Inferior Fixed Factors and the Competitive Supply Response to Price Changes: Long Run *vs.* Short Run

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It has been widely accepted among economists that, in a competitive market, the supply response to price changes will be greater in the long run than in the short run (for instance see Becker[1] or Hirshleifer[3]). Logically this proposition is supported by two other propositions: the one claiming that in the long run a larger number of firms will be producing at a higher price, and the other claiming that, when price rises, a competitive firm increases its supply more in the long run than in the short run. Both propositions seem intuitively convincing, but after a careful examination, we can find that the latter is not fully proven yet.

The purpose of present paper is to clarify the conditions on which the latter proposition is correct. It will be shown that the proposition crucially depends upon whether the fixed factor(s) is(are) normal or inferior. More precisely, a competitive firm will be shown to exhibit a supply response no less in the long run than in the short run, provided that the fixed factor(s) is(are) not inferior for any factor wages. Therefore, if at least one fixed

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factor is inferior, then it may not be true any more that the competitive supply response to price changes will be greater in the long run than in the short run. Let us call the proposition "When price changes, a competitive firm will exhibit a supply response greater in the long run than in the short run or at least an equal response in both the long run and the short run." as *Proposition B-H.*

In Section I it will be proven that, for an output level in the neighborhood of the long run optimum, the price elasticity of supply for a competitive firm is always greater in the long run than in the short run or at least equal to each other. But an example will be provided to show that this does not necessarily imply that the *Proposition B-H* is true. In Section II we shall derive necessary conditions for the case that the supply response of a competitive firm (to price changes) can be smaller in the long run than in the short run, i.e., for the case that the *Proposition B-H* is false. In Section III we will show that this necessary condition implies, for the case of one variable and one fixed factor, that the fixed factor must be inferior for some output level and factor wages. In Section IV some aspects of the short run cost functions will be investigated in order to generalize the result of Section III to the case of an arbitrary number of factors. It will be shown that the short run cost functions are (strictly) convex in the arguments representing the fixed factors, if the production functions are (strictly) quasi-concave in the arguments representing both fixed and variable factors. In Section V the result of Section III will be generalized to the case of an arbitrary (finite) number of factors. And finally in Section VI some concluding remarks will be given.

I. The Slopes of Marginal Cost Curves: Long Run *vs.* Short Run

In his famous lecture note *Price Theory*, Milton Friedman provided a geometric proof for the proposition that if a competitive firm is currently minimizing its long run total costs, then the relevant short run marginal cost curve will always possess a slope steeper than that of the long run in neighborhood of the current level of output. In this Section we will begin by providing an algebraic proof for this proposition.

We assume a twice differentiable, strictly quasi-concave, and strictly monotone increasing production function $y=f(v,z)$ defined over $n+m$ factors $x=(v,z)=(v_1,v_2,\cdots,v_n;z_1,z_2,\cdots,z_m)$ where the level of output is denoted by y. The symbols v and z represent the vectors of variable and fixed factors respectively. Let $x^0(y)=(v^0(y),z^0(y))$ denote the conditional factor demand function for given output level y and the present factor wages. Let $C(y)$ denote the long run total costs and $C_s(y,\bar{z})$ the short run total costs with the fixed factor $\bar{z}$ for producing the output level y under current factor wages. Clearly the inequality

$$D(y,\bar{z})=C_s(y,\bar{z})-C(y)\geq 0 \tag{1}$$

holds for all non-negative values of y.

If it holds that $\bar{z}=z^0(\bar{y})$ for some $\bar{y}$, i.e., the fixed factor holding $\bar{z}$ is optimal for producing the level of output $\bar{y}$, then obviously the inequality (1) becomes an equality. Therefore at the output level $\bar{y}$ the difference $D(y,\bar{z})$ will assume its minimum value, that is, the values of first and second order derivatives of $D(y,\bar{z})$ will be

$$\frac{\partial D(\bar{y},\bar{z})}{\partial y}=\frac{\partial C_s(\bar{y},\bar{z})}{\partial y}-\frac{\partial C(\bar{y})}{\partial y}=0, \tag{2}$$

and

$$\frac{\partial^2 D(\bar{y},\bar{z})}{\partial y^2}=\frac{\partial^2 C_s(\bar{y},\bar{z})}{\partial y^2}-\frac{\partial^2 C(\bar{y})}{\partial y^2}\geq 0, \tag{3}$$

