Causes of Inefficiency in the Theory of Local Public Goods

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I. Introduction

Since the classic Tiebout [23] paper, a vast literature has accumulated on the theory of local public goods. One line of development is the theory of clubs, commenced by Buchanan [6] and generalized by, among others, Pauly [17, 18] and McGuire [11, 12]. The present paper investigates the individualistic incentives arising in varying sizes of clubs, and comes up with following two propositions:

(1) The "rich" are less vulnerable to misprovision of local public goods than the "poor."

(2) Dynamic instability of jurisdiction structure leads to "cycles" of migration, jurisdictional mergers and fragmentation.

In the theory of voluntary club formation, the following have to be determined simultaneously: the number and size of clubs, the level of provision of the collective good in question, and the distribution of households among jurisdictions. Under the identical taste and equal-cost-sharing assumptions, the problem can be

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set up as maximizing the utility of a representive individual $U\left(x, \bar{y}-\frac{C(x,n)}{n}\right)$, where x is the collective good consumption and $\bar{y}$ is given private good income, and $C(x, n)$ is the cost of the collective good x. Here the club size n enters as an argument of the cost function reflecting the scale economies of joint consumption and/or the congestion cost. As first order conditions for maximizing utility, we obtain, $\frac{C}{n}=\frac{\partial C}{\partial n}$ and $n\frac{\partial U/\partial x}{\partial U/\partial y}=\frac{\partial C}{\partial x}$. These are McGuire's[12] "normative supply function" and "normative demand function" respectively, and depicted as SS and DD curves in Fig. 1a. For both functions, $\frac{dx}{dn}<0$ is assumed solely for illustrative purpose. Qualitative nature of the analysis below remains unchanged with $\frac{dx}{dn}>0$.

The only restriction needed is the existence of a set of non-intersecting utility contours in the (x, n) space. The circles in Fig. 1a depict the utility mapping of those who have the normative demand function D_bD_b, with the inner circle indicating higher utility than the outer one and the point B the highest utility. The slopes of the utility contours are vertical at the points of intersection with the D_bD_b curve and horizontal at the intersection with the SS curve by the nature of the curves DD and SS. Likewise when we have consumers of different tastes, we can envision the SS curve as a sort of "contract" curve.

When we have a population composed of different taste groups (e.g. with demands of D_aD_a and D_bD_b in Fig. 1a) and the size of each homogeneous

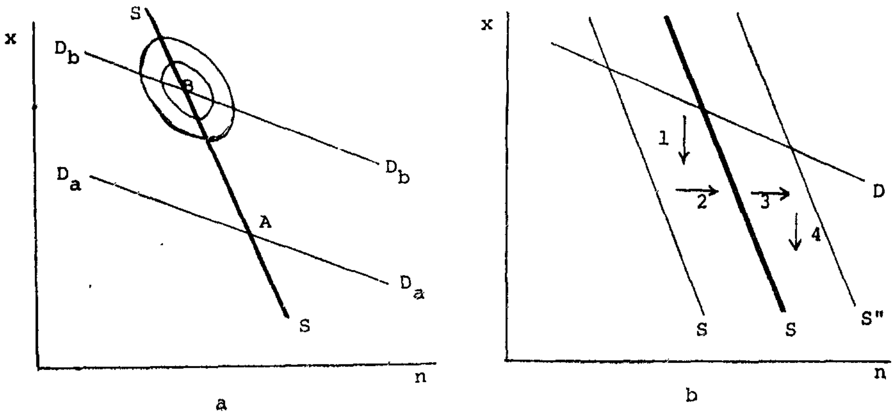

Fig. 1.

subpopulation is big relative to the corresponding optimal club size, perfect mobility leads to a voluntary segregation which is Pareto-efficient (points A and B in Fig. 1a). (1)

However, for this to be true we have to rule out the individualistic incentives causing inefficiencies. As in Fig. 1b the free-rider behavior and the resulting underprovision of the local public good shifts the SS curve down to $S'S'$ (arrow 1), and the entry restriction results in overpopulated jurisdictions shifting the SS curve rightward to $S''S''$ (arrow 3). These can be seen as disequilibrating forces rather than shifts of the SS curve, without changing the substance of the main arguments. Underprovided groups have the incentive to attract more members (arrow 2), and overpopulated groups have the incentive to lower their production of the local public goods (arrow 4). "Thus, the combination of these two sorts of non-optimal individualism just might result in an *efficient* range of jurisdictionals, sizes, and public-good outputs, but with each and every jurisdiction *inhabited* by the *wrong* people" (McGuire[12, pp. 129–130]).

