1.04)	1.206 (11.58)	−0.0190 (−2.47)*	−0.0004 (−0.21)	0.0032 (1.70)*	0.0001 (0.13)	0.000 (0.11)	0.210	0.079	0.192	1.60
(4) β_c (1.2)	2.033 (7.65)	−0.0388 (−1.98)*	−0.0006 (−0.12)	−0.0041 (0.86)	−0.0023 (−0.31)	−0.0050 (−1.69)*	0.204	0.071	0.489	1.53
(5) β_i (1.04)	1.212 (13.08)	−0.0142 (−2.09)*				0.00006 (0.06)	0.120	0.067	0.193	2.25
(6) β_c (1.2)	1.994 (8.80)	−0.0330 (−1.99)*				−0.0054 (−2.14)*	0.184	0.134	0.472	3.71

Note: α_i=the dependent variable, default risk, is measured in ordinary metric values.
α_c=default risk is measured in categorical values. If $\alpha \leq 3.25\%$, then $\alpha_c=1$, if $\alpha > 3.25\%$, then $\alpha_c=2$.
β_i=the dependent variable, systematic risk, is measured in ordinary metric values.
β_c=systematic risk is measured in categorical values. If $\beta \leq 1.0$, then $\beta_c=1$, if $\beta > 1.0$, then $\beta_c=2.0$
The numbers in parentheses below the dependent and independent variables are their mean values, and numbers in parentheses below the regression coefficients are their t-ratios.
* Significant either at the 5% level or 1% level.

is assigned the value of 1.0, and if systematic risk is greater than 1.0, the systematic risk level of the firm is assigned the value of 2.0.

In Regression Equation (3) of Table 11, the nominal value of systematic risk β_i is regressed on the five independent variables as in the discriminant analysis of Table 8. Out of the five independent variables, only the earnings/price ratio (E/P) and the expected return $E(R)$ are significant at the 5% level. The earnings/price ratio has a negative sign, and the expected return has a positive sign. These signs are consistent with the signs of the discriminant analysis in Table 8. However, the correlation coefficient between the two variables are significant ($r=0.401$). In Regression Equation (4), the categorical value β_c is regressed on the five independent variables. Only the earnings/price ratio and default risk are significant. The earnings/price ratio and default risk both have negative signs. In other words, as both the earnings/price ratio and default risk increase, the firm tends to have a low systematic risk.

In Regression Equations (5) and (6), only two independent variables are included to reduce multicollinearity. In both equations, the earnings/price ratio is significant and has negative signs. However, default risk has a positive sign but not significant in Regression Equation (5), but it has a negative sign and is significant in Regression Equation (6).

In effect, in the above regression equations, default risk is either insignificant with a positive sign, or significant with a negative sign. These results suggest that systematic risk does not necessarily accurately reflect the risks associated with the business and financial conditions of the firm. In other words, a firm with a low systematic risk β could be a firm on the brink of default and bankruptcy. Thus a portfolio of low systematic risk values could be a portfolio of "risky" firms.[28] In such a case, default risk of a firm must be considered as an additional factor in portfolio selection.

4. The Ratings of Common Stocks

In the *Moody's Handbook of Common Stocks*, each corporation or its stock is

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(28) See footnote (2). Although default risk is not significantly correlated with systematic risk β_i, it is significantly correlated with total risk $\hat{s}(R)$ and unsystematic risk $\hat{s}(e)$. See Footnote (26).

$$\hat{s}(e) = 178.2 + 9.431\alpha \qquad R^2 = 0.162 \quad \bar{R}^2 = 0.138 \quad S = 6.90 \quad F = 6.59$$
$$(1.36) \quad (2.57)^*$$

$$\hat{s}(R) = 18.12 + 0.1307\alpha \qquad R^2 = 0.107 \quad \bar{R}^2 = 0.081 \quad S = 12.1 \quad F = 4.07$$
$$(7.82) \quad (2.02)^*$$

However, we note that the correlation coefficients are extremely low.

