The Choice of a Monetary Instrument: Some Theoretical and Empirical Results

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I. Introduction

Although Poole [22] published the pioneering paper on the optimal choice of monetary instruments in 1970, the subject is still hotly debated by academics and policymakers; e.g., in October of 1979 the Fed announced that they had switched the operating procedures from primarily controlling an interest rate to primarily controlling a monetary aggregate measure.

In his original paper, Poole showed that the variance of the endogenous variables in a macroeconomic model depended on which instrument, the money stock or the interest rate, the Central Bank chose to control. He also showed that intermediate information, i.e., current observations, could be used to formulate a combination policy which achieved the minimum variance. The instrument choice and intermediate information literature has been formalized and extended by Bryant [3], Friedman [8], Kareken, Muench, and Wallace

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〔11〕, LeRoy and Lindsey 〔14〕, Craine and Havenner 〔6〕, Turnovsky 〔25〕, and others. Taking a somewhat different tack, Sargent and Wallace 〔23〕 showed that if prices are endogenous and expectations are rational, then the probability distribution of real output is independent of known policy rules.

This paper shows that the previous works on the static problem of instrument choice can be nested in a unifying framework. The special cases depend on the particular partition of the information structure that conditions the probability distribution of the endogenous variables. This framework enables us to see more clearly that the problem of instrument choice still exists even under the assumption of rational expectation unless the current information on the endogenized policy variable is available. By generalizing into a dynamic framework we also show that the choice of instrument and the decision of instrument setting in general are interdependent and depend on the stationary-state probability distribution of the endogenous variables. Finally, we present empirical evidence on the relative importance of (1) the correct choice of an instrument, (2) the optimal multiperiod policy, and (3) the value of current information.

Our discussion is divided into three main sections. Based on a simple aggregate model, Section II develops the contemporaneous probability distribution of the endogenous variables for the traditional static optimization. It is shown that the distribution depends on the assumed portion of the available information set. Rational and other forms of expectations can be modeled by considering the difference in the portions of information between the public and private sectors. Traditionally it has been assumed that the public sector (Central Bank) conditioned its decisions on private behavior (the model), but the private sector often assumed to condition its decisions on a smaller information set, specifically one that included neither the policy variables nor the model itself. Rational expectations assumes that the public and private sectors use the same conditioning information. Although decisions are in general conditioned on lagged information, sometimes current endogenous variables are observable and can be used to condition current decisions. Current conditioning information, like any other information, reduces the variance of the distribution. The choice of instrument and the resulting optimal policy rule for each information structure are discussed.

In Section III we show the criteria for the optimal multiperiod instrument choice and policy rule decision in a linear quadratic control framework. If the vector of endogenous variables follows a stationary stochastic process, then the analog to the static instrument choice criteria of choosing the instrument that minimizes the preference weighted reduced-form covariance is to choose the instrument that minimizes the stationary-state variance. It is shown that the stationary-state variance depends on the feedback rule, so that in general the instrument choice and instrument setting decisions are interdependent, regardless of how expectations are formed.

Section IV presents empirical results paralleling the discussions in Sections II and III. We have estimated a rational expectations version and an autoregressive version of a simple linear aggregate macroeconomic model. The models are used to evaluate the relative importance of the correct choice of instrument, the optimal policy rule, and the gain from the use of current information. In these models, current information provides the greatest improvement in terms of reducing the variance. The final section summarizes our conclusions.

II. Static Problems of Instrument Choice

1. The Model and the Decision Makers

The model used in this section combines a traditional dynamic IS-LM demand structure with a Lucas-type aggregate supply equation so that prices are endogenous and price expectations influence both demand and supply. The model is very similar to the model used by Sargent and Wallace [23]. Although it is highly aggregated, the basic macroeconomic target variables, inflation and real output, are endogenous.

The following equations describe the behavioral structure of the economy: an "IS" or commodity demand equation,

$$z_t=a_1[i_t-(p_{t+1}^*-p_t^*)]+a_2z_{t-1}+a_3(m-p)_{t-1}+e_{1t}, \tag{1}$$

an "LM" or portfolio balance equation,

$$i_t=b_1z_t+b_2z_{t-1}+b_3(m-p)_t+b_4(m-p)_{t-1}+e_{2t}, \tag{2}$$

and, an aggregate supply equation,

$$p_t=c_1z_t+c_2z_{t-1}+p_t^*+e_{3t}. \tag{3}$$

Here z, m, p and i are the natural logs of real output, the nominal money stock, the aggregate price level, and the nominal interest factor (one plus the interest rate). The variable p_{t+j}^*, $j=0,1$, represents the private sector's conditional expectation of p_{t+j} formed with the appropriate information set which will be specified below. The error terms, e_j, $j=1,2,3$, are assumed to be independent Gaussian errors.(1)

The structural demand and supply equations in the model represent the aggregation of private agents' optimal decision rules, e.g., individuals decide the optimal level of current vs. future consumption as a function of real interest rate (the relative price of current vs. future consumption) and a change in the real rate triggers a decision to rearrange their optimal consumption path. These decisions may depend on unobservable variables, such as anticipated inflation, in which case the agents must form expectations. We assume the agents' expectations are optimal linear forecasts conditional on the available information. Thus the private agents' expectations are always "rational" in the sense that they incorporate all available information.

The Central Bank's decisions are not specified in the behavioral equations of the model. The Central Bank has two decisions. One is the choice of a monetary instrument, $x_t, x_t \epsilon \{i_t, m_t\}$, and the other is the choice of parameters in the monetary decision rule. We assume the policy maker is concerned about the outcome for the time period t only; that is, the problem is a static one.