respectively. The equation (2) implies that if the currently held fixed factors $\bar{z}$ is the long run optimum for producing the output $\bar{y}$, then the relevant short run marginal cost is equal to the long run marginal cost, i.e.,

$$\mathrm{SMC}(\bar{y},\bar{z})=\mathrm{LMC}(\bar{y}). \tag{2'}$$

This result is well-known and is derived in standard textbooks with the help of Envelope Theorem (for instance see Varian[6] or Malinvaud[4]). The inequality (3) implies that the slope of $\mathrm{SMC}(y,\bar{z})$ curve is greater than or at least equal to that of LMC(y) curve at the output level $\bar{y}$, i.e.,

$$\frac{\partial \mathrm{SMC}(\bar{y},\bar{z})}{\partial y} \geq \frac{\partial \mathrm{LMC}(\bar{y})}{\partial y}, \tag{3'}$$

which has been proven by Milton Friedman geometrically. Therefore if a competitive firm is presently minimizing its long run total costs, then at this point the price elasticity of supply for this firm is not less in the long run than in the short run.

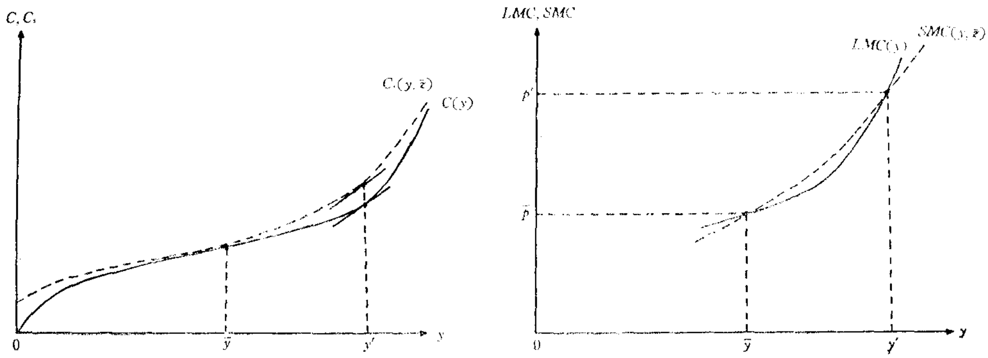
This result supports the *Proposition B-H*, if the price changes are sufficiently small, but not otherwise. Suppose for some output $y'(>\bar{y})$ the following relations hold:

$$\mathrm{SMC}(y',\bar{z}) = \mathrm{LMC}(y'),$$

and (4)

$$\frac{\partial \mathrm{SMC}(y',\bar{z})}{\partial y} < \frac{\partial \mathrm{LMC}(y')}{\partial y}.$$

Then the difference $D(y,\bar{z})$ of equation (2) will assume its maximum value at the output y'. The relation (4) implies that the LMC(y) curve has a slope steeper than the SMC($y,\bar{z}$) curve at the output y'. The Figure 1 (a) and (b) illustrate the case. If the price of output is set at $\bar{p}$ in Figure 1 (b), then the firm will maximize its profit by holding the fixed factors $\bar{z}$ and producing the output $\bar{y}$. Suppose the output price rises. If the new price is higher than the old price $\bar{p}$ but lower than p' in Figure 1 (b), then the supply increase of this firm will be greater in the long run than in the short run. But if the



(a) (b)

Fig. 1.