Also, we have obtained the following regression equations in order to evaluate the effect of default risk on the return variables:

$$\frac{E}{P} = 12.5378 - 0.0223\alpha \qquad R^2 = 0.023 \quad \bar{R}^2 = 0.007 \quad SE = 4.87 \quad F = 0.74$$
$$(15.44)(-0.86)$$

$$\frac{D}{P} = 3.5923 - 0.0086\alpha \qquad R^2 = 0.024 \quad \bar{R}^2 = -0.005 \quad SE = 1.78 \quad F = 0.82$$
$$(12.12)(-0.91)$$

$$\frac{\Delta P}{P} = 18.3624 - 0.0465\alpha \qquad R^2 = 0.005 \quad \bar{R}^2 = 0.054 \quad SE = 19.95 \quad F = 2.99$$
$$(5.31)(-0.42)$$

$$E(R) = 21.9521 - 0.0482\alpha \qquad R^2 = 0.006 \quad \bar{R}^2 = -0.024 \quad SE = 20.75 \quad F = 0.19$$
$$(6.35)(-0.44)$$

$$\frac{Y}{V} = 19.83 \quad -0.0827\alpha \qquad R^2 = 0.125 \quad \bar{R}^2 = 0.099 \quad SE = 7.04 \quad F = 4.84$$
$$(14.80)(-2.20)^*$$

Table 12. The Rating of Common Stocks

	Intercept	Risk variables	Leverage B/S	R^2	$\bar{R}^2$	SEE	F
(1) M.R. (20.97)	22.54 (14.91)	-0.0911α $(-2.15)^*$		0.120	0.094	0.795	4.64
(2)	27.16 (11.36)	$-0.3040\ \hat{s}(R)$ $(-3.04)^*$		0.211	0.190	7.51	9.22
(3)	29.18 (12.72)	$-0.2014\ \hat{s}(R)$ $(-2.05)^*$	$-0.0968\ B/S$ $(-2.82)^*$	0.366	0.328	6.85	9.53
(4)	19.97 (2.63)	$0.9614\ \beta$ (0.13)		0.001	-0.029	8.47	0.02
(5)	29.37 (4.21)	$-2.943\ \beta$ (-0.47)	$-0.1257\ B/S$ $(-3.67)^*$	0.290	0.247	7.25	6.75

Note: * Significant either at the 5% level or at the 1% level.

evaluated by the following six grades: high grade ($A=40$), investment grade ($B=30$), high medium grade ($C^+=25$), medium grade ($C=20$), low medium grade ($C^-=15$), and speculative grade ($D=10$). [29]

In order to examine the statistical significance of risk-return variables in the Moody's corporate ratings, we have calculated a number of regression equations with various combinations of the risk-return variables. Some of the results are presented in Table 12.

In Regression Equation (1), the only independent variable is default risk. It has a significant negative sign, as is expected. In Regression Equation (2), total risk is the only independent variable, and it is significant. In Regression Equation (3), total risk and the debt/equity ratio (B/S) are the two independent variables, and both variables are significant. It suggests that if a firm has a higher total risk and a higher debt/equity ratio, the firm tends to have a low grade in common stock.

In Regression Equation (4), the only independent variable is systematic risk β, but it is not significant. In Regression Equation (5), systematic risk and

(29) According to the *Moody's Handbook of Common Stocks*, "the assigned grade is based on analysis of each company's financial strength, ability to withstand economic or business reversals, stability of earnings, and record of dividend payments. Other considerations include conservativeness of capitalization, depth and caliber of management, accounting practices, technological capabilities and industry position" (1979 ed., p. 4a). The alphabetical and numerical grades are assigned by this author.

For studies on the bond and stock ratings, see Pogue and Soldofsky (1969), West (1970), Pinches and Mingo (1973), and O'Connor (1973).

the debt/equity ratio are included. Only the debt/equity ratio is significant and it has a negative sign.

