# Incidence of Taxes in a Growing Economy

Joon Koo Lee*

< Contents >

I. Introduction
II. Structure of the Model
III. Equilibrium Factor Prices
IV. Responsiveness of Factor Supplies
V. Incidence of the Wage Income Tax
VI. Incidence of the Interest Income Tax
VII. Conclusions

## I. Introduction

There have been several attempts to study the incidence of various taxes in the framework of general equilibrium (Feldstein [3][4], Diamond [2], Kotlikoff and Summers[5]). They have succeeded in drawing more general conclusions than the previous partial equilibrium analyses could do. But one common drawback of these studies is that their assumptions on factor supplies are of *ad hoc* nature to a great extent. For example, they generally assume that the supply of each factor is either inelastic or an increasing function of its net remuneration. This may be a harmless, frequently used macroeconomic assumption, but it is not a necessary consequence of microeconomic theory. In fact, it is theoretically possible that the supply of a factor is a decreasing function of its net remuneration. Even though the general conclusions drawn from these studies is that the incidence of a certain tax significantly depends on the elasticities of factor supplies, no plausible explanation is given as to what determines these elasticities and how they are affected by a change in

* The author is Assistant Professor of Economics, Seoul National University.

economic environments, and so on. What they commonly lack is microeconomic foundations on which their analyses should have been based.

The starting point of this study is individuals' life-cycle allocation processes. This microeconomic phenomena will be linked to such macro-economic phenomena as the supplies of labor and capital through aggregation and steady-state relationships. To highlight the long-run interdependence of labor supply and capital accumulation, the analyses will be done in the framework of a growing economy. Especially I will focus on the tax incidence along steady-state growth paths.[(1)]

Section II presents the basic structure of the tax incidence model. It is shown how individuals' allocation processes can be linked to aggregate factor supplies in a steady-state. Section III describes how equilibrium factor prices respond to changes in parameters. Section IV discusses the responsiveness of factor supplies to parameter changes in detail. Section V presents the generalized conditions of the incidence of the wage income tax, while Section VI analyzes the incidence of the interest income tax. The last section concludes this study with the brief summary of the results.

## II. Structure of the Model.

The present model is the simplest of general equilibrium with one product and two factors of production, labor and capital. The technology of the economy may be summarized by this simple neoclassical production function.

$$y=F(L, K), \tag{1}$$

where $y$ denotes total output, and $L$ and $K$ denote th einputs of labor and capital. I assume that the function $F$ is linearly homogenous and that $F_i>0$, $F_{ii}<0$, and $F_{ij}>0$ for $i\neq j$. It is assumed throughout this study that the economy in

(1) The change in tax rates affect the supply of labor in two ways; through changed leisure choice on the one hand and through changed human investment on the other hand. The long-run is a suitable time horizon for the impact of tax rate changes on the accumulation of human capital to be fully appreciated.

question is perfectly competitive. Therefore, both factors are paid what they contribute, that is, their marginal products,

$$w=F_L(L,K) \tag{2}$$

$$r=F_K(L,K). \tag{3}$$

It is to be noted that $w$ and $r$ denote gross returns to factors of production. These two equations represent the demand side of the factor market.

The supplies of factors are determined by individuals' optimizing behavior. A representative individual, who is assumed to live two periods, begins his (or her) economic life with exogenously given wage rate ($w_1$) and an inheritance ($I$) which he has not yet received but which he knows will be given at the beginning of the second period. He first decides how to allocate his total available time in the first period, normalized to be 1, into human investment ($m_1$) and work ($h_1=1-m_1$). It is assumed that such human investment augments one's supply of labor measured in efficiency units by a factor of $g(m_1)$, where $g$ is a strictly concave function. Therefore, his wage rate in the second period is increased to

$$w_2=w_1\cdot g(m_1). \tag{4}$$

It is also assumed that he divides his time in the second period between work ($h_2$) and leisure ($l_2=1-h_2$).

If his consumption expenditure in each period is given by $C_1$ and $C_2$ respectively, his stock of non-human wealth at the end of his life is given by,

$$A_2=(1+(1-t_2)r)\ \{(1-t_1)w_1(1-m_1)-C_1+I\} \tag{5}$$
$$+(1-t_1)w_1g(m_1)(1-l_2)-C_2$$

where $r$=interest rate,

$t_1$=rate of wage income tax,

$t_2$=rate of interest income tax.