The completion of the model requires the specification of the information set on which both the private and the public sectors base their decisions. Table 1 summarizes this information structure. We now turn to each case.

(1) We have assumed uncorrelated errors and omitted the uncontrollable exogenous variables for simplicity. These assumptions may be relaxed without any substantive change in the results.

Table 1. Information Structure for Decision Makers*

	Central Bank	Private Sector
Case 1	S_{t-1}	$\{p_{t-j}\}$
Case 2	S_{t-1}	$S_{t-1}, x_t,$ Policy Rules
Case 3	S_{t-1}, m_t, i_t	S_{t-1}, m_t, i_t

* $S_{t-1}=\{$model structure, $\{p_{t+j}{}^*\}$, all past observations of variables in the model$\}$.
$\{p_{t+j}{}^*\}=\{p_t{}^*,\ p_{t+1}{}^*\}$.
$\{p_{t-j}\}=\{p_{t-1}, p_{t-2}, \ldots\}$.
$x_t \epsilon \{i_t, m_t\}$.

2. The Traditional Instrument Choice Decision

Information is unevenly distributed in Case 1. The private sector only uses past prices to condition its forecasts of the current and future unobservable log of the aggregate price level,

$$p_t{}^*=E[p_t \mid \{p_{t-j}\}]=w_1 p_{t-1}+w_2 p_{t-2}+\cdots,$$
$$p_{t+1}{}^*=E[p_{t+1} \mid \{p_{t-j}\}]=w_1 p_t{}^*+w_2 p_{t-1}+\cdots \tag{4}$$

where the w's are constant weights. We assume that the model is stable so that p_t is a stationary stochastic process; therefore, the forecasts are the mathematical expectation of the log of the price conditional on the log of past prices. (2)

The Central Bank has access to complete information about the economy and will use this information in making its decision. The conditional distribution of the endogenous variables depends on the setting of the monetary instrument and the instrument selected so that the Central Bank considers the effect of its decisions on the distribution of the endogenous variables. Using i_t as the instrument the model can be written in matrix notation as

$$\begin{bmatrix} 1 & 0 & 0 \\ -b_1 & b_3 & -b_3 \\ -c_1 & 1 & 0 \end{bmatrix}\begin{bmatrix} z \\ p \\ m \end{bmatrix}_t=\begin{bmatrix} a_2 & -a_3 & a_3 \\ b_2 & -b_4 & b_4 \\ c_2 & 0 & 0 \end{bmatrix}\begin{bmatrix} z \\ p \\ m \end{bmatrix}_{t-1}+\begin{bmatrix} a_1 & -a_1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p_t{}^* \\ p_{t+1}{}^* \end{bmatrix}+\begin{bmatrix} a_1 \\ -1 \\ 0 \end{bmatrix} i_t+\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_t. \tag{5}$$

Since $p_t{}^*$ and $p_{t+1}{}^*$ are known to policy makers, the reduced form becomes

$$y(i)_t=\begin{bmatrix} z \\ p \\ m \end{bmatrix}_t=\begin{bmatrix} a_1 \\ a_1 c_1 \\ a_1(c_1-b_1/b_3)+1/b_3 \end{bmatrix} i_t+\begin{bmatrix} d(i)_1 \\ d(i)_2 \\ d(i)_3 \end{bmatrix}+\begin{bmatrix} 1 & 0 & 0 \\ c_1 & 0 & 1 \\ c_1-b_1 b_3 & -1/b_3 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_t \tag{6}$$

(2) For example, see Granger and Newbold [9, Chapter 6]. These equations are the same form as those given by Muth [19, p. 324], whose model contained no exogenous policy variables.

where $[d(i)_1\ d(i)_2\ d(i)_3]'$ summarizes all the known non-policy effects including p_t^* and p_{t+1}^*. On the other hand, if m_t is the policy variable, we obtain the following reduced form

$$y(m)_t=\begin{bmatrix} z \\ p \\ i \end{bmatrix}_t = -\alpha \begin{bmatrix} a_1b_3 \\ a_1b_3c_1 \\ b_3 \end{bmatrix} m_t + \begin{bmatrix} d(m)_1 \\ d(m)_2 \\ d(m)_3 \end{bmatrix} + \alpha \begin{bmatrix} -1 & -a_1 & a_1b_3 \\ -c_1 & -a_1c_1 & a_1b_1-1 \\ c_1-b_1/b_3 & -1/b_3 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_t \tag{7}$$

where $\alpha=1/(a_1b_1-a_1b_3c_1-1)$ and $[d(m)_1\ d(m)_2\ d(m)_3]'$ is similarly defined as above.

The reduced form gives the transformation from the random structural errors to the random endogenous variables (the last terms in (6) and (7)) conditional on the predetermined variables and price expectations (the middle terms on the right hand side of (6) and (7)) and the policy variable (the first terms on the right hand side of (6) and (7)). In general, all of the reduced form coefficients depend on the monetary instrument selected since the instrument determines the slope of the aggregate demand combining the IS and LM. The setting of the instrument determines the expected intercept. Notice that either instrument can be set so that the conditional expectation of any single endogenous variable of interest will be the same. This is guaranteed by the nonzero coefficient vectors of i_t and m_t in (6) and (7). The conditional covariance, however, does vary with the monetary instrument selected as can be seen in the reduced form error structure of (6) and (7).

If the interest rate is selected as the monetary instrument, then the model is recursive and the current probability distribution of the real output is independent of the other current endogenous variables.(3) Real output is demand determined and the IS equation gives current aggregate demand. The conditional covariance matrix of z_t and p_t is obtained from (6):

$$\text{cov}\left(\begin{bmatrix} z \\ p \end{bmatrix}_t \middle| S_{t-1}, i_t \right) = (\sigma_{kl}(i)) = \begin{bmatrix} \sigma_1^2 & c_1\sigma_1^2 \\ c_1\sigma_1^2 & c_1^2\sigma_1^2+\sigma_3^2 \end{bmatrix} \tag{8}$$

where $E(e_j^2)=\sigma_j^2$, $j=1,2,3$.