In effect, the Moody's ratings are consistent with theoretical effects of default risk, total risk and financial leverage. However, the systematic risk β does not appear to play an important role in the Moody's corporate ratings. Also, we note that the coefficient of determination is at most 0.366 for Regression Equation (3).

X. Summary and Conclusions

In this paper, we have placed the major emphasis on the empirical methods of measuring business risk, financial risk and particularly, default risk. Default risk may be measured in various ways. But in this paper, we have shown the method of measuring default risk in terms of probability. In Sections II and III, the methods of measuring the above risks were described, and in Section V, the empirical results were presented for the 36 corporations. For instance, 75% of the 36 corporations in the sample had default risk less than 10%, but about 11% of the firms (4 firms) had default risk greater than 90%. The average default risk was 17.24%.

The familiar formulas for measuring the risks associated with the rate of return on common stocks are listed in Section IV, and the empirical results are also presented in Section V, together with the risks associated with the business and financial conditions of the firms. In Sections VI~IX, factor analysis, discriminant analysis, and regression analysis were used to examine the statistical relationships among the various risk and return measures.

The regression results suggest the following conclusions: First, as to the hypotheses on the cost of capital, the regression results are consistent with the Modigliani-Miller model and the Miller equilibrium market model, in which the cost of equity capital is a linear increasing function of the debt/equity ratio. However, when the cost of equity capital is tested as a quadratic function of the debt/equity ratio, the two debt/equity ratio variables are both significant, and the nonlinear regression equation shows a higher $\bar{R}^2$ than the linear regression equation. These results are consistent with the nonlinear versions of the Modigliani-Miller model and Miller equilibrium market model. These results suggest that the cost of debt capital may not be constant, but may be an increasing function of the debt/equity ratio.

As to the average cost of capital, the debt/value ratio is not significant. In other words, the average cost of capital tends to stay constant regardless of the level of financial leverage. These results are consistent with the Miller equilibrium market model.

Second, as to the capital market hypotheses, the regression results are not consistent with the capital asset pricing model (the security market line hypothesis), but they are consistent with the capital market line hypothesis. However, these results do not suggest that the *a priori* capital asset pricing model is wrong, but that the practical usefulness of an *ex post* security market line may depend upon the sampling period. That is, if a sample is taken during the period of a stagnant or bearish stock market, the empirical security market line may be horizontal or negatively sloped. In such a case, the security market line could not be used to evaluate the expected return.

Third, as stated in the above, an empirical systematic risk β may be not only ineffective in evaluating the *ex ante* expected return on common stock, but also it may not necessarily reflect accurately the risks associated with the business and financial conditions of the firm. As we have seen in the regression equations and in the discriminant functions, a higher default risk can be correlated with a lower systematic risk β. In such a case, a low risk portfolio selected on the basis of β values can be actually a portfolio of a high default risk. Thus, in order to select a "safe" security or portfolio, default risk of each security should be evaluated as an additional factor.

However, the above conclusions are necessarily tentative because of statistical and data problems. For instance, the sample size and the sample period could be further increased, and variables may be defined with more refined data. Whether or not the findings are due to sampling and other statistical bias may depend upon further empirical studies.

References

Altman, E.I., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*, Sept. 1968, pp. 589-609.

Altman, E.I., R.G. Haldeman, and P. Narayanan, "Zeta Analysis: A New Model to Identify Bankruptcy Risk of Corporations," *Journal of Banking and Finance*, June 1977, pp. 29-54.

Barges, A·, *The Effects of Capital Structure on the Cost of Capital*, 1963.

Beaver, W.H., "Financial Ratios and Prediction of Failure," *Empirical Research in Accounting: Selected Studies*, supplement to *Journal of Accounting Research*, 1966, pp. 71-111.

Beaver, W.H., "Market Prices, Financial Ratios, and the Prediction of Failure," *Journal of Accounting Research*, Autumn 1968, pp. 179-192.