Following McGuire's [12] analysis on incentives tending toward inefficient configuration, namely the free-rider behavior and the monopolistic restriction on the number of jurisdictions, this paper attempts some further exercises on the positive feature of the model. In section II it will be argued that due to two kinds of asymmetric effects of the causes of inefficiency, those with a certain preference (the poor) are less likely to fully realize their potential utility than others (the rich), and accordingly, integration rather than segregation may be

(1) "[T]his assumption [of efficiently sized jurisdictional options] together with free, utility-maximizing search and marginal cost 'membership charges,' implies an overall-efficient constellation of segregated jurisdictions." (McGuire [12, p. 127]).

This might be called the ***voluntary-segregation-optimum-theorem***, which contains both normative (segregation is more efficient than integration when feasible) and positive (voluntary segregation) features.

However, we may reach the same conclusion by replacing his efficient size assumption plus marginal cost charge with the equal cost sharing scheme; since by the nature of the U-shaped average cost curve, equal-cost-sharing entrants will be welcomed up to the efficient club size where the average cost equals the marginal cost. Without loss of generality we continue to assume equal cost sharing throughout this paper. It is implicitly presumed that restructuring jurisdictions involves no transactions cost in the long run.

desirable. In a generalization to a continuously heterogeneous population model, section III shows that the above tendency is strengthened by an additional cause of inefficiency, viz., incentive for excessive group proliferation. Section IV considers a dynamic process with interjurisdictional mobility and the median-voter decision making, and suggests that unless strict exclusion is possible, the process of self-selection and segregation is a highly unstable one involving congestion in every jurisdiction, and that the convergence to an optimal state is very unlikely. A brief summary and policy consideration follow in section V.

II. Two Asymmetries in Homogeneous Subpopulation Model

Fig. 1 is redrawn as Fig. 2 with tilted shifts of the *SS* curve rather than parallel shifts to allow for the asymmetric effects of free rider behavior and entry restriction. That the free rider problem and resulting under-provision of collective goods are more serious in large groups than in small groups is widely contended.(2) Large groups are at disadvantage in the costs of organization and communication, burden of strategic interaction among members, and reward for group-oriented action. Likewise, the entry restriction problem affects larger groups more heavily. Large groups typically have enough bargaining power such

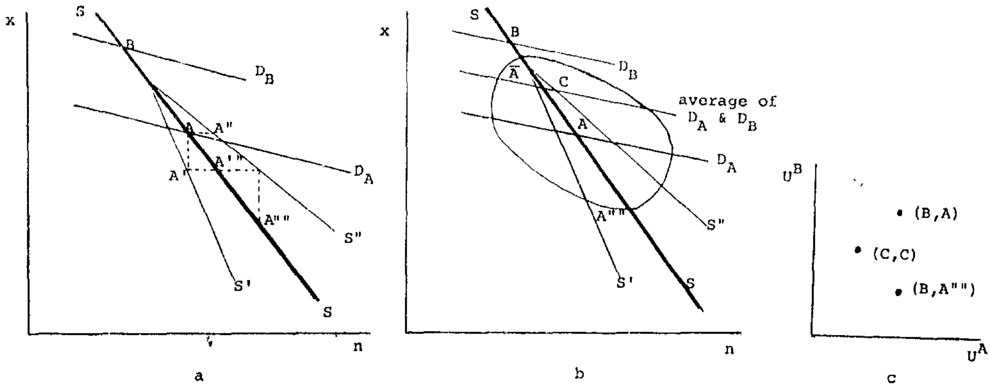

Fig. 2.