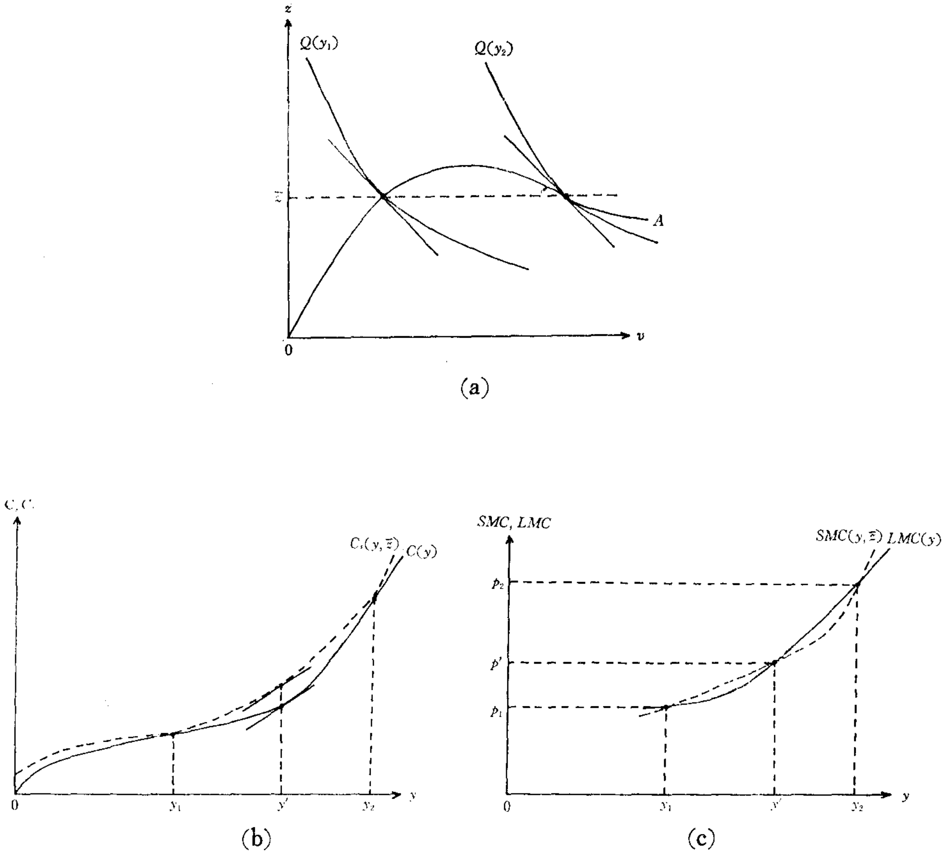


Fig. 2

new price is higher than p', then the supply increase will be, on the contrary, greater in the short run than in the long run.

Thus we must examine if there can exist such a level of output y' under standard assumptions about the production function. The answer is yes, and an example can be provided for this purpose in the case of one fixed factor and one variable factor. The Figure 2 illustrates the case. In Figure 2 (a) the curve OA represents the expansion path under given factor wages, and the curves $Q(y_1)$ and $Q(y_2)$ denote the isoquants for the outputs y_1 and y_2 respectively. We can see from the shape of expansion path that the fixed factor is inferior for some range of output levels. Suppose the level of presently held fixed factor is $\bar{z}$. We can see in Figure 2 (a) that $\bar{z}$ is the common long run optimum level of fixed factor holding for producing both outputs y_1 and y_2. Consequently the long run and short run total cost curves should be tangent to each other at the levels of output y_1 and y_2 as in Figure 2 (b). Then it is clear that there must exist an output y' between y_1 and y_2 at which the difference $D(y, \bar{z})$ of equation (1) assumes its local maximum. The graphs of relevant marginal cost curves are drawn in Figure 2 (c). So in this case a rise of output price from p_1 to some level bewteen p' and p_2 will lead to a supply increase greater in the short run than in the long run.

II. Necessary Conditions for the Failure of Proposition B-H

The preceding example suggests that the long run and short run competitive supply response to price changes may depend upon whether the fixed factor(s) is(are) inferior or not. In this Section we will derive necessary conditions for the failure of the *Proposition B-H* to prevail, i.e., for the case that the competitive supply response can be greater in the short run than in the long run when the price of output changes. And in subsequent Sections we will show that indeed these necessary conditions hold only if at least one fixed

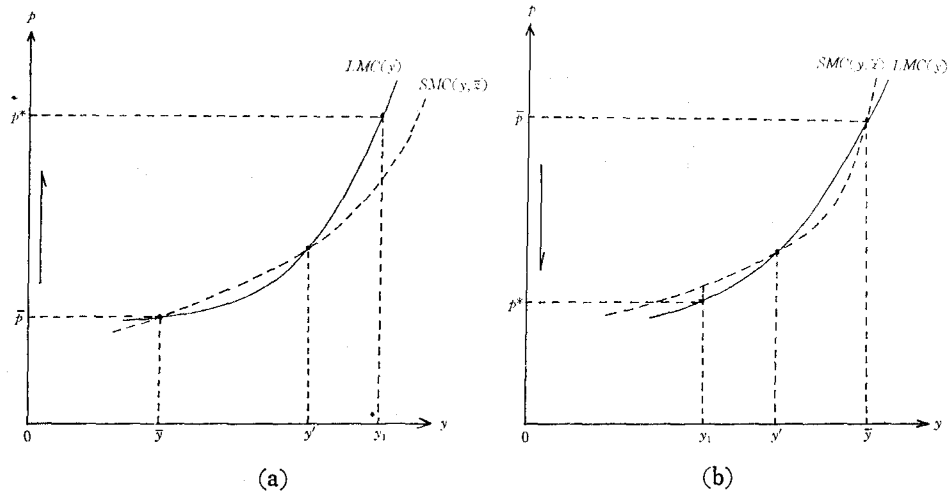


Fig. 3

factor is inferior at some level of output for a certain structure of factor wages.