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(3) The recursive structure is broken if current prices enter the IS equation, e.g., through the real interest rate. Turnovsky [25] examines an alternative specification which is not recursive.

On the other hand, if the money stock is chosen as the instrument the system is fully integrated. Aggregate demand depends on both price level and real income so that real output is given by the solution of the aggregate demand and supply equations. The conditional covariance matrix in this case is

$$\operatorname{cov}\left(\begin{bmatrix} z \\ p \end{bmatrix}_t \middle| S_{t-1}, m_t\right) = (\sigma_{kl}(m))$$
$$= \alpha^2 \begin{bmatrix} \sigma_1^2 + a_1^2\sigma_2^2 + a_1^2 b_3^2 \sigma_3^2 & c_1\sigma_1^2 + a_1^2 c_1 \sigma_2^2 + a_1 b_3 (a_1 b_1 - 1)\sigma_3^2 \\ \sigma_1^2(m) & c_1^2\sigma_1^2 + a_1^2 c_1^2 \sigma_2^2 + (a_1 b_1 - 1)\sigma_3^2 \end{bmatrix}. \tag{9}$$

In both cases the aggregate supply curve is identical and independent of the monetary instrument. If expected equilibrium output and price are target variables of policy makers, it is clear that either instrument can be utilized to attain a point on the supply curve by shifting the demand curve. But the combination of price and output off the supply curve cannot be achieved; in other words, the system is not static-controllable(e.g., Norman and Jung [20]). Thus the problem of instrument choice arises when the policy objective is in terms of covariance structure. If, for example, minimizing the variance of current output were the only objective, then the comparison of $\sigma_{11}(i)$ and $\sigma_{11}(m)$ would be the selection criterion of instrument. Thus if

$$\sigma_{11}(i) = \sigma_1^2 < (\sigma_1^2 + a_1^2\sigma_2^2 + a_1^2 b_3^2 \sigma_3^2)/(a_1 b_1 - a_1 b_3 c_1 - 1)^2 = \sigma_{11}(m),$$

the interest rate is the optimal instrument; otherwise the money stock is preferred.[4] However, since prices are endogenous a more comprehensive criteria would be based on the moments of the bivariate distribution of price and output.

3. Policy Neutrality Solution

The second information structure is prevalent in the rational expectations literature. Here the private and public sectors have the same information set, so the distribution of information is symmetric. Both sectors consider how public and private behavior (the relevant economic theory) affect the distribution of the variables they are interested in. In this case both the private sector's and the Central Bank's subjective expectations are given by the objective pro-

(4) In fixed-price systems considered in the earlier literature, $\sigma_{11}(i) = \sigma_1^2$ and $\sigma_{11}(m) = (\sigma_1^2 + a_1\sigma_2^2)/(a_1 b_1 - 1)^2$.

bability distribution conditioned on realizations through period $t-1$.

Since Case 1 and Case 2 differ only in terms of the forecasting rule, p_t^* and p_{t+1}^*, the reduced form of the present information structure is the same as (6) or (7) depending on the instrument selected. We are assuming that policy decisions do not contain random (or surprise) components. Therefore, if the instrument choice decision is based on the covariance matrix of the endogenous variables, then the results of the previous subsection apply here with no modification. In other words, the problem of instrument choice may exist regardless of how expectations are formed.

The next question to ask then is whether any particular instrument is efficient in attaining some desired combinations of expected equilibrium endogenous variables. Lucas [15] and Sargent and Wallace [23] have shown that if the private sector conditions its decisions (price forecasts) on the behavior of the Central Bank then expected real output is supply determined. As a result, Government policies which only shift the expected demand curve (policy setting) have no impact on expected supply and expected real output. This can be seen most easily by investigating the supply equation (3). Because $p_t^*=E[p_t|S_{t-1}, x_t]$, $p_t-p_t^*=\sum_{j=1}^{3}\beta_j e_{jt}$ where β_j's are determined by the way p_t^* and p_{t+1}^* are formed. Thus we can write the equation as

$$\begin{aligned} z_t &= (1/c_1)(p_t-p_t^*)-(c_2/c_1)z_{t-1}-(1/c_1)e_{3t} \\ &= -(c_2/c_1)z_{t-1}+(1/c_1)(\beta_1 e_{1t}+\beta_2 e_{2t}+(\beta_3-1)e_{3t}). \end{aligned} \tag{10}$$

The expected output is completely determined at $-(c_2/c_1)z_{t-1}$ and the demand can affect only the expected price level at best. Only unanticipated shifts in policy or demand can feed through the structure and affect the real output.

Sargent and Wallace [23] pointed out that their model does not have a unique solution of expected price level when the interest rate is used as the policy instrument. This is because, in (1), $p_{t+1}^*-p_t^*$ is reduced to a random error and the aggregate demand becomes a vertical line coinciding with the aggregate supply. Thus only the money stock policy can achieve the desired level of expected price with the usual negatively sloped demand curve, whereas the interest rate policy leaves the price level undetermined.(5)

As in Case 1, the choice of monetary instrument matters when the Central Bank is concerned with the covariance structure of target endogenous variables. The only difference is that in Case 1 the output can be manipulated. Thus the choice of instrument and instrument setting are in both cases separable. The situation drastically changes when current information on some variables is available as we will see next.