(2) Most convincingly Olson [15]. For long-run dynamic implications, see Olson[16]. The tilted (downward) shift of the *SS* curve to *S′S′* in Fig. 2 reflects the vulnerability of larger groups to free riders. Here, existence of free riders implies not only the untrue revelation of public goods demand by members but also various forces leading against the group interest like those mentioned above.

as: capacity for lobbying, threat, etc., to influence institutional changes in their interests.(3) Hence they are more likely to exercise monopolistic entry restrictions than smaller groups. Furthermore, formation as a large group is more difficult than as a small group, due to informational and administrative cost disadvantages, free rider problems, possible fixed costs, and so on.(4)

Clearly those (who are probably poor) whose preference would have led them to choose point A (Fig. 1) could end up at points A', A'', A''', or at worst, at point A'''' when both disadvantages reinforce; while those (probably wealthy) who would choose higher levels of production and smaller group sizes like point B can remain immune to either free-riders or monopolistic behavior. This implies efficient procurement of the local collective good by the wealthy on the one hand, and the lower provision of it in a congested group for the poor on the other. Instead of getting into the complications of equity arguments, let us examine the segregation-integration issue briefly. Fig. 2b is a reproduction of Fig. 2a with utility mapping of A-type consumer-voters. Points A and B present segregated solutions for A-type and B-type consumer-voters respectively, and the point C is the solution for integrated jurisdictions. So long as it is possible for $\bar{A}$, the point at which the utility contour passing A'''' cuts the SS curve above, to be located on a lower utility contour than C, an integration solution is not strictly Pareto-inferior to a segregated one. Fig. 2c plots the actual utility levels of the consumers of type A (U^a) and those of type B (U^b), where (B, A) corresponds to the segregation optimum, (B, A'''') to the segregation with two kinds of asymmetric inefficiency, and (C, C) to the integrated solution. As shown in Fig. 2c, integration (C, C) might be more desirable than segregation (B, A'''') according to the choice of a social welfare function.

(3) It is interesting to note that V. Goldberg proposed the inclusion of the self-interest motive in influencing institutional changes in optimal decentralization models. See Goldberg [9, esp. p. 474, fn. 25].

(4) For related arguments in the context of Industrial Organization, see, e.g., Williamson [26]. The tilted (rightward) shift of the SS curve to $S''S''$ reflects the greater vulnerability of larger groups to the entry restriction.

III. Continuously Heterogeneous Population Model

So far we have dealt with the homogeneous-subpopulation model developed by McGuire [12]. He conjectured that "This theorem [the voluntary-segregation-optimum theorem referred to in section I (fn. 1)] is also valid for the case of populations with continuous distributions of income tastes."(5) We now proceed to examine this conjecture.(6) In this continuously heterogeneous population model, each jurisdiction comprises members of different preferences (income-tastes), only one (most probably the median voter)(7) of whose preference is exactly realized in collective goods provision. Therefore the larger the group size, the greater the divergence between the preference of the marginal (boundary) member and the actual pattern of local collectives (x, n).

Before going into details about the implication of this different divergence pattern by different group sizes, it seems helpful to concentrate our attention on a special case where the distribution of income-taste is appropriately skewed so that each group has the same degree of variability in the preference of its members (as in Fig. 3).(8) Under the above condition, mobility and voluntary segregation will probably lead to (or at least maintain, once reached) the optimum jurisdictional constellation with the following properties:

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(5) McGuire [12, p. 122, fn. 16].

(6) There are several local public good models which allow for continuously heterogeneous populations. Westhoff[24] is the most noteworthy, but his model deals with "Samuelsonian pure public goods [financed by a proportional income tax]...and consequently increasing returns to scale with respect to community size are never exhausted." (pp. 85-86).

Wheaton [25] compares alternative taxing schemes in a model considering consumer heterogeneity. Berglas [2] extends the comparison to include diverse tastes and ***skills***, although the diversity is restricted to the two homogeneous subpopulations. Here we assume equal-cost-sharing which seems more relevant to the present discussion.