Consider a competitive firm currently maximizing his long run profit by producing the level of output $\bar{y}$. If a price rise has led this firm's supply to increase less in the long run than in the short run, then there must exist a level of output y_1, with $y_1>\bar{y}$, at which the long run marginal cost is greater than that of the relevant short run. See Figure 3 (a) for this. Similarly Figure 3 (b) depicts the case in which the *Proposition B-H* fails to hold when the output price falls.

Summing up, it is necessary, for the supply of a competitive firm to respond greater in the short run than in the long run to price changes, that there exists a level of output y_1 so that

$$\begin{aligned} &\mathrm{SMC}(y_1, \bar{z})<\mathrm{LMC}(y_1) \text{ if } y_1>\bar{y}, \\ \text{and} \\ &\mathrm{SMC}(y_1, \bar{z})>\mathrm{LMC}(y_1) \text{ if } y_1<\bar{y}, \end{aligned} \tag{5}$$

where the difference $D(y, \bar{z})$ assumes its minimum value zero at the level of output $\bar{y}$.

III. Two-Factor Case

In this section we will consider the simple case of two factors: the one factor v is variable and the other z is fixed. Let $v(y, \bar{z})$ denote the quantity of variable factor required to produce the output y with the holding of fixed factor $\bar{z}$, that is, the inverse of short run production function $y=f(v, \bar{z})$. Let w_v and w_z represent the prices of the variable and fixed factors respectively. Then the short run total costs $C_s(y, \bar{z})$ can be written as

$$C_s(y, \bar{z})=w_v v(y, \bar{z})+w_z\bar{z}. \tag{6}$$

Denote $z^0(y)$ for the long run optimum level of fixed factor for producing the output y. Then clearly the long run optimum level $v^0(y)$ of variable factor can be expressed as $v^0(y)=v(y, z^0(y))$. Accordingly the long run total costs $C(y)$ can be written as

$$C(y)=w_v v(y, z^0(y))+w_z z^0(y). \tag{7}$$

Now let the first inequality of (5) hold for the output y_1, i.e., $\mathrm{SMC}(y_1, \bar{z})<\mathrm{LMC}(y_1)$ and $y_1>\bar{y}$. Also write $z_1=z^0(y_1)$. Then the long run marginal cost $\mathrm{LMC}(y_1)$ can be derived as

$$\begin{aligned}\mathrm{LMC}(y_1)&=\frac{\partial C(y_1)}{\partial y}=w_v\frac{\partial v(y_1, z_1)}{\partial y}+w_v\frac{\partial v(y_1, z_1)}{\partial z}\frac{\partial z^0(y_1)}{\partial y}+w_z\frac{\partial z^0(y_1)}{\partial y}\\&=w_v\frac{\partial v(y_1, z_1)}{\partial y}+w_v\frac{\partial z^0(y_1)}{\partial y}\left(\frac{\partial v(y_1, z_1)}{\partial z}+\frac{w_z}{w_v}\right).\end{aligned} \tag{8}$$

The factor bundle $(v(y_1, z_1), z_1)$ is the conditional factor demand for producing the output y_1 under factor wages (w_v, w_z). And the derivative $\frac{\partial v(y_1, z_1)}{\partial z}$ represents the slope of isoquant $Q(y_1)$ at the factor bundle $(v(y_1, z_1), z_1)$. Therefore the second term on the right hand side of equation (8) must vanish to zero.

Consequently, for the first inequality of (5), we have

$$\mathrm{SMC}(y_1, \bar{z})=w_v\frac{\partial v(y_1, \bar{z})}{\partial y}<w_v\frac{\partial v(y_1, z_1)}{\partial y}=\mathrm{LMC}(y_1), \tag{9}$$

where $y_1>\bar{y}$.