He leaves $A_2$ to his child as a bequest. To highlight the differential incidence of the wage income and interest income taxes, I will assume away taxes on intergenerational wealth transfers. Since it is very improbable that an individual leaves a debt to his child, I will assume that $A_2$ is non-negative.

A representative individual tries to maximize his lifetime utility.

$$U(C_1, C_2, l_2)+B(A_2) \tag{6}$$

subject to the end-of-life wealth constraint. The function $B$ denotes the utility of bequest. Both $U$ and $B$ functions are assumed to be strictly concave.

It should be noted that an individual solves this optimization problem assuming that the wage rates and interest rates are exogenously given. He takes the prices of factors of production as a signal and determines his factor supply thereupon. To avoid the complicated problem of aggregation, I will assume that every person in one generation is identical. In a certain period of time, there are two generations of people: one in their first period of economic life and the other in their second period. As population is assumed to grow by the rate of $n$ from generation to generation, the number of young people is $(1+n)$ times that of old people in every period. The labor supply by a young person in the $i$-$th$ period is given by $1-m^i_1$, where $m_1{}^i$ denotes the time spent for accumulating human capital. The labor supply by an old person in the $i$-$th$ period is $g(m_1{}^{i-1})\ (1-l_2{}^i)$ where $m_1{}^{i-1}$ and $l_2{}^i$ denote his human investment in the previous and leisure choice in the current period respectively. Provided there are $(1+n)P_0$ of young people and $P_0$ of old people in that particular period, the total supply of labor will be given by

$$L^i=P_0\{(1+n)(1-m_1{}^i)+g(m_1{}^{i-1})(1-l_2{}^i)\}. \tag{7}$$

Capital is supplied wholly by the old generation. Total capital stock consists of old people's net worth at the beginning of that period $(A_1{}^i)$ and their receipts of inheritances $(I^i)$:

$$K^i=P_0(A_1{}^i+I^i). \tag{8}$$

We have to rewrite the optimization problem of a representative individual who begins his economic life in the $i$-$th$ period with proper superscripts. He wants to maximize

$$U(C_1{}^i,\ C_2{}^{i+1},\ l_2{}^{i+1})+B(T^i,\ A_2{}^{i+1}) \tag{9}$$

subject to

$$A_2{}^{i+1}=(1+(1-t_2)r^{i+1})\{(1-t_1)w_1{}^i(1-m_1{}^i)-C_1{}^i+I^i\}$$

$$+(1-t_1)w_1^{i+1}g(m_1^i)(1-l_2^{i+1})-C_2^{i+1}. \tag{10}$$

The solution of this optimization problem consists of the followings:

$$C_1^i=\alpha(I^{i+1}, w^i, r^i, t), \tag{11}$$

$$C_2^{i+1}=\beta(I^{i+1}, w^i, r^i, t), \tag{12}$$

$$m_1^i=r(I^{i+1}, w^i, r^i, t), \tag{13}$$

$$l_2^{i+1}=\tau(I^{i+1}, w^i, r^i, t), \tag{14}$$

$$A_2^{i+1}=\theta(I^{i+1}, w^i, r^i, t). \tag{15}$$

One must note that $l_2^{i+1}$ presented above should be distinguished from $l_2^i$ in Eq. (7) which is what we need to find out the aggregate supply of labor. Actually $l_2^i$ was determined a period earlier when the current old generation began its economic life. The situation is the same for the supply of capital. We must solve two optimization problems, one for each generation, to find out the aggregate factor supplies.

However, the assumption of a steady-state by solving only one optimization problem, instead of two. In a steady-state, $W^i=W^{i+1}$ and $r^i=r^{i+1}$ hold, for the capital-effective labor ratio is assumed to be kept constant over time. The amount and composition of intergenerational wealth transfers are also same for each generation, and so each generation repeats the same resource allocation pattern in a steady-state. Therefore the solutions of one individual's optimization problem can be substituted into Eqs. (7) and (8) to find out the aggregate labor supplies regardless of superscripts. But we should not directly substitute the solutions we get by maximizing Eq. (9) subject to Eq. (10) even in a steady-state. For a steady-state to prevail, however, the following relationships should be satisfied to keep the capital-output ratio constant and make each generation repeat the same resource allocation pattern.(4)

$$A_2^{i+1}=(1+n)I^{i+2}. \tag{16}$$

---

(2) Notice that there has been a slight change in notation—$w_1^i$ is replaced by $w^i$. Also remember that the superscript $i$ refers to $i$-th time period, not $i$-th generation.