Bierman, H., Jr., and J.E. Hass, *An Introduction to Managerial Finance*, 1973, pp. 96-98.

Black, F., "Capital Market Equilibrium with Restricted Borrowing," *Journal of Business*, July 1972, pp. 444-455.

Black, F., M.C. Jensen, and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in *Studies in the Theory of Capital Markets*, ed. by M.C. Jensen, 1972, pp. 79-124.

Brealey, R., and S. Myers, *Principles of Corporate Finance*, 1980.

Brigham, E.F., and R.C. Crum, "On the Use of the CAPM in Public Utility Rate Cases," *Financial Management*, Summer 1977, pp. 7-15.

Brigham, E.F., and M.J. Gordon, "Leverage, Dividend, and the Cost of Capital," *Journal of Finance*, March 1968, pp. 85-104.

Chen, K.H., and T.A. Shimerda, "An Empirical Analysis of Useful Financial Ratios," *Financial Management*, Spring 1981, pp. 51-60.

Comrey, A.L., *A First Course in Factor Analysis*, 1973.

Cooley, W.W., and P.R. Lohnes, *Multivariate Data Analysis*, 1971.

Copeland, T.E., and J.F. Weston, *Financial Theory and Corporate Policy*, 1979.

Donaldson, G., *Corporate Debt Capacity*, 1961.

Donaldson, G., "New Framework for Corporate Debt Policy," *Harvard Business Review*, March/April 1962, pp. 117-131.

Donaldson, G., "Financial Goals: Management vs. Stockholders," *Harvard Business Review*, May/June 1963, pp. 116-129.

Donaldson, G., "Strategy for Financial Emergencies," *Harvard Business Review*, November/December 1969, pp. 67-79.

Douglas, G., "Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency," *Yale Economic Essays*, No. 1, 1968, pp. 3-45.

Durand, D., "Cost of Debt and Equity Funds for Business: Trends and Problems of Measurement," in *Conference on Research in Business Finance*, NBER, N.Y., 1952, repr. in E. Solomon, ed., *The Management of Corporate Capital*, 1959, pp. 91-116, and in S.H. Archer and C.A. D'Ambrosio, eds., *The Theory of Business Finance: A Book of Readings*, 2nd ed., 1976, pp. 420-452.

Fama, E.F., "Risk, Return and Equilibrium: Some Clarifying Comments," *Journal of Finance*, March 1968, pp. 29-40.

Fama, E.F., "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, May 1970, pp. 383-417.

Fama, E.F., and J.D. MacBeth, "Risk, Return and Equilibrium: Empirical Tests," *Journal of Political Economy*, May 1973, pp. 607-636.

Findlay, M.C., and E.E. Williams, "Towards More Adequate Debt Service Coverage Ratios," *Financial Analysts Journal*, Autumn 1975, pp. 54-61.

Findlay, M.C., E.E. Williams, and L.A. Gordon, "Towards More Adequate Measures of Lender Protection," *Financial Management*, Autumn 1975, pp. 54-61.

Friend, I., and M. Blume, "Measurement of Portfolio Performance under Uncertainty," *American Economic Review*, Sept. 1970, pp. 561-575.

Gonedes, N.J., "A Test of the Equivalent-Risk Class Hypothesis," *Journal of Financial and Quantitative Analysis*, June 1969, pp. 159-177.

Gordon, M.J., "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-57: Comment," *American Economic Review*, 1967, pp. 1267-77.

Grauer, R.R., "Generalized Two Parameter Asset Pricing Models: Some Empirical Evidence," *Journal of Financial Economics*, March 1978, pp. 11-32.

Grauer, R.R, "Investment Policy Implications of the Capital Asset Pricing Model," *Journal of Finance*, March 1981, pp. 127-141.

Hair, J.F., R.E. Anderson, R.L. Tatham, and B.J. Grablowsky, *Multivariate Data Analysis with Readings*, 1979.