Obviously $\bar{z}$ must be different from z_1. If $\bar{z}>z_1$, then the fixed factor is inferior, since as the level of output increases from $\bar{y}$ to y_1, the use of fixed factor is reduced from $\bar{z}$ to z_1. So consider the case $\bar{z}<z_1$.

If $\bar{z}<z_1$, then, by the inequality (9), there must exist a level of fixed factor z' between $\bar{z}$ and z_1 so that the inequality

$$\frac{\partial}{\partial z}\left(\frac{\partial v(y_1, z')}{\partial y}\right)>0 \tag{10}$$

holds. Let (w_v, w_z') be the wages at which the factor bundle $(v(y_1, z'), z')$ minimizes the cost for producing the output y_1. Then we can write $z'=z^0(y_1, w_v, w_z')$, making the associated wage structure explicit this time. Consequently the equality

$$\frac{\partial v(y_1, z')}{\partial z}=-\frac{w_z'}{w_v}, \tag{11}$$

where $z'=z^0(y_1, w_v, w_z')$, must hold. Now differentiate the equation (11) with respect to the output y and obtain

$$\frac{\partial^2 v(y_1, z')}{\partial y \partial z} + \frac{\partial^2 v(y_1, z')}{\partial z^2} \quad \frac{\partial z^0(y_1, w_v, w_z')}{\partial y} = 0. \tag{12}$$

The equation (12) describes the feature of expansion path when the level of output y is increased, keeping the wages (w_v, w_z') constant.

Now the first term on the left hand side of the equation (12) is positive by the inequality (10) and Young's rule. Therefore the second term must be negative in order for the equation (12) to hold. When the isoquants are strictly convex toward the origin, the technical rate of substitution should be strictly diminishing, or equivalently, the inequality $\frac{\partial^2 v(y_1, z')}{\partial z^2} \geq 0$ must hold. Now it follows that $\frac{\partial z^0(y_1, w_v, w_z')}{\partial y}$ must be negative, which implies that the fixed factor is inferior at the output level y_1 and the wages (w_v, w_z'). Therefore even though the inferiority of the fixed factor is not directly observed in the inequality (9), i.e., $\bar{z} < z_1$, this inequality (9) implies that the fixed factor should be inferior at some other wages such as (w_v, w_z'). The second case of the inequalities (5) can be treated in the same way.

IV. The Short Run Cost Functions

The properties of cost functions are well-known. (For instance see Shephard [5] or Varian[6].) But it does not seem to be fully investigated yet how the short run cost functions are related to the current holding of fixed factors. In this Section we will show that, if the production function is (strictly) monotone increasing and (strictly) quasi-concave in all the factors, then the short run variable cost functions are (strictly) monotone decreasing and (strictly) convex in the fixed factors. This result will be used to generalize the result of the preceding Section.

We will consider the case of one output, m fixed inputs and n variable inputs. All the previous notations will be naturally extended to this case. But this time the notation $v(y, z)$ will denote the optimal bundle of n variable factors for producing the output y with the fixed factors z at the given wages.

And the notation $C_v(y, z)$ denotes the short run variable cost $w_v v(y, z)$.

Let $z' \geq z (z' \neq z)$ and $v'' = v(y, z)$. The factor bundle (v'', z) can produce the output y by definition. By monotonicity of the production function, the factor bundle (v'', z') can also produce the output y. Since the bundle $v(y, z')$ minimizes the variable cost $w_v v$ for producing the output y with the fixed factors z', we have, for $z' \geq z$,

$$C_v(y, z') = w_v v(y, z') \leq w_v v'' = w_v v(y, z) = C_v(y, z). \tag{13}$$

Therefore the short run variable cost functions are monotone non-increasing in the fixed factors z. (If the production funtion is strictly monotone, then the factor bundle (v'', z') can produce strictly more than the level of output y. Hence the inequality (13) becomes a strict one.)