(3) Since the population is assumed to grow by the rate of $n$, this relation is necessary to provide the same initial conditions for each generation. Unless each generation is provided with the same initial condition, there will not be a steady-state.

(4) However, the denominator, $(l+n-\theta_l)$, is unambiguously positive.

Solutions of the individuals' maximization problem, represented by Eqs. (11) through (15), were obtained without regard to this steady-state relationship. Now we have to make these solutions consistent with the condition of a steady-state. Substituting Eq. (16) into Eq. (15), we have,

$$(1+n)I^{i+2}=\theta(I^{i+1}, w^i, r^i, t). \tag{17}$$

In a steady-state, superscripts become meaningless, for one variable has the same value over time. Hence,

$$(1+n)I=\theta(I, w, r, t). \tag{17$'$}$$

Solving this equation for $I$, we can express steady-state inheritance as a function of steady-state wage rate, interest rate and tax rate.

$$I=\phi(w, r, t). \tag{18}$$

This can be obtained by totally differentiating Eq. (17)',

$$(1+n-\theta_I)dI=\theta_w dw+\theta_r dr+\theta_t dt. \tag{19}$$

It is straightforward from comparative statics that

$$1+n-\theta_I>0. \tag{20}$$

Also we know that $\theta_w$ is positive while $\theta_r$ is ambiguous since an increase in $t_1$ is in effect the same as a decrease in $w$, $\theta_{t_1}$ is negative. A similar relationship exists between $r$ and $t_2$, and hence $\theta_{t_2}$ is ambiguous. Obviously the following holds:

$$\frac{dI}{dw}=\frac{\theta_w}{1+n-\theta_I}>0. \tag{21}$$

The derivative $\frac{dI}{dr}$ has the following expression:

$$\frac{dI}{dr}=\frac{\theta_r}{1+n-\theta_I}. \tag{22}$$

The ambiguity in $\theta_r$ makes the sign of the derivative ambiguous. This ambiguity had better be removed somehow, since it turns out that we cannot get any definite result concerning the incidence of the interest income tax with such ambiguity. Close examination of the expression for $\theta_r$ seems to allow us to assume that $\frac{dI}{dr}$ is zero.

Our next task is to derive steady-state consumption expenditures, leisures,

**Table 1. Steady-state Consumption Expenditures ($C_1, C_2$), Leisure Choice ($l_2$) and Human Investment ($m_1$).**

| | $dC_1$ | $dC_2$ | $dm_1$ | $dl_2$ |
|---|---|---|---|---|
| $dw$ | + | + | ? | ? |
| $dr$ | ? | ? | − | + |
| $dt_1$ | − | − | ? | ? |
| $dt_2$ | ? | ? | + | − |

leisure choice and human investment as functions of $w$,$r$ and $t$. Substituting the results we obtained above in Eqs. (11) through (14), we have the following results summarized in Table 1.

Finally we have reached the point where we can derive the steady-state supplies of factors of production utilizing all the results of previous analyses. Substituting the results obtained above in Eqs. (7) and (8), we can exress the steady-state supplies of labor and capital in terms of $w$, $r$ and t. Since this step deserves more close examination, I will postone it to a later section.

## III. Equilibrium Factor Prices

Suppose that we have finished all the calculations necessary for the supply side analysis described above. Then we are able to write the steady-state supplies of labor and capital as follows.

$$L=L(w, r, t), \tag{23}$$

$$K=K(w, r, t). \tag{24}$$

Substituting these into the functions of factor demands, Eqs. (2) and (3), we have the following two equilibrium relations.

$$w=F_L(L(w, r, t),\ K(w, r, t)), \tag{25}$$

$$r=F_K(L(w, r, t),\ K(w, r, t)). \tag{26}$$

For a moment, I will drop the assumption that the function $F$ is linearly homogenous. The reason for this will be made clear soon. This system of two equations can be solved for $w$ and $r$, i.e., equilibrium wage rate and interest rate in a steady-state. Total differentiation of Eqs. (25) and (26) will yield the following system of equations.

$$\begin{bmatrix} F_{LL}L_w+F_{LK}K_w-1 & F_{LL}L_r+F_{LK}K_r \\ F_{KL}L_w+F_{KK}K_w & F_{KL}L_r+F_{KK}K_r-1 \end{bmatrix} \cdot \begin{bmatrix} dw \\ dr \end{bmatrix} = \begin{bmatrix} -(F_{LL}L_t+F_{LK}K_t) \\ -(F_{KL}L_t+F_{KK}K_t) \end{bmatrix} dt. \quad (27)$$