Hakansson, N.H.,"Optimal Investment and Consumption Strategies under Risk for a Class of Utility Functions," *Econometrica*, Sept. 1970, pp. 587-607.

Hakansson, N.H., "Capital Growth and the Mean Variance Approach to Portfolio Selection," *Journal of Financial and Quantitative Analysis*, Jan. 1971, pp. 517-557.

Haley, C.W., and L.D Schall, *The Theory of Financial Decisions*, 2nd ed., 1979.

Hamada, R.S., "Portfolio Analysis, Market Equilibrium and Corporation Finance," *Journal of Finance*, March 1969, pp. 13-31.

Hamada, R.S., "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance*, May 1972, pp. 435-452.

Harman, H.H., *Modern Factor Analysis*, 1976.

Hoel, P.G., *Introduction to Mathematical Statistics*, 1971.

Holland, D. M., "Profitability and Capital Costs for Manufacturing Corporations and All Nonfinancial Corporations," *American Economic Review*, May 1980, pp. 320-325.

Hong, H., and A. Rapport, "Debt Capacity, Optimal Capital Structure, and Capital Budgeting," *Financial Management*, Autumn 1978, pp. 7-11.

Jensen, M.C., "Capital Markets: Theory and Evidence," *The Bell Journal of Economics and Management Science*, Autumn 1972, pp. 357-398.

Kim, J., and C.W. Mueller, *Introduction to Factor Analysis*, 1978.

Kim, J., and C.W. Mueller, *Factor Analysis*, 1978.

Kim, E.H., W.G. Lewellen, and J.J. McConnell, "Financial Leverage Clienteles: Theory and Evidence," *Journal of Financial Economics*, No. 7, 1979, pp. 83-109.

Klecka, W.R., *Discriminant Analysis*, 1980.

Lewellen, W.G., "Management and Ownership in the Large Firm," *Journal of Finance*, May 1969, pp. 299-322.

Lintner, J., "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Reveiw of Economics and Statistics*, Feb. 1965, pp. 13-47.

Mao, J.C.T., *Quantitative Analysis of Financial Decisions*, 1969, pp. 432-452.

Maksoudian, Y.L., *Probability and Statistics with Application*, 1969.

Markowitz, H.M., "Portfolio Selection," *Journal of Finance*, March 1952, pp. 77-91.

Markowitz, H.M., *Portfolio Seletion: Efficient Diversification of Investment*, 1959.

Martin, J.D., and D.F. Scott, Jr., "Debt Capacity and the Capital Budgeting Decision," *Financial Management*, Summer 1976, pp. 7-14.

Mayers,D., and E.M. Rice, "Measuring Portfolio Performance and the Empirical Content of Asset Pricing Models," *Journal of Financial Economics*, No. 7, 1979, pp. 3-28.

Melnyk, Z. Lew, "Cost of Capital as a Function of Financial Leverage," *Decision Science*, July/August 1970, pp. 327-356.

Miller, M., "Debt and Taxes," *Journal of Finance*, May 1977, pp. 261-275.

Miller, M., and M. Scholes, "Rates of Relation to Risk: A Reexamination of Some Recent Findings," in *Studies in the Theory of Capital Markets*, ed. by M. Jensen, 1972, pp. 47-78.

Modigliani, F., and M.H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, June 1958, pp. 261-297.

Modigliani, F., and M.H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review*, June 1963, pp. 433-443.

Modigliani, F., and M.H. Miller, "Reply," *American Economic Review*, June 1965, pp. 524-527.

Modigliani, F., and M.H. Miller, "Some Estimates of the Cost of Capital to the Electric Utility Industry," *American Economic Review*, June 1966, pp. 333-391.

Modigliani, F., and G.A. Pogue, "An Introduction to Risk and Return," *Financial Analysts Journal*, March/April 1974, pp. 68-80; May/June 1974, pp. 69-88.