(7) cf. Black [4]. For applications of the median-voter outcome to local public goods models, see e.g. Bradford & Oates [5] and Bergstrom & Goodman [3].

(8) In other words, if the number of people who want higher n is big enough relative to the number of those who prefer lower n (the density of the taste distribution is higher in the higher-n-preferring group), then it can exactly offset the tendency of the greater divergence beween the preference of the marginal member and the actual pattern of local collectives in larger groups than in smaller groups. Now the larger groups have more inframarginal members than the smaller groups.

(1) the population is segregated into quasi-homogeneous collections of the most similar income-tastes,[9]

(2) each jurisdiction with median-voter-determined level of (x, n) encompasses exactly n members,

(3) consumer-voters at boundaries are indifferent to either direction because their losses are the same relative to the median-voters of either side. Here it is assumed that the welfare loss of a representative consumer depends

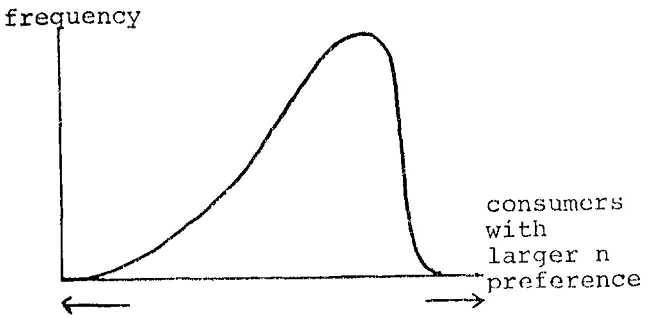


Fig. 3.

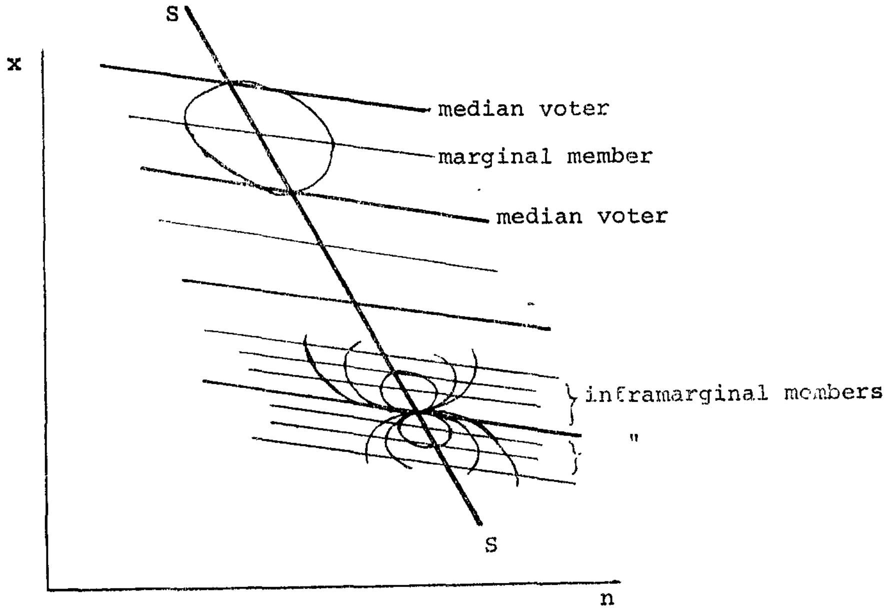


Fig. 4.

(9) This result is comparable to a more formal statement of Westhoff. "Theorem 1. If there exists an equilibrium, then each community is formed from a single interval of consumers." Westhoff [24, p. 93].

only on the magnitude of the divergence of the actual pattern of (x, n) from his desired pattern, and not on whether the divergence is positive or negative.

(4) specified jurisdictions are in such numbers as to exhaust the population.

This situation is depicted in Fig. 4 where the line SS is divided into equal segments by construction, and hence the "rents" accrued to members with median preference and likewise those at the comparable level of inframargin are the same across jurisdictions.