4. Intermediate Information

In previous sections it was assumed that the Central Bank and the private sector conditioned their forecasts only on information available at the beginning of the period. Sometimes current market information also is available and can be used to condition expectations. The intermediate information or intermediate target literature, e.g., LeRoy [13], Friedman [8] and Kareken *et al.* [11], examines how the Central Bank can optimally use this information. However, the private sector also could condition its forecasts on current information; e.g., see Woglom [26] and Canzoneri, Henderson, and Rogoff [4].

In this section we analyze the effect of current conditioning information on the distribution of endogenous variables. We make the realistic assumption that aggregate output and price level are unobservable in the current period. These aggregate statistics require collection and processing of data which takes time and money. The current market interest rate and/or the money stock, however, are potentially observable to at least some market participants.

The effect of current conditioning information can be examined in terms of deviations from the equilibrium expected at the beginning of the period. The reduced form residuals, the last terms on the right hand side of (6) and (7), give the impact of the unobservable structural errors on the endogenous variables. Treating these as deviations from the expected equilibrium, the levels of the endogenous variables can be written as

$$y(x)_t = y(x)_t^* + B^{-1}(x)e_t, \quad x \in \{m, i\}. \tag{11}$$

(5) See McCallum [18] for some methods of avoiding this indeterminacy problem.

The mean and covariance matrix for the bivariate distribution of the log of real output and the aggregate price conditioned on the currently observed endogenous variable, the money stock or the interest rate, are:(6)

$$\begin{bmatrix} z(i) \\ p(i) \end{bmatrix}_t = \begin{bmatrix} z(i) \\ p(i) \end{bmatrix}_t^* + \begin{bmatrix} v_{zm}(i) \\ v_{pm}(i) \end{bmatrix} v_{mm}^{-1}(i)[m(i)_t - m(i)_t^*]$$

$$= \begin{bmatrix} z(m) \\ p(m) \end{bmatrix}_t^* + \begin{bmatrix} v_{zi}(m) \\ v_{pi}(m) \end{bmatrix} v_{ii}^{-1}(m)[i(m)_t - i(m)_t^*]$$

$$= \begin{bmatrix} z(m) \\ p(m) \end{bmatrix}_t, \tag{12}$$

where

$$E[B^{-1}(i)e_t, e_t'B^{-1\prime}(i)] = (v_{kl}(i)), \quad k, l = z, p, m,$$

$$E[B^{-1}(m)e_t, e_t'B^{-1\prime}(m)] = (v_{kl}(m)), \quad k, l = z, p, i,$$

and

$$\text{cov}\left(\begin{bmatrix} z \\ p \end{bmatrix}_t \middle| i_t, S_{t-1} \right) = \begin{bmatrix} v_{zz}(m) - v_{zi}(m)v_{ii}^{-1}(m)v_{iz}(m) & v_{zp}(m) - v_{zi}(m)v_{ii}^{-1}(m)v_{iz}(m) \\ v_{zp}(m) - v_{zi}(m)v_{ii}^{-1}(m)v_{iz}(m) & v_{pp}(m) - v_{pi}(m)v_{ii}^{-1}(m)v_{ip}(m) \end{bmatrix}$$

$$= \text{cov}\left(\begin{bmatrix} z \\ p \end{bmatrix}_t \middle| m_t, S_{t-1} \right). \tag{13}$$

Notice that the mean and the covariance of the real output and the aggregate price are the same when the interest rate is the instrument (exogenous) and money stock is used as the conditioning information variable as when the money stock is the instrument (exogenous) and the interest rate is the conditioning information variable. The bivariate distribution is the same in either case because it is conditioned on both the interest factor and the money stock in each case.(7) Also notice that, since the bivariate distribution of endogenous variables is conditioned on a larger information set compared with Case 1 and Case 2, their variances are necessarily smaller.

When the Central Bank uses current information the optimal monetary rule will be continuously updated during the current period. These policies have

(6) These are standard results for the conditional mean and covariance of the multivariate normal, Anderson [1, Chapter 2].

(7) This can be verified directly with some tedious algebra by showing that the forecast errors are identical for two information variables using (11) and elements of the covariance matrices.

been labeled combination policies since there is no longer any need to distinguish which variable is the exogenous policy variable and which variable is the observed information variable. When the private sector conditions its price expectations on the current information and the current policy response, then the distribution of the real output will be independent of the policy rule and the choice of instrument. This follows using the argument presented in Case 2 (Section 3) with $p_t^*=E[p_t|S_{t-1}, x_t]$ and $p_{t+1}^*=E[p_{t+1}|S_{t-1}, X_t]$ replaced by $p_t^*=E[p_t|S_{t-1}, m_t, i_t]$ and $p_{t+1}^*=E[p_{t+1}|S_{t-1}, m_t, i_t]$. The aggregate supply equation becomes

$$z_t=-(c_2/c_1)z_{t-1}+(1/c_1)[p_t-E(p_t|S_{t-1}, m_t, i_t)]-(1/c_1)e_{3t}.$$

The forecasting error $p_t-E[p_t|S_{t-1}, m_t, i_t]$ is independent of the policy rule (which is included in the conditional mean) and the choice of policy instrument since the current conditional variance is the same for either instrument. Thus, in this case the distribution of real output as well as the price level is independent of policy.

III. Dynamic Problem of Instrument Choice

The basic monetary decision problem in a static setting, as in the previous section, is to choose a monetary instrument, either the interest factor or the money stock, and optimal values for the selected instrument that minimize the current welfare loss function. In most of the monetary instrument choice literature these decisions are treated as if they are separable. The covariance matrix of the endogenous variables determines the choice of instrument and the model parameters the instrument value (monetary rule). By generalizing the traditional instrument choice problem to a dynamic setting we show that in general these decisions are not separable. If the model is linear and loss function is quadratic, the instrument choice and instrument setting decisions are separable if the model is statically controllable (the conditions for static controllability are given in the Appendix); in nonlinear models even static controllability is not a sufficient condition for separating the decisions.[8] This section derives the contribution of the current decision to the multiperiod loss in a linear quadratic control framework.