The determinant of the left-hand side matrix is given by,

$$|G|=1-(F_{KL}L_r+F_{KK}K_r+F_{LL}L_w+F_{LK}K_w) + (F_{LL}F_{KK}-F_{LK}F_{KL})(L_wK_r-K_wL_r). \quad (28)$$

If the aggregate production function $F$ is not homogenous of degree one in $L$ and $K$ and the term $(F_{LL}F_{KK}-F_{LK}F_{KL})$ is strictly positive, the stability conditions for the system require that $|G|$ is strictly positive (See Appendix). We do not yet know the signs of individual terms. The comparative static results of this system are,

$$\frac{dw}{dt}=\frac{1}{|G|}\{(F_{LL}F_{KK}-F_{LK}F_{KL})(K_tL_r-L_tK_r)+F_{LL}L_t+F_{LK}K_t\}, \quad (29)$$

$$\frac{dr}{dt}=\frac{1}{|G|}\{(F_{LL}F_{KK}-F_{LK}F_{KL})(K_wL_t-L_wK_t)+F_{KK}K_t+F_{KL}L_t\}. \quad (30)$$

These results are what we are ultimately interested in. Note that the responses of wage rates and interest rates are expressed in very general terms. These formulas can be applied to the analysis of tax incidence of any kind of tax. All that we should know is the shape of the aggregate production function ($F_{ij}$'s) and the supply responses of labor and capital ($L_w$, $K_w$, $L_r$ and $K_r$).(5) Also note that these equations do not depend on the time horizon. They can be applied to studying short-run tax incidence as well as long-run one if we know the supply responses of labor and capital for the relevant time horizon. Feldstein's [4] argument that the long-run incidence of the wage income tax is independent of the supply response of labor while the short-run incidence is sensitive to it depends heavily on the set of specific assumptions he made about the factor supplies. His argument is not necessarily true in a more general setting—there is no logical reason for the asymmetry. In the next section, I will demonstrate that his result is obtained as a special case of the general

(5) As will be shown later, $L_t$ and $K_t$ are especially related to $L_w$, $L_r$, $K_w$ or $K_r$.

formulation presented above.

When we come back to our original assumption of the linearly homogenous aggregate production function, we have to rely on a slightly different argument of stability to sign $|G|$.(6) If the function $F$ is homogenous of degree one in $L$ and $K$, we cannot derive the demand functions for $L$ and $K$ in terms of $w$ and $r$ since the matrix $|G|$ becomes singular. In this case, Eqs. (2) and (3) are not linearly independent. However, equilibrium equations (25) and (26) are still linearly independent. These equilibrium equations can be tranformed to excess demand price equations (For example, $F_L(L(\cdot),\ K(\cdot))-w=0$ is an excess demand price equation.) Since Eqs. (25) and (26) are linearly independent, excess demand functions can be implicitly defined by excess demand price functions.(7) As a result, we have two linearly independent excess demand equations even though we start from two linearly dependent factor demand functions. The stability of the system is guaranteed by,

$$|G|^{*}=1-(F_{KL}L_r+F_{KK}K_r+F_{LL}L_w+F_{LL}K_w)>0 \tag{31}$$

where $|G|^{*}$ denotes the determinant of $G$ when the function $F$ is linearly homogenous.(8) And the comparative static results will be rewritten as

$$\frac{dw}{dt}=\frac{1}{|G|^{*}}(F_{LL}L_t+F_{LK}K_t), \tag{32}$$

$$\frac{dr}{dt}=\frac{1}{|G|^{*}}(F_{KK}K_t+F_{KL}L_t). \tag{33}$$

(6) It is impossible to have a steady-state without assuming the constant returns to scale technology. But Feldstein [3] deals with the tax incidence in a steady-state under the assumption of general aggregate production function without proving that the steady-state is possible in that case.

(7) It should be mentioned in passing that this approach can be applied to the former case where the aggregate production function is not linearly homogenous. But the former approach, which cannot be applied here, has an advantage in that the stability conditions are derived in a more conventional way.

(8) Since the matrix $G$, like the matrix $A$ in the appendix, is the Jacobian of the set of excess demand functions, the stability conditions are:

Trace of $G<0$

Determinant of $G>0$.

But the trace condition,

$2-(F_{KL}L_r+F_{KK}K_r+F_{LL}L_w+F_{KL}K_w)>0$

is subsumed by the determinant condition represented by Eq. (31).

In fact, Eqs. (32) and (33) are special cases of Eqs. (29) and (30) respectively. The discussion of various supply responses we have to know to find out the incidence of a certain tax is presented in the next section.