Thus, if there are no free-riders and if entry is allowed up to the optimal level, "voting-by-feet" leads to an overall segregation equilibrium optimum. However, in the absence of forces restricting the number of jurisdictions to the optimum, there arises another cause of inefficiency, i.e. an incentive to excessive entry. Every marginal (boundary) or near-marginal consumer-voter will choose a strategy to create a new jurisdiction in the hope that the neighboring inframarginal members will join in, and that through the memberships relocation (by virtue of mobility) a parish could be established with almost the same size as the previously prevailing efficient ones.(10) By the nature of this incentive,

(10) Suppose there are two jurisdictions: There arises an incentive to form another jurisdiction (shaded area in Fig. a) at the neighborhood of the marginal member, Mr. B. Instead of

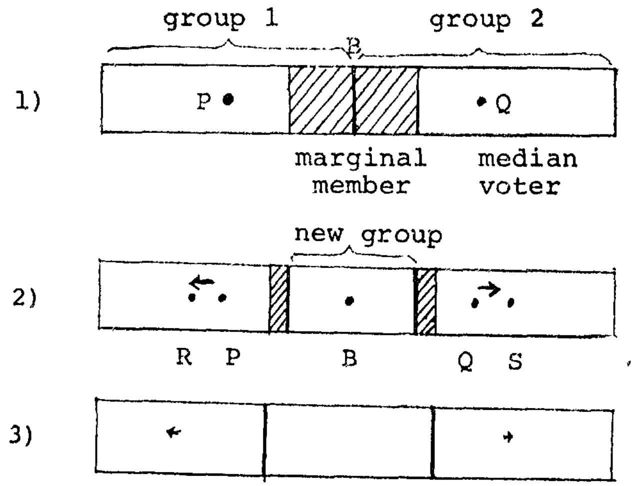


Fig. a.

every entrant imposes an external diseconomy on the others; and the whole community ends up with excessive proliferation of small-size high-cost jurisdictions (the *SS* curve shifts leftward).(11) This is the same result that McGuire mentioned [12, p. 130]. But it is doubtful that the conclusion can come from his homogeneous-subpopulation model. The incentive to overproliferate stems

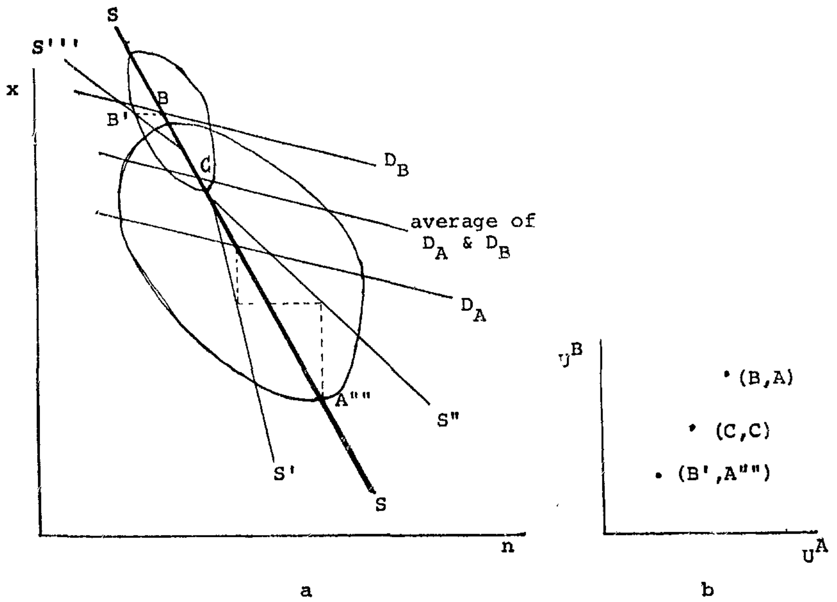


Fig. 5.

paying for the pattern of local collectives reflecting the preferences of Mr. P or Mr. Q, those marginal members wish their own jurisdiction to reflect their preferences. Once they belong to another club, the median-voter-determined patterns of local collectives of the established groups will change as their median-voters move further away from each other in preference (Mr. R and Mr. S are now median-voters rather than Mr. P and Mr. Q, and so on). As a result, additional members of the established groups are attracted to the new group. The process of relocation-production revision interaction continues until the size of the new group expands to the same size as the other two.