Now consider two different bundles of fixed factors, z^0 and z^*. Let $v^0 = v(y, z^0)$ and $v^* = v(y, z^*)$. For an arbitrary real number α with $0 < \alpha < 1$, the factor bundle $(\alpha v^0 + (1-\alpha) v^*, \ \alpha z^0 + (1-\alpha) z^*)$ can produce the output y if the production function is quasi-concave, and can produce strictly more if the production function is strictly quasi-concave. It follows that

$$\begin{aligned} C_v(y, \alpha z^0 + (1-\alpha) z^*) &= w_v v(y, \alpha z^0 + (1-\alpha) z^*) \\ &\leq w_v (\alpha v^0 + (1-\alpha) v^*) \\ &= \alpha C_v(y, z^0) + (1-\alpha) C_v(y, z^*). \end{aligned} \tag{14}$$

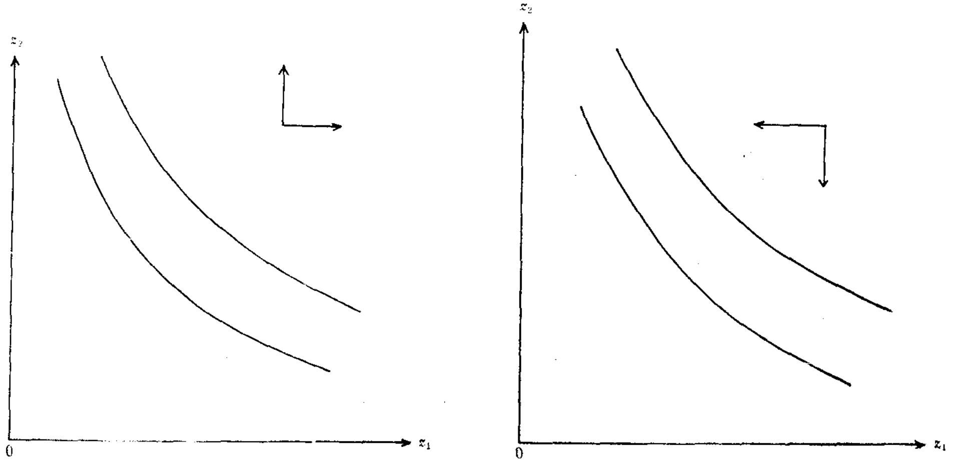
The inequality (14) implies that the short run variable cost function $C_v(\cdot)$ is convex in the fixed factors. Of course the inequality (14) becomes strict, if the production function is strictly quasi-concave. Figure 4 depicts this result.

Summing up, we have the following lemma.

Lemma 1. If the production function is (strictly) monotone increasing and (strictly) quasi-concave in all the factors, then the short run variable cost functions are (strictly) monotone decreasing and (strictly) convex in the fixed factors.

The following corollary follows immediately from the lemma.

Corollary 1. If the production function is strictly monotone increasing and strictly quasi-concave in all the factors, then it holds that $\frac{\partial^2 C_v(y, z)}{\partial z_i^2} \geq 0$ for all



(a) Isoquants $y=f(\bar{v}, z_1, z_2, \bar{z}_3, \cdots, \bar{z}_m)$ with other (barred) factors fixed

(b) Isocosts $c=C_v(z_1, z_2, \bar{z}_3, \cdots, \bar{z}_m, y)$ with other (barred) fixed factors fixed

Fig. 4. Isoquants and Isocosts

$i=1, 2, \cdots, m$.

Now let us investigate the long run and short run responses in the use of a variable factor when level of output increases. Let z^0 denote the long run optimal holding of the fixed factors for producing the output y at the current factor wages. Then for each variable factor i, we have

$$v_i^0(y)=v_i(y, z^0), \text{ where } z^0=z^0(y). \tag{15}$$

Differentiate the equation (15) with respect to the output y and obtain

$$\frac{\partial v_i^0(y)}{\partial y}=\frac{\partial v_i(y, z^0)}{\partial y}+\left(\frac{\partial v_i(y, z^0)}{\partial z}\right)\left(\frac{\partial z^0(y)}{\partial y}\right). \tag{16}$$

Since the factor bundle $(v(y,z^0), z^0)$ minimizes the long run cost for producing the output y at the current factor wages (w_v, w_z), we must have

$$\frac{\partial v_i(y, z^0)}{\partial z_j}=-\frac{w_j^z}{w_i^v}<0, \text{ for } j=1, 2, \cdots, m. \tag{17}$$

So if each fixed factor is not inferior, i.e., $\frac{\partial z_j^0(y)}{\partial y} \geq 0$ for $j=1, 2, \cdots, m$, then it follows that

$$\frac{\partial v_i^0(y)}{\partial y} \leq \frac{\partial v_i(y, z^0)}{\partial y}, \text{ for } i=1, 2, \cdots, n. \tag{19}$$

The inequality (19) implies that, if no fixed factor is inferior, then the use of each variable factor increases more in the short run than in the long run. This result is summarized in the following lemma.