Let the quadratic welfare loss function be

$$W(x)_{t|t-1}=\min_{x_t\in(m_t,i_t)} E[\sum_{j=0}^{T}(y(x)-\bar{y}(x))'_{t+j}K(y(x)-\bar{y}(x))_{t+j}|S_{t-1}] \tag{14}$$

where $y(x)$ is a vector of state or endogenous variables, $\bar{y}(x)$ is a vector of desired target values and K is a positive semi-definite weighting matrix. The state vector is governed by the stable linear model

$$y(x)_t=A_x y(x)_{t-1}+C_x x_t+b(x)_t+u(x)_t \tag{15}$$

where x is a vector of controllable exogenous variables, b is a vector of deterministic uncontrollable exogenous variables and u is a stochastic white noise error vector. The subscript x and x in the parenthesis indicate that the corresponding variable or matrix depends on the instrument selected.

When equation (15) is derived with the first information structure (Case 1), the above policy evaluation scheme is subject to the well-known criticisms by Lucas [15], and Kydland and Prescott [12]. Policies obtained in this way are not only suboptimal but also conducive to a greater economic instability. One way to avoid this problem is to incorporate the second information structure, in which case we have $C_x=0$. However, models with overlapping contracts and rational expectations are shown to be consistent with the reduced-form representation as in equation (15) (for example, see Aoki and Canzoneri [2]).

It can be shown (see Chow [5, Chapter 8]) that the minimum expected value of the welfare loss function is expressed as a quadratic form in the just current state vector,

$$W(x)_{t|t-1}=E[y(x)_t'H(x)_t y(x)_t-2y(x)_t'h(x)_t+c(x)_t|S_{t-1}] \tag{16}$$

where[9]

$$H(x)_t=K+(A_x+C_xG(x)_{t+1})'H(x)_{t+1}(A_x+C_xG(x)_{t+1}),$$

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(8) See Turnovsky [24] or Kareken [10].

(9) The notation follows Chow [5, Chapter 8], except for a few obvious changes. These are standard Riccati difference equations which can be solved backward starting from T.

$$h(x)_t=k(x)_t+(A_x+C_xG(x)_{t+1})'(h(x)_{t+1}-H(x)_{t+1}b(x)_{t+1}),$$
$$c(x)_t=\bar{y}(x)_tK\bar{y}(x)_t+(b(x)_{t+1}+C_xg(x)_{t+1})'(b(x)_{t+1}+C_xg(x)_{t+1})$$
$$-2(b(x)_{t+1}+C_xg(x)_{t+1})'h(x)_{t+1}+c(x)_{t+1} \tag{17}$$
$$+E[u(x)_{t+1}'H(x)_{t+1}u(x)_{t+1}|S_{t-1}],$$
$$H(x)_T=K,\ \ h(x)_T=k(x)_T=K\bar{y}(x)_T,$$

and, that the optimal control rule is

$$x_t=G(x)_ty(x)_{t-1}+g(x)_t \tag{18}$$

where

$$G(x)_t=-[C_x'H(x)_tC_x]^{-1}C_x'A_x,$$
$$g(x)_t=-[C_x'H(x)_tC_x]^{-1}C_x'[H(x)_tb(x)_t-h(x)_t]. \tag{19}$$

Then the current state vector $y(x)_t$ can be separated into two orthogonal components, the conditional mean

$$E[y(x)_t|S_{t-1}]\equiv y(x)_t^*=A_xy(x)_{t-1}+C_xx_t+b(x)_t$$
$$=(A_x+C_xG(x)_t)y(x)_{t-1}+b(x)_t, \tag{20}$$

and the deviations from the conditional mean

$$u(x)_t=y(x)_t-y(x)_t^*. \tag{21}$$

In addition, notice that the error $u(x)_t$ is distributed independently of the parameters in the loss function (16); therefore, the loss also can be divided into a portion which depends on the conditional expectation of the state vector (the certainty-equivalence portion)

$$W^{CE}(x)_{t|t-1}=y(x)_t^{*\prime}H(x)_ty(x)_t^*-2y(x)_{t-1}^*h(x)_t+c(x)_t \tag{22}$$

plus the variance of the state vector

$$W^S(x)_{t|t-1}=W[u(x)_t'H(x)_tu(x)_t|S_{t-1}]=\text{trace}(H(x)_xV(x)) \tag{23}$$

where

$$V(x)=E[u(x)_tu(x)_t'|S_{t-1}].$$

The certainty-equivalence portion of the loss is identical to the loss in a deterministic problem with a similar structure and thus is unaffected by the choice of monetary instruments.[10]

The variance portion of the loss, equation (23), is then the criterion for the choice of a monetary instrument. Equation (23) measures the expected loss

(10) This implies $y(i)_t^{*\prime}H(i)_ty(i)_t^*=y(m)_t^{*\prime}H(m)_ty(m)_t^*$ where H's are from (17).

from the current stochastic errors in the *current* and *future* period,

$$W^S(x)_{t|t-1}=\text{trace}[KV+(A_x+C_xG(x)_{t+1})'H(x)_{t+1}(A_x+C_xG(x)_{t+1})V(x)]. \quad (24)$$

The first term on the right hand side measures the preference weighted loss from the current variance of the target variables (the standard criteria for the static choice of a monetary instrument) and the second the loss in the future as the current errors influence future values of the state variables.