This can be easily generalized to many jurisdictions cases. Each period, group size equalization comes about in a wave-like pattern starting from the point of entry, until all groups are of the same size, slightly smaller than the original ones.

(11) The analogy is from the monopolistic competition model of Chamberlin [7]. For a more recent and sophisticated version. "Theorem 6: Under increasing returns to scale and with a sufficiently uniform and evenly distributed market, 'monopolistic competition' will lead to a greater degree of product differentiation than is socially optimal" (Lancaster [10, p. 582]). Also see Spence [22], and Schmalensee [21].

The analogy between the theory of clubs and the theory of industrial market has been discussed for some time. See, for example, McGuire [12, p. 130] and Berglas [1, p. 121].

only from continuously-heterogeneous preferences.(12)

By now, we are ready to investigate the effect of asymmetry mentioned in section II. In this version of model, Fig. 5 illustrates the situation. A leftward shift (tilt) of S to S''' reflects excessive entry, where this effect also is envisioned to have asymmetry, i.e., entry with smaller size is easier, therefore smaller size jurisdictions proliferate more. Here Fig. 5 is drawn such that integration (C, C) is Pareto-superior to segregation (B', A'''').(13) This possibility definitely enhances the case for integration.

Another interesting result can be observed from Fig. 5. By moving down points like A along the SS curve, and by proliferation in the left part of the curve (in other words making small groups smaller and large groups larger), the variability of income-tastes within groups are no longer equal across the jurisdictions, even if we start from the situation where they are exactly the same. That is, a large jurisdiction tends to encompass relatively greater variation of member-tastes than a small group. This leads to a wholly different and interesting story.

IV. Mobility and Voting—A Dynamic Consideration

We proceed with simplified assumptions of uniform distribution of income-tastes, optimal number of jurisdictions, no free-riders, no monopolists, no excessive entry.(14) As is shown in Fig. 6a and mentioned in section III, those who choose larger production in the smaller group have an advantage in realizing their potential utility maximum over those who want a lower level of production

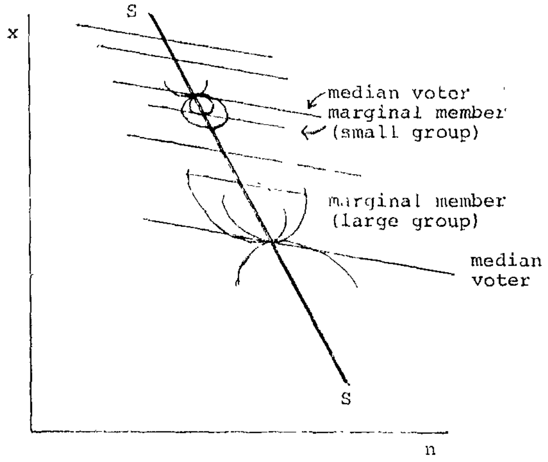
(12) Or possibly when the sizes of homogeneous subpopulations are small relative to the optimal sizes. See fn. 16 infra.

(13) The relation between Fig. 5a and Fig. 5b is the same as in Fig. 2.

(14) The assumption of uniform distribution is only for simplicity. Qualitatively similar results hold as long as the skewness of the taste distribution does not fully offset the force that "the larger group size one chooses, the greater the probability of his utility loss due to the divergence of the actual provision of local collectives from his desired pattern." Furthermore, even if the two forces just offset, or skewness overrules, free-rider and entry restricting behavior at the larger group level, and excess proliferation at the smaller group level will make the analysis of this section applicable.

in the larger group, because any marginal member in the smaller groups can enjoy a utility level *pari passu* closer to the median-voter's than his counterpart in larger groups. This result is all the more striking if we recall that the overall-efficient optimality condition ($\sum MRS = MC = AC$) still holds, and all those inefficiency generating problems like free-riders, monopolistic entry restriction, excess proliferation are assumed away. Such advantages in smaller groups are the key factors in the following dynamic considerations.