## IV. Responsiveness of Factor Supplies.

We have seen in the previous section that the responsiveness of factor supplies to changes in parameters plays a critical role in the determination of tax incidence. The thrust of this study is that this should be derived rigorously from microeconomic models rather than being simply assumed. This section looks into the determining factors of the supplies of labor and capital to find out what conditions are necessary to get specific results of tax incidence.

The response of aggregate labor supply to a change in wage rate ($L_w$) is given by

$$\frac{dL}{dw}=P_0\{g'h_2-(1+n))\frac{dm_1}{dw}-g\frac{dl_2}{dw}\}. \tag{34}$$

We can see that the assumption of positive $L_w$, which is made by almost everyone in the study of tax incidence and looks very innocuous, actually depends on the satisfaction of two conditions at the same time. That the substitution effect should dominate the income effect in the decision of leisure choice to have an upward sloping labor supply curve is well-known. However, this is not enough to guarantee a positive $\frac{dm_1}{dw}$ and a negative $\frac{dl_2}{dw}$ in the setting of a steady-state. Remember that a third effect, i.e., the steady-state adjustment effect, is also at work. Therefore, a stronger condition is necessary to obtain such results.

(C. 1) The substitution effect is dominant over the sum of the income effect and the steady-state adjustment effect.

Even if $\frac{dm_1}{dw}$ is positive and $\frac{dl_2}{dw}$ is negative, however $L_w$ is not necessarily positive since we do not know whether

$$g'h_2-(1+n)\gtreqless 0. \tag{35}$$

Note that the condition

$$\text{(C. 2)}\quad (g'h_2-(1+n))\geqq 0$$

is sufficient for $L_w$ to be positive if the condition (C. 1) is already met. From the first order condition for an optimum, we know

$$g'h_2-(1+r)=0 \tag{36}$$

holds for everybody. Hence the above condition requires that the rate of interest ($r$) be greater than or equal to the rate of population growth ($n$) in a steady-state. In other words, the economy should take efficient growth paths. The intuitive reason behind this condition is as follows. If people decide to invest more in human capital, this will decrease labor supply in the current period since the work force is diverted from job markets to schools. But the accumulation of human capital increases labor supply measured in efficiency units in the next period. Since everybody equates the marginal return on human capital to that on non-human capital ($r$), the marginal increase of labor supply in the next period will be equal to $g'h_2(=1+r)P_0dm_1$. In a steady-state, this is also equal to the marginal increase in labor supply by older generation in the current period due to an increase in steady-state $m_1$. Since the number of younger generation is $(1+n)P_0$, the decrease in labor supply due to an incremental increase in human investment is given by $(1+n)P_0dm_1$. Therefore, $(r-n)P_0dm_1$ denotes the net increase in labor supply when human investment ($m_1$) increases infinitesimally. What the condition requires after all is that the net contribution of human investment to the aggregate labor supply be positive. If the term $(g'h_2-(1+n))$ happens to be negative, even the substitution effect will give a conflicting signal.[9]

The response of aggregate labor supply to change in interest rate($L_r$) is given by,

$$\frac{dL}{dr}=P_0\left\{(g'h_2-(1+n))\frac{dm_1}{dr}-g\frac{dl_2}{dr}\right\}. \tag{37}$$

(9) The substitution effect on $m_1$ will tend to decrease aggregate labor supply if $(g'h-(1+n))$ is negative, while the substitution effect on $l_2$ leads to an increase in labor supply by reducing leisure consumption.

We know that $\frac{dm_1}{dr}$ is negative and $\frac{dl_2}{dr}$ is positive. Hence the condition (C. 2) we have seen in the previous section is sufficient for $L_r$ to be negative.

As I mentioned before, a change in the rate of wage income tax ($t_1$) is the mirror image of a change in wage rate. And a similar relationship holds between the changes in $t_2$ and $r$. It follows from this;

$$\frac{dL}{dt_1}=-\frac{w}{1-t_1}\cdot\frac{dL}{dw}, \tag{38}$$

$$\frac{dL}{dt_2}=-\frac{r}{1-t_2}\cdot\frac{dL}{dr}. \tag{39}$$

Hence $L_{t_1}$ is negative if both conditions (C. 1) and (C. 2) are met. If condition (C. 1) is met, $L_{t_2}$ will be positive.