To illustrate the dynamic instrument choice criteria, assume that the state vector follows a stationary stochastic process with the constant unconditional (steady-state) mean $\mu(x)$. The analog to the standard static welfare loss function is the stationary-state welfare loss function,

$$W=\min_{x\in\{i,m\}} E[(y(x)-\bar{y}(x))'K(y(x)-\bar{y}(x))] \quad (25)$$

where $y(x)$ is described by the unconditional stationary-state distribution. The current state vector $y(x)_t$ can be separated into orthogonal components as before: the unconditional mean $\mu(x)$, plus deviations from the unconditional mean, $d(x)_t=y(x)_t-\mu(x)$. As a result, the welfare loss function can be written as the sum of a component which depends on the deviation of the expected value of the state vector from the desired value

$$W^{CE}=[\mu(x)-\bar{y}(x)]'K[\mu(x)-\bar{y}(x)],$$

and a component which depends on the unconditional steady-state variance

$$W^{SS}(x)=E[d(x)_t'Kd(x)]=\text{trace}[KE(d(x)_t d(x)_t')]. \quad (26)$$

Let the steady-state monetary rule have the form

$$x_t=G(x)y(x)_{t-1}+g(x), \quad (27)$$

then

$$\begin{aligned} d(x)_t &= A_xy(x)_{t-1}+C_x(G(x)y(x)_{t-1}+g(x))+b(x)+u(x) \\ &\quad -[A_x\mu(x)+C_x(G(x)\mu(x)+g(x))+b(x)] \\ &= [A_x+C_xG(x)]d(x)_{t-1}+u(x), \end{aligned}$$

and the loss W^{SS} can be rewritten as

$$\begin{aligned} W^{SS} &= \text{trace}[KE(d(x)_t d(x)_t')] \\ &= \text{trace}[K\{V(x)+(A_x+C_xG(x))'V(x)(A_x+C_xG(x)) \\ &\qquad +(A_x+C_xG(x))^{2\prime}V(x)(A_x+C_xG(x))^2+\dots\}] \\ &= \text{trace}[H(x)V(x)]. \end{aligned} \quad (28)$$

The firstt term in the infinite series KV represents the contribution of the current error to the current loss. The contemporaneous covariance $V(x)$ is a function of the choice of a monetary instrument and of course influences the steady-state variance. The steady-state variance also depends, however, on the parameters in the feedback rule (27). Thus, in general, the choice of the feedback rule (the monetary instrument setting) and the choice of a monetary instrument *cannot* be made independently.

IV. Empirical Evidence

This section presents quantitative evidence on the relative importance of the issues discussed in Sections II and III. It is based on our estimates of a modified version of the model used by Friedman [8] to assess the value of intermediate information. His original specification is based on autoregressive expectations which corresponds to Case 1, Section II. We have reestimated this specification and a rational expectations version of the model which corresponds to our Case 2.

1. Autoregressive Expectations

The model is slightly more general than the dynamic IS-LM model presented in Section II. Neither we nor Friedman, however, intended this model as a complete econometric description of the U.S. economy; instead, it is a simple, highly aggregated, model designed basically to yield some quantitative evidence on the relevance of the theoretical issues in the choice of a monetary instrument.

The model was estimated with quarterly data from the period 1960I∼1976II; the definitions of the variables and the data sources are given below. The parameters were estimated using instrumental variables with a correction for first order serial correlation suggested by Fair [7].

The equations with the parameter estimates and their asymptotic t-ratios, in parenthesis, for the autoregressive expectations version of the model are:

the IS equation,

$$\Delta z_t = .007 + .413\Delta z_{t-1} - .056^*\Delta i_{L_t} + 0.73^*\Delta E_t - .098\Delta I_{t-1}$$
$$\quad (3.87)\ (4.03) \quad (-1.82) \quad (1.36) \quad (-2.69)$$

$$SEE = .0083,\ \bar{R}^2 = .43,\ \rho = -.209, \tag{29}$$

the LM equation,

$$\Delta(m-p)_t = .576\Delta(m-p)_{t-1} + .399\Delta z_t - .028\Delta i_{S_t}$$
$$\quad (4.85) \quad (3.01) \quad (-4.17)$$

$$SEE = .0058,\ \bar{R}^2 = .58,\ \rho = .020, \tag{30}$$

the term structure,

$$i_{L_t} = .932 i_{L_{t-1}} + .156 i_{S_t} - .065 i_{S_{t-1}} + .053^*\Delta(L-S)_{t-1}$$
$$\quad (50.42) \quad (4.25) \quad (-2.03) \quad (1.79)$$

$$SEE = .023,\ \bar{R}^2 = .97,\ \rho = .511, \tag{31}$$

and aggregate supply,

$$\Delta p_t = .830\Delta p_{t-1} + .066\Delta I_{t-1} + .093\Delta z_{t-1}$$
$$\quad (21.82) \quad (4.60) \quad (3.53)$$

$$SEE = .003,\ \bar{R}^2 = .89,\ \rho = -.412, \tag{32}$$

where

E=high-employment expenditures,

I=price deflator for dollar-denominated imports (1972=1.0),

L=face amount of outstanding federal securities maturing in more than one year (beginning of quarter),

m=money stock, m_2 (currency plus demand and time deposits plus certificates of deposit under $100,000),

p=price deflator for gross national product (1972=1.0),

i_L=long-term interest rate (Moody's Baa corporate bonds), in percent,

i_S=short-term interest rate (three month Treasury bills), in percent,

S=face amount of outstanding federal-government securities maturing in less than one year (beginning of quarter),

z=real gross national product,

$\Delta p = p_t - p_t^*$, and

Δ for other variables denote the first differences.