Mossin, J., "Equilibrium in a Capital Asset Market," *Econometrica*, Oct. 1966, pp. 768-783.

Nie, N.H. and others, eds., *SPSS: Statistical Package for the Social Sciences*, 2nd ed., 1975.

O'Connor, M.C., "On the Usefulness of Financial Ratios to Investors in Common Stock," *Accounting Review*, April 1973, pp. 339-352.

Pinches, G.E., and K.A. Mingo, "A Multivariate Analysis of Industrial Bond Ratings," *Journal of Finance*, March 1973, pp. 1-18.

Pogue, T.F., and R.M. Soldofsky, "What's in a Bond Rating," *Journal of Financial and*

Quantitative Analysis, June 1969, pp. 201-228.

Reinganum, M.R., "Abnormal Returns in Small Firm Portfolios," ***Financial Analysts Journal***, March/April 1981, pp. 52-56.

Roll, R., "Evidence on the Growth Optimum Model," ***Journal of Finance***, June 1973, pp. 551-566.

Roll, R., "A Critique of the Asset Pricing Theory's Tests: Part I," ***Journal of Financial Economics***, March 1977, pp. 129-176.

Roll, R., "Ambiguity When Performance Is Measured by the Securities Market Line," *Journal of Finance*, Sept. 1978, pp. 1051-1069.

Rubinstein, M.E., "A Mean Variance Synthesis of Corporate Financial Theory," ***Journal of Finance***, March 1973, pp. 167-181.

Rubinstein, M., "An Aggregation Theorem for Securities Markets," ***Journal of Financial Economics***, Sept. 1974, pp. 225-244.

Rubinstein, M., "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," ***Journal of Finance***, May 1976, pp. 551-571.

Sharpe, W.F., "Simplified Model for Portfolio Analysis," ***Management Science***, Jan. 1963, pp. 277-293.

Sharpe, W.F., "Capital Asset prices: A Theory of Market Equilibrium under Conditions of Risk," ***Journal of Finance***, Sept. 1964, pp. 425-442.

Sharpe, W.F, "Risk Aversion in the Stock Market: Some Empirical Evidence," ***Journal of Finance***, Sept. 1965, pp. 416-422.

Silvers, J.B., "Liquidity, Risk, and Duration Patterns in Corporate Financing," ***Financial Management***, Autumn 1976, pp. 54-64.

Soldofsky, R.M., and D.F. Max, ***Holding Period Yields and Risk-Premium Curves for Long-Term Marketable Securities: 1910-1976***, New York University Monograph Series in Finance and Economics, 1977.

Solomon, E., ***The Theory of Financial Management***, 1963.

Stuerle, E., ***Is Income from Capital Subject to Income Taxation?***, OTA Paper 42, U.S. Department of the Treasury, Oct. 1980.

Tatsuoka, M.M., ***Multivariate Analysis***, 1971.

Tinic, S.M., and R.R. West, ***Investing in Securities: An Efficient Markets Approach***, 1979.

U.S. Council of Economic Advisers, ***Economic Report of the President***, Jan. 1980.

Van de Geer, J.P., *Introduction to Multivariate Analysis for the Social Sciences*, 1971.

Van Horne, J.C., *Financial Management and Policy*, 5th ed., 1980.

West, R.R., "An Alternative Approach to Predicting Corporate Bond Ratings," *Journal of Accounting Research*, Spring 1970, pp. 15-21

Weston, J.F., "A Test of Cost of Capital Propositions," *Southern Economic Journal*, Oct. 1963, pp. 105-112.

Weston, J.F., and E.F. Brigham, *Managerial Finance*, 7th ed., 1981.

Wippern, R.F., "A Note on the Equivalent Risk Class Assumption," *Engineering Economist*, Spring 1966, pp. 13-22.