Substituting the expression for $A_1$ into the capital supply equation (8) and totally differentiating it, we have

$$\frac{dK}{dw}=P_0\left\{(1-t_1)(1-m_1)-(1-t_1)w\frac{dm_1}{dw}-\frac{dC_1}{dw}+\frac{dI}{dw}\right\}. \tag{40}$$

The first and fourth term in the braces are positive while the third term is negative. The second term is negative provided condition (C. 1) is satisfied. Thus it is very difficult to tell the sign of the derivative $\frac{dK}{dw}$.

The responsiveness of the supply of capital to a change in interest rate is given by

$$\frac{dK}{dr}=P_0\left\{(1-t_1)w\frac{dm_1}{dr}-\frac{dC_1}{dr}+\frac{dI}{dr}\right\}. \tag{41}$$

The first term in the brace is positive and the second term is ambiguous. The last term is positive by assumption. Unless $\frac{dC_1}{dr}$ has a very large positive value, which is very unlikely, $\frac{dK}{dr}$ will be positive.

We have seen that there is a specific relation between $\frac{dL}{dr}$ and $\frac{dL}{dt_2}$ as well as between $\frac{dL}{dw}$ and $\frac{dL}{dt_1}$. Analogous relations exist in case of the supply response of capital:

$$\frac{dK}{dt_1}=-\frac{w}{1-t_1}\frac{dK}{dw}, \tag{42}$$

$$\frac{dK}{dt_2}=-\frac{r}{1-t_2}\frac{dK}{dr}. \tag{43}$$

Therefore $K_{t_2}$ is likely to be negative while $K_{t_1}$ is ambiguous.

Now we are ready to find out the incidence of taxes in the context of a growing economy. In the next two sections, I will analyze the incidence of a wage income tax and an interest income tax in a growing economy using the results obtained thus far.

## V. Incidence of the Wage Income Tax

In the following discussion of tax incidence in a steady-state, I will resume the assumption of a linearly homogenous aggregate production function. The incidence of the wage income tax in an economy which is characterized by a constant returns to scale technology can be analyzed by looking at only $\frac{dw}{dt_1}$. As $w$ denotes gross wage rate, labor bears at least a part of the burden if

$$\frac{1-t_1}{w}\frac{dw}{dt_1}<1 \tag{44}$$

holds.[10] Labor actually gains from the increase in the wage income tax rate if the inequality goes the other way.

Moreover, how much of the whole burden labor bears, if it does, depends on the sign of $\frac{dw}{dt_1}$. To be specific, labor bears more than the whole burden if $\frac{dw}{dt_1}$ is negative, while only a part of the burden is borne by labor if it is positive. In case $\frac{dw}{dt_1}=0$, labor bears exactly 100% of the burden.

From Eq. (32), we know that the response of the steady-state wage rate to a change in the proportional wage income tax rate is given by,

$$\frac{dw}{dt_1}=\frac{1}{|G|^*}(F_{LL}L_{t_1}+F_{LK}K_{t_1}). \tag{32}$$

To facilitate the determination of the sign of $\frac{dw}{dt_1}$, we may rewrite Eq. (32)

(10) Labor does not bear the burden of the wage income tax at all if the net wage rate remains constant after the change, i.e.,

$$\frac{d((1-t_1)w)}{dt_1}=(1-t_1)\frac{dw}{dt_1}-w=0.$$

like the following using the relations between $L_w$, $K_w$, $L_{t_1}$ and $K_{t_1}$ (See Eqs. (38) and (39));

$$\frac{dw}{dt_1}=\frac{w}{1-t_1}\pi \Big/ \{1-(F_{KL}L_r+F_{KK}K_r)+\pi\} \tag{45}$$

where

$$\pi=-(F_{LL}L_w+F_{LK}K_w).$$

The sign of $\pi$ is not yet known, but we know that the incidence of the wage income tax critically depends on the sign and magnitude of $\pi$. The relations between the incidence of the wage income tax and the signs of $\pi$ and $(1-F_{KL}L_r-F_{KK}K_r)$ are summarized in the table 2.

At first, the possibility that labor gains from the wage income tax (Case (4)) is excluded since the analysis of the supply responses of labor and capital presented in the previous section suggests that it is very unlikely to have

$$1-(F_{KL}L_r+F_{KK}K_r)\leqq 0. \tag{46}$$

We have seen that $L_r$ is negative if there is no capital glut, that is, $r>n$. It has also been shown that $K_r$ will be positive unless $\frac{dC_1}{dr}$ has a very large positive value. Since $F_{KL}>0$ and $F_{KK}<0$, it is very likely that the above expression is positive. Therefore, only Cases (1), (2) and (5) remain as plausible cases.