All other variables are natural logs except for the interest rate. The specification of this model is essentially the same as the model estimated by Friedman [8, p. 322], except we dropped his "money supply" equation which related the money stock to nonborrowed reserves and the short-term interest rate.(11) The estimated coefficients all have the theoretically correct sign and the magnitudes seem reasonable. The insignificant (at the 5% level) coefficients are marked with an asterisk. The values of the estimated coefficients are quite close to Friedman's estimates; but they are not identical since our sample period was slightly longer and the data had been revised. The interpretation of the model is fairly straightforward and Friedman [8, pp. 322–7] gives an excellent description which we will not repeat.

If this model adequately approximates the economy, then given a policy welfare loss function we can quantitatively evaluate the relative importance of (1) choosing the correct monetary instrument, (2) using the correct multiperiod decision rule, and (3) conditioning on current information. We chose as a policy objective to minimize the preference weighted steady-state covariance matrix of the growth rate of real output (Δz) and the inflation rate (Δp).

Table 2, (1), gives the contemporaneous covariance matrix of the target variables conditional on the monetary instrument. The variances of the growth of real output and inflation conditional on an exogenous money stock are less than the comparable variances conditional on an exogenous interest rate. In fact for all positive preference weighting matrices, K, the money stock would be the optimal instrument in a static problem.

In a multiperiod planning problem, the instrument choice and instrument setting decisions are interdependent and both depend on the preference weights in the loss function. Table 2, (2), gives minimum values of the preference weighted steady-state covariance matrix for a range of preference weights.(12)

(11) The coefficient on the short-term rate was very small and insignificant, but extremely important in the control exercises when the interest rate was the instrument. Since we did not trust this estimate, we decided to drop this equation and assume the Fed could directly control the money stock. Friedman [8, p. 325] also acknowledges problems with this equation.

(12) Parameter uncertainty was ignored in deriving the feedback rules and evaluating the loss function. Thus the solutions are really first order certainty equivalence approximations. The actual computation used the algorithm developed by Norman and Jung [20].

Table 2

(1) Contemporaneous Covariance Matrix of the Target Variables*

	$V(m)$		$V(i)$	
	Δz	Δp	Δz	Δp
Δz	3314	−5.7	4,436	35
Δp	−5.7	615	35	616

(2) Preference Weighted Steady-state Covariances of the Target Variables *,**

k_{22}	$W^{SS}(m)$	$W^{SS}(i)$
.6	4,053 (9%)	5,201 (8.4%)
1.0	4,531 (8.4%)	5,695 (9.2%)
2.0	5,690 (7.3%)	6,891 (9.2%)

* Units of the reported values do not bear any significance on our interest.
** For the methodology of infinite-horizon optimal policy rule see Chow [5, Chapter 7].

The weight (k_{11}) on the growth rate of real output is normalized at one and the weight (k_{22}) on the inflation rate varies. Here too we see that the money stock is the dominant instrument.(13) Furthermore, the money stock does relatively better as the weight on inflation is increased indicating that, at least in this model, money stock policies do a better job of stabilizing the inflation rate.

The values in parentheses below the preference weighted steady-state covariances show the reduction due to the optimal feedback rule. That is, the loss from the multiperiod planning policy is approximately 7 to 9 percent less than the minimum loss from any static policy which is repeated over the horizon, e.g., a fixed money growth rate.

Theoretically the best policy, of course, is not to select an instrument and set its value for even a period, but to use all observable intermediate (current) information to continuously modify the control variable. As shown before, when current information is used, there is no instrument choice problem since the conditioning information includes at least both potential instruments. We have used the long and short term interest rate and the money growth rate

(13) We have tried a wider range of weights including off-diagonal weights and the money stock has always been the preferred instrument so that this result seems fairly robust in this model.

Table 3*

(1) Covariance Matrix of the Target Variables Conditional on Current Information

$V(m_t, i_{L_t}, i_{S_t})$

	Δz	Δp
Δz	2,443 (26%)	− 81.9
Δp	−81.9	602 (2.3%)

(2) Preference Weighted Steady-state Covariance of the Target Variables Conditional on Current Information

k_{22}	$W^{SS}(m)$
.6	3,139 (22.5%)
1.0	3,591 (20.8%)
2.0	4,685 (17.7%)

* See the footnotes of Table 2.

to condition the contemporaneous covariance of the target variables. Table 3 gives the contemporaneous covariance matrix of the target variables conditional on current information and the preference weighted steady-state covariance matrix conditional on current information. The numbers in parentheses below the variances give the percentage reduction due to the current conditioning information. For example, in Table 3, $V(m, i_L, i_S))$ gives the elements of the contemporaneous covariance matrix conditional on the money stock; conditioning on all currently observable information reduces the variance of the growth rate of real output by 26% and the inflation rate by 2.3% from the values in Table 2, (1). The percentage reduction in the preference weighted steady-state covariance matrix are from the minimum values in Table 2, (2).

Table 2 and 3 indicate that in a dynamic quarterly model where expectations are autoregressive, the correct choice of a monetary instrument and the correct setting of feedback rule are important. Table 3 shows that the additional gain from using current information is so large that this policy should be followed.