Appendix Figures. Histograms of Risk-Return Variables

Middle of interval	Number of observations
0.	2 **
10.	8 ********
20.	11 ***********
30.	8 ********
40.	1 *
50.	1 *
60.	3 ***
70.	1 *
80.	1 *

(a) Business risk, $\hat{s}/Y$

Middle of interval	Number of observations
0.	1 *
1.	11 ***********
2.	12 ************
3.	4 ****
4.	2 **
5.	1 *
6.	2 **
7.	0
8.	2 **
9.	0
10.	0
11.	0
12.	1 *

(b) Financial risk, $\hat{s}(E/P)$

Middle of interval	Number of observations
0.	23 ***********************
10.	6 ******
20.	1 *
30.	0
40.	0
50.	1 *
60.	0
70.	1 *
80.	0
90.	1 *
100.	3 ***

(c) Default risk, α

Middle of interval	Number of observations
5.	5 *****
10.	3 ***
15.	9 *********
20.	9 *********
25.	4 ****
30.	0
35.	2 **
40.	0
45.	2 **
50.	2
55.	2 **

(d) Total risk, $\hat{s}(R)$

Middle of interval	Number of observations
.7	1 *
.8	3 ***
.9	8 ********
1.0	6 ******
1.1	9 *********
1.2	5 *****
1.3	1 *
1.4	1 *
1.5	1 *
1.6	1 *

(e) Systematic risk, β

Middle of interval	Number of observations
0.	6 ******
20.	12 ************
40.	6 ******
60.	4 ****
80.	2 **
100.	4 ****
120.	2 **

(f) Debt/equity ratio, B/S

Middle of interval	Number of observations
0.	2 **
5.	4 ****
10.	3 ***
15.	3 ***
20.	6 ******
25.	4 ****
30.	2 **
35.	2 **
40.	2 **
45.	2 **
50.	4 ****
55.	2 **

(g) Debt/value ratio, B/V

Middle of interval	Number of observations
0.	1 *
5.	4 ****
10.	4 ****
15.	7 *******
20.	10 **********
25.	4 ****
30.	0
35.	2 **
40.	0
45.	2 **
50.	0
55.	2 **

(h) Standard deviation of the rate of change in stock price, $\hat{s}(\Delta P/P)$

Middle of interval	Number of observations
.0	2 **
.1	2 **
.2	9 *********
.3	4 ****
.4	3 ***
.5	1 *
.6	3 ***
.7	4 ****
.8	3 ***
.9	2 **
1.0	1 *
1.1	1 *
1.2	0
1.3	1 *

(i) Standard deviation of the dividend yield, $\hat{s}(D/P)$

Middle of interval	Number of observations
4.	1 *
6.	4 ****
8.	8 ********
10.	4 ****
12.	4 ****
14.	2 **
16.	8 ********
18.	2 **
20.	1 *
22.	1 *
24.	1 *

(j) Earnings/price ratio, E/P

Middle of interval	Number of observations
−10.	1 *
0.	10 **********
10.	5 *****
20.	5 *****
30.	9 *********
40.	4 ****
50.	0
60.	0
70.	1 *
80.	0
90.	1 *

(k) Expected return on common stock, *E*(*R*)

Middle of interval	Number of observations
−10.	1 *
0.	13 *************
10.	3 ***
20.	5 *****
30.	8 ********
40.	4 ****
50.	0
60.	1 *
70.	0
80.	0
90.	1 *

(l) Rate of change in stock price, $\Delta P/P$

Middle of interval	Number of observations
.0	2 **
.5	2 **
1.0	1 *
1.5	1 *
2.0	3 ***
2.5	1 *
3.0	4 ****
3.5	8 ********
4.0	2 **
4.5	2 **
5.0	2 **
5.5	4 ****
6.0	4 ****

(m) Dividend yield, D/P

Middle of interval	Number of observations
5.	1 *
10.	6 ******
15.	10 **********
20.	11 ***********
25.	3 ***
30.	3 ***
35.	2 **

(n) Average rate of return on capital Y/V