Lemma 2. If no fixed factor is inferior at the current level of output and the present factor wages, and if $z^0=z^0(y)$, then it holds that

$$\frac{\partial v_i{}^0(y)}{\partial y}\leq\frac{\partial v_i(y, z^0)}{\partial y}, \text{ for each } i=1, 2, \cdots, n.$$

V. The General Case of the n Variable and m Fixed Factors

In this Section we will show that, for the case of m fixed and n variable factors, the necessary conditions of Section II imply that at least one fixed factor is inferior either at current wages or at some other wages. Suppose that the first inequality of (5) holds, i.e., $\mathrm{SMC}(y_1, \bar{z})<\mathrm{LMC}(y_1)$ and $y_1>\bar{y}$, $\bar{z}=z^0(\bar{y})$. Let the bundle of fixed factors z_1 be the long run optimum for producing the output y_1, i.e., $z_1=z^0(y_1)$. Then we have

$$C(y_1)=w_v v(y_1, z_1)+w_z z^0(y_1). \tag{20}$$

So the long run marginal cost $\mathrm{LMC}(y_1)$ can be derived as

$$\begin{aligned}\mathrm{LMC}(y_1)&=\frac{\partial C(y_1)}{\partial y}\\&=\frac{\partial}{\partial y}(w_v v(y_1, z_1))+\left(\frac{\partial}{\partial z}(w_v v(y_1, z_1))\right)\left(\frac{\partial z^0(y_1)}{\partial y}\right)+w_z\left(\frac{\partial z^0(y_1)}{\partial y}\right)\\&=\frac{\partial}{\partial y}(w_v v(y_1, z_1))+\left(\frac{\partial}{\partial z}(w_v v(y_1, z_1))+w_z\right)\left(\frac{\partial z^0(y_1)}{\partial y}\right).\end{aligned}\tag{21}$$

Obviously the fixed factor bundle z_1 minimizes $C_s(y_1, z)$, since it is the long run optimum for producing the output y_1. Therefore one can derive

$$\frac{\partial C_s(y_1, z_1)}{\partial z}=\frac{\partial}{\partial z}(w_v v(y_1, z_1))+w_z=0. \tag{22}$$

This shows that the second term in the final expression of (21) vanishes to zero. Consequently, the first inequality of the necessary conditions (5) can be rewritten as

$$\frac{\partial}{\partial y}(w_v v(y_1, \bar{z}))>\frac{\partial}{\partial y}(w_v v(y_1, z_1)), \text{ where } y_1>\bar{y}. \tag{23}$$

Again it is obvious that $\bar{z} \neq z_1$. If one fixed factor is inferior, then the conclusion of Section III is readily generalized. Consider the case in which no fixed factor is inferior in the relevant range. Then the inequality $z_1 \geq \bar{z}$ must hold.

From the inequality (23) we see that there exists a real number β with $0<\beta<1$ such that at $z^*=\bar{z}+\beta\ (z_1-\bar{z})$ the inequality

$$\left(\frac{\partial^2}{\partial z \partial y}(w_v v(y_1, z^*))\right)(z_1-\bar{z})>0 \tag{24}$$

holds. This inequality implies that there must exist at least one fixed factor (for convenience let it be mth one) so that the inequality

$$\frac{\partial^2}{\partial z_m \partial y}(w_v v(y_1, z^*))>0 \tag{25}$$

holds, since it holds that $z_1-\bar{z} \geq 0$.

Now remember that the notation $v(y_1, z^*)$ denotes the bundle of variable factors which minimizes the short run variable costs for producing the output y_1 with the current holding of fixed factors z^* at the wages (w_v, w_z). Let the wage vector (w_v^*, w_z^*) be a wage structure at which the factor bundle $(v(y_1, z^*), z^*)$ becomes the long run conditional factor demand for producing the output y_1. Evidently it must hold that $w_v^*=cw_v$ for some positive scalar c, since the bundle of variable factors $v(y_1, z^*)$ minimizes the related variable cost evaluated both at the wages w_v and w_v^*. So without loss of generality we can write this wage vector as (w_v, w_z^*).