However, it is very hard to figure out the sign of $\pi$ at this stage. We know that $L_w$ is positive if two conditions are simultaneously met—no capital glut

**Table 2. Conditions of the Incidence of the Wage Income Tax**(11)

| | $\pi$ | $(1-F_{KL}L_r-F_{KK}K_r)$ | Incidence($L$ bears) |
|---|---|---|---|
| (1) | 0 | + | The whole burden |
| (2) | − | + | More than the whole burden |
| (3) | + | 0 | Nothing at all |
| (4) | + | − | $L$ gains |
| (5) | + | + | Only a part of the burden |

(11) Since the stability condition requires the denominator of the derivative, $(1+\xi+\pi)$, be greater than zero, the term $(1+\xi)$ should be strictly greater than zero when $\pi$ is zero or less than zero.

and dominant substitution effect. Then the first term in $\pi$ will be positive. Unless the positive terms in the expression for $K_w$ (See Eq. (40)) are dominant, $\pi$ will be positive, and so labor can shift a part of the burden of the wage income tax to capital. However, we cannot exclude the possibility that $\pi$ is either zero or negative. The inclusion of bequest motives, which distinguishes the present model from that of Kotlikoff and Summers, obviously raises the likelihood of $\pi$ being negative.(12) If bequest motives play an important role in the formation of capital stock, as Kotlikoff and Summers themselves argue in a different paper, labor might bear more than 100 percent of the wage income tax.(13)

Note that Feldstein's [4] result that labor bears exactly the whole burden of the wage income tax in a steady-state regardless of its elasticity of supply holds true only in the razor-edge case where $\pi=0$ in the context of this model. Apart from the fact that $\pi$ being zero is rather a rarity than a rule, it is clear that the long-run incidence depends on the supply elasticity of labor contrary to his assertion. Actually, the assumptions in his model are far stricter than they look. They require the supply responses of labor and capital to have specific magnitudes as well as specific signs. The Kotlikoff-Summers result that labor bears exactly 100 percent of the tax is obtained also under specific assumptions of fixed leisure and lump-sum rebate of tax revenues. Their more general result that labor will always shift some part of the tax when leisure is variable also critically depends upon the assumption of lump-sum rebate and the absence of bequest motives. As we have seen so far, in a more general setting, labor bears at least a part of the burden and sometimes it bears more than 100 percent of the burden.

## VI. Incidence of the Interest Income Tax

The analysis of the incidene of the interest income tax ($t_2$) can be performed

(12) The last term in Eq. (40), $\frac{dI}{dW}$, which represents the bequest motives is positive and therefore raises the likelihood of $\pi$ being negative.

(13) See Kotlikoff and Summers [5].

in exactly the same manner as we dealt with the incidence of the wage income tax. This time, however, we observe the response of interest rate, rather than that of wage rate, to the change in $t_2$. It is given by,

$$\frac{dr}{dt_2}=\frac{1}{|G|^*}(F_{KK}K_{t2}+F_{KL}L_{t2}) \tag{47}$$

which can be rewritten as

$$\frac{dr}{dt_2}=-\frac{r}{1-t_2}\phi\Big/\{1-(F_{LL}L_W+F_{LK}K_W)+\phi\} \tag{48}$$

where

$$\phi=-(F_{KK}K_r+F_{KL}L_r).$$

Analogously to the previous section, we can eliminate a few implausible cases based upon our knowledge of the signs of derivatives. We can show that

$$\phi=-(F_{KK}K_r+F_{KL}L_r)>0 \tag{49}$$

is the most likely outcome and therefore capital bears *at most* a part of the burden of the interest income tax. This result may be contrasted with the one we obtained in the previous section, that is, labor's bearing *at least* a part of the burden of the wage income tax.

## VII. Conclusions

This paper has demonstrated that the rigorous derivation of factor supplies supported by appropriate microeconomic foundation could result in a different pattern of the incidence of taxes in a growing economy than those suggested by previous works. In particular, it has been made clear that, contrary to Feldstein's assertion, the long-run tax incidence critically depends upon the supply elasticities of factors of production. Furthemore, the possibility that labor bears even more than 100% of the burden of the wage income tax, which had been rejected both by Feldstein and Kotlikoff and Summers, has been shown to be real. The introduction of bequest motives has turned out to be the major factor in generating such a result. It should be emphasized, however, that I merely pointed out the theoretical possibility that labor bears more than 100% of the burden of the wage income tax. Further empirical studies on the shape of the aggregate production function and the supply responses of factors of production are necessary to obtain more definite answer to the question. This will also be true for the case of incindlence of the interest income tax.