2. Rational Expectations

When expectations are rational the price forecasts must be replaced with the rational expectations price forecasts. We substituted the forecasts, p_t^*, from

Table 4*

(1) Contemporaneous Covariance Matrix of the Target Variables

	$V(m)$ Δz	$V(m)$ Δp	$V(i)$ Δz	$V(i)$ Δp
Δz	2,847	−11.7	4,704	228
Δp	−11.7	69.1	228	88

(2) Preference Weighted Steady-state Covariance of the Target Variables

k_{22}	$W^{SS}(m)$	$W^{SS}(i)$
.6	38,618	63,792
1.0	38,646	63,827
2.0	38,715	63,915

(3) Covariance Matrix of the Target Variables Conditional on Current Information

$V(m_t, i_{L_t}, i_{S_t})$

	Δz	Δp
Δz	2,485 (12.7%)	1.1
Δp	1.1	57.2 (17.2%)

* See the footnotes of Table 2.

regression of the price on all the lagged values in the system plus time series forecasts of the current exogenous variables for the unobservable expectations. These variables entered through the real rate in the IS equation

$$r_{L_t}=\log[(1+i_{L_t})(p_{t-1}/p_t^*)-1]),$$

and through a Lucas-type supply function we used in place of the quasi-Phillips curve (32); the other equations remain the same. The new equations were estimated using the same method as before.(14)

The modified equations with the parameter estimates and their asymptotic t-ratios, in parentheses, are:

the IS equation,

$$\underset{(4.72)}{\Delta z_t= .008}+\underset{(2.16)}{.238}\Delta z_{t-1}+\underset{(.81)}{.043}\Delta E_t-\underset{(-2.53)}{.112}\Delta r_{Lt}-\underset{(-2.62)}{.108}\Delta I_{t-1} \tag{33}$$

$$SEE=.008,\ \bar{R}^2=.33,\ \rho=-.02,$$

and the aggregate supply equation,

(14) This rational expectations estimation method belongs to a class of weakly rational expectations approaches, e.g., see McCallum [17]

$$z_t = .323 + .0004t + 1.262z_{t-1} - .315z_{t-2} + 4.731(p_t - p_t^*) \qquad (34)$$
$$(1.53)\ (1.30)\quad (11.01)\quad (-2.71)\qquad (2.59)$$

$$SEE = 0.09,\ \bar{R}^2 = .997,\ \rho = .095.$$

This version of the IS equation does not fit quite as well as equation (29), but the real interest rate has a coefficient almost twice as large as the nominal rate in the previous version (as theory would predict) and it is significant at the 95% confidence level. Furthermore, the lagged value of the growth rate of output is less important and the coefficient on the lagged error term is extremely small. There is no direct comparison for the supply equation (34); however, the crucial variable, the error in price expectations, enters significantly with the theoretically correct sign.

When expectations are rational, the growth rate of real output is independent of any monetary rule as shown in Section II, but the variance of the growth rate does depend on the choice of instrument. Table 4 gives the contemporaneous covariance matrix of target variables conditional on the monetary instrument, and the preference weighted steady-state covariance. It shows that the money stock policy is dominant for both the static and the dynamic problems. Also the table reports the contemporaneous covariance matrix conditional on the long and short interest rate and the money stock. The numbers in parentheses below the variances give the percentage reduction due to the current conditioning information. Again, there is a substantial reduction in the variance from the current information.

V. Conclusions

We conclude by summarizing some of the main results. First, the previous literature on the static problem of instrument choice and optimal monetary rule is unified into a simple framework based on the information structure conditioning decision-making of both the private and the public sectors. In the absence of current information on the endogenized monetary variable, both the symmetric and the asymmetric information structure have the identical error structure given the same instrument. Thus, whatever instrument yields a better covariance matrix is selected as the preferred monetary control variable. Since the policy rule of either instrument does not enter into the error structure, finding its optimal value is made independent of this choice process. When the private and the public sectors do not share the same information set, either the expected equilibrium output or the price level, but not both, can be attained by whichever instrument is selected. On the other hand, when information set is symmetrically partitioned to both decision makers, the output level is completely determined by the aggregate supply, and only the expected price level may be affected by the instrument. It was shown, then, that the availability of current information on the endogenized instrument makes the choice problem unnecessary. Also we show that variances of the output and the price are uniformly reduced by this additional information.

Second, the problem is dynamized in a linear quadratic optimization. It was found that the stationary-state welfare loss contains a component depending on both the instrument selected and its optimal policy rule, which indicates that the choice of the instrument setting and the choice of the instrument itself may not be separable as in the static problem.

Finally, a simple, highly aggregated, four-equation quarterly macroeconometric model for the U.S. is estimated to evaluate the relative importance of (1) choosing the correct instrument, (2) using the optimal multiperiod monetary rule, and (3) conditioning on the current information. In all cases considered, the money stock turned out to be the preferred instrument. The optimal multiperiod policy rule improves the welfare approximately 7 to 9 percent whereas the use of the current information showed the rather significant improvement up to 26 percent.

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Appendix

Clearly when the weighting matrix $H(x)_t$ is equal to K, then the current instrument choice decision only depends on the preference weighted current conditional covariance matrix. The matrix Riccati equation (17) which gives $H(x)_t$ is reduced to the simple static equation $H(X)_t=K$ if there exists a feedback matrix $G(x)_{t+1}$ such that $K(A_x+C_xG(x)_{t+1})=0$. If this is satisfied the system is statically controllable.

When a model is statically controllable the series of decision which minimizes the current period loss $E\{(y(x)-\bar{y}(x))_t'K(y(x)-\bar{y}(x))_t|S_{t-1}\}$ also minimizes the multiperiod loss. Any static linear model is statically controllable since $A=0$ (i.e., $G=0$) and the errors do not accumulate through the dynamics. Tinbergen's condition that the number of instruments equal or exceed the number of targets generally also gives a statically controllable model. For example, assume that the rank of K equals the rank of C (n targets and n instruments), then the optimal feedback matrix is $G(x)_t=-C_x^{-1}A_x$ which satisfies the condition $K(A_x+C_xG(x)_{t+1})=0$.

In practice these conditions are seldom met in realistic problems. The economy is dynamic, policymakers usually control a single instrument and have multiple targets, and even if policies were coordinated these conditions only hold for linear systems.