Now let $s^*=(z_1^*, z_2^*, \cdots, z_{m-1}^*)$. And denote $z_m(y_1, s^*)$ for the short run optimal level of the z_m for producing the output y_1 with the $(m-1)$ fixed factors s^* at the wages (w_v, w_z^*). That is, let us treat the mth fixed factor as a variable one. As the bundle of original fixed factors $z^*=(s^*, z_m^*)$ is the long run optimum level for producing the output y_1 at these wages (w_v, w_z^*), it is clear that $z_m^*=z_m(y_1, s^*)$. Therefore if we solve the minimization problem

$$\min_{z_m}\ w_v v(y_1, s^*, z_m)+w_m{}^{z*} z_m, \tag{26}$$

then the solution for z_m must be $z_m^*=z_m(y_1, s^*)$. So the first order condition for the minimization problem (26) will give

$$\frac{\partial}{\partial z_m}(w_v v(y_1, s^*, z_m^*)) + w_m{}^{z*} = 0. \tag{27}$$

Now keep the holding of $(m-1)$ fixed factors s^* constant, and differentiate the equation (27) with respect to y to obtain

$$\frac{\partial^2}{\partial y \partial z_m}(w_v v(y_1, z^*)) + \frac{\partial^2}{\partial z_m{}^2}(w_v v(y_1, z^*)) \frac{\partial z_m(y_1, s^*)}{\partial y} = 0. \tag{28}$$

The equation (28) describes how, keeping the holding of $(m-1)$ fixed factors constant at s^*, the 'short run' optimum level of the mth 'fixed' factor $z_m(y, s^*)$ changes in the neighborhood of the output y_1 at the wages $(w_v, w_z{}^*)$. The inequality (25), the Corollary 1 of the preceding Section, and the equation (28) together imply that

$$\frac{\partial z_m(y_1, s^*)}{\partial y} < 0. \tag{29}$$

Assume that the first $(m-1)$ fixed factors are all non-inferior. Then the inequality (29) and the Lemma 2 of the preceding Section give

$$\frac{\partial z_m{}^0(y_1)}{\partial y} < 0, \tag{30}$$

which implies that the mth fixed factor z_m is inferior at the output level y_1 and the wages $(w_v, w_z{}^*)$.

When the second inequality of (5) holds, again we can derive the same conclusion in the similar way. Therefore if either one of the necessary conditions (5) holds, then at least one fixed factor must be inferior either at the current wages (w_v, w_z) or at some other wages $(w_v, w_z{}^*)$.

Summing up, we obtain the following theorem.

Theorem. Let the production function be strictly monotone increasing and strictly quasi-concave. If the supply response to price changes happenes to be greater in the short run than in the long run for a competitive firm, then at least one fixed factor is necessarily inferior at some level of output and wage structures.

VI. Concluding Remarks

According to our analysis, if every fixed factor is always normal, then the

Proposition B–H always holds. Therefore *Proposition B–H* is consistent with all kinds of the production functions, which are widely used in empirical studies, such as Leontief, Cobb-Douglas, CES, and Translog production functions.

It will be interesting to investigate whether there are empirical evidences which contradict the *Proposition B–H*. But we cannot successfully carry on this kind of econometric studies simply by estimating a linear model. In order to see this, consider an econometric model for the competitive supply function $y_t = \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + u_t$, where the notations y, P and u denote the quantity supplied, the price of the commodity and the disturbance term respectively. And the subscript denotes the time period in which the observation is made. The model must be a dynamic one (as it is), if we are to isolate the short run supply response from the long run one when the price changes. If it turns out that $\alpha_2 < 0$, then the competitive supply response to price changes will be greater in the short run than in the long run. But this result contradicts the theoretical conclusion that if the price change is sufficiently small, then the sign of coefficient α_2 should be always positive. Therefore if we are going to pursue after the empirical studies about our theoretical analysis, then we need a nonlinear model, which assures that the *Proposition B–H* holds for sufficiently small changes of the price but allows for its possible failure for a significant change. But there are almost no empirical studies on the supply functions even using linear models! So for the moment we do not have any empirical evidence or counter-evidence concerning the *Proposition B–H*.

Finally let us think about the possibilities for fixed factors to be inferior. The inferiority of the fixed factor is certainly not plausible, if there are only one fixed factor. But if we allow for the multiple fixed factors, then one of the fixed factors can be inferior. For instance consider a technology which uses two kinds of equipments as the fixed factors; the one equipment, A, is for the specialized use in small scale production and the other, B, is for the specialized use in large scale production. Then of course the use of the equipments A will be reduced as the level of output increases, which implies that the fixed factor A is inferior in this case.

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