## References

[1] Blinder, A.S., *Toward an Economic Theory of Income Distribution*, Cambridge, Mass.: M.I.T. Press, 1974.

[2] Diamond, P.A., "Incidence of an Interest Income Tax," *Journal of Economic Theory*, 2, Sep. 1970, pp. 211-224.

[3] Feldstein, M., "Incidence of a Capital Income Tax in a Growing Economy with Variable Savings Rate," *Review of Economic Studies*, 41, Oct. 1974, pp. 505-513.

[4] Feldstein, M., "Tax Incidence in a Growing Economy with Variable Factor Supply," *Quarterly Journal of Economics*, 88, Nov. 1974, pp. 551-573.

[5] Kotlikoff L.J., and L.M. Summers, "Tax Incidence in a Life Cycle Model with Variable Labor," *Quarterly Journal of Economics*, 93, Nov. 1979, pp. 705-718.

[6] Kotlikoff L.J., and L.M. Summers, "The Role of Intergenerational Transfers in Aggregate Capital Accumulation," National Bureau of Economic Research Working Paper, No. 445, Cambridge, Mass.: Feb. 1980.

[7] Samuelson, P.A., *Foundations of Economic Analysis*, Cambridge, Mass.: Harvard University Press, 1947.

[8] Samuelson, P.A., "An Exact Consumption-Loan Model of Interest with or without Social Contrivance of Money," *Journal of Political Economy*, 66, Dec. 1978, pp. 467-482.

## Appendix: Stability Conditions of the System

Samuelson [7] proves that the necessary and sufficient condition for stability is that the real part of every latent root of the matrix A whose *i*-*j*th element is the partial derivative of the $i$-th excess demand function with respect to the $j$-th price evaluated at the equilibrium set of prices be negative. In the context of this model, the matrix A can be derived from the following system of dynamic equations:

$$\dot{w}=\lambda_1(L^D-L^S) \tag{A. 1}$$

$$\dot{r}=\lambda_2(K^D-K^S) \tag{A. 2}$$

where $\lambda_1$ and $\lambda_2$ are some increasing functions. Then the matrix $A$ is given by,

$$A=\begin{bmatrix} \lambda_1'(L_w{}^D-L_w{}^S) & \lambda_1'(L_r{}^D-L_r{}^S) \\ \lambda_2'(K_w{}^D-K_w{}^S) & \lambda_2'(K_r{}^D-K_r{}^S) \end{bmatrix}. \tag{A. 3}$$

The system will be stable if the following conditions are met:

$$\text{Trace of } A<0, \tag{A. C. 1}$$

$$\text{Determinant of } A>0. \tag{A. C. 2}$$

Note that $L_i{}^S$ and $K_i{}^S$ in the above expression are denoted by $L_i$ and $K_i$ respectively in this study. However, $L_i{}^D$ and $K_i{}^D$ are not yet known. These can be derived from the following relations if the function $F$ is not linearly homogenous.

$$w=F_L(L, K), \tag{2}$$

$$r=F_K(L, K). \tag{3}$$

If the function $F$ is not linearly homogenous, i.e., $F_{LL}F_{KK}-F_{LK}F_{KL}\neq 0$, $L_i{}^D$ and $K_i{}^D$ can be written as,

$$L_w{}^D=\frac{F_{KK}}{F_{LL}F_{KK}-F_{LK}F_{KL}}, \tag{A. 4}$$

$$L_r{}^D=\frac{F_{KL}}{F_{LL}F_{KK}-F_{LK}F_{KL}}, \tag{A. 5}$$

$$K_w{}^D=\frac{F_{KL}}{F_{LL}F_{KK}-F_{LK}F_{KL}}, \tag{A. 6}$$

$$K_r{}^D=\frac{F_{LL}}{F_{LL}F_{KK}-F_{LK}F_{KL}}. \tag{A. 7}$$

Substituting these into the expression for the matrix $A$, we can see that the condition (A. C. 2) requires that

$$1-(F_{KL}L_r+F_{KK}K_r+F_{LL}L_w+F_{LK}K_w)$$

$$+(F_{LL}F_{KK}-F_{LK}F_{KL})(L_wK_r-K_wL_r)>0, \tag{A. 8}$$

which is equivalent to

$$|G|>0. \tag{A. 9}$$