As shown in Fig. 6b, the stability of the optimal size distribution of jurisdictions is lost unless water-tight exclusion devices are feasible and exercised because every marginal member is not indifferent to either direction. In fact, those (shaded in Fig. 6b) members who want more collective goods and a smaller group than the midpoint member between the median-voters of their group and their adjacent jurisdiction, have every incentive to relocate. This is



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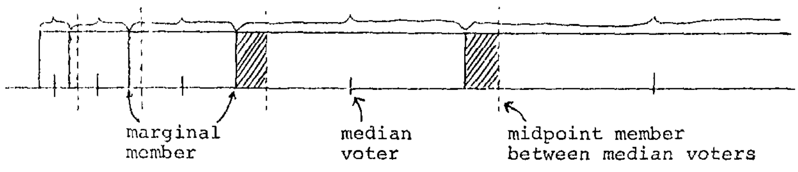

b

Fig. 6.

because their preferences are closer to the actual pattern of local provisions in the neighboring smaller group than to that of their own group. This holds true for all sizes of jurisdictions. Hence, at the same time every group has a certain portion of its members endeavoring to relocate to the neighboring smaller group, and has an ultimate task of excluding immigrants from the next larger group, no matter what devices it relies on, e.g., zoning regulation or implicit realtor control.(15)

It is a reasonable assumption that strict exclusion could not be achieved. If that is the case, the aspirants (shaded portion of Fig. 6b) will move to the adjacent smaller groups with higher levels of production. Accordingly, every jurisdiction except one (the largest economical jurisdiction) suffers congestion (the *SS* curve shifts rightward to *S′S′* in Fig. 7). If this were the whole story, then it might be tempting to accept the resulting jurisdictional constellation as a second best configuration. But the Buchanan-McGuire model (of simultaneous determination

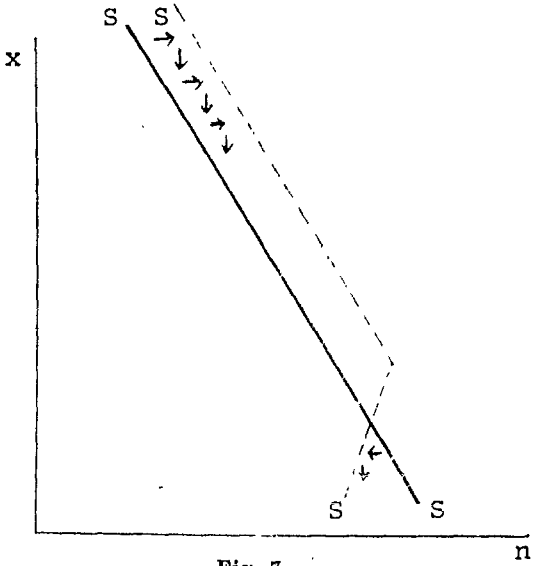

Fig. 7.

(15) This is a quite different phenomenon from the widely discussed tendency of horizontal inequity and immigration from the poor community to the rich. Recall that we are dealing with equal-cost sharing (benefit taxation) and that the framework artfully avoids spatial consideration by postulating a jurisdictional constellation for every local public good. For analyses of incentives to relocate questing for a tax base, see Oates [14, esp. p. 83] and references cited there.

of group size and the level of production) does not stop here. The problem of deciding the pattern of collective good provision (x, n) reoccurs. As a result, every single jurisdiction revises its level of production downward to meet the new median-voter's preference (downward arrows in Fig. 7).

Unfortunately, this process of adding congestion and downward revision of the level of production does not end at the first round. The size distribution of jurisdictions is highly unstable and consumer-voters suffer everincreasing congestion and an everdecreasing collective goods level. After a certain period of degeneration, however, the smallest group with the highest level of production gets large enough to break down, and launches the birth of a new, still smaller (with still higher provision level) group. Likewise, the lowest two production groups, still losing membership, will eventually merge to one, and then merge to another as they lose more members. In the meantime, degeneration continues and repeats its cycle.(16)

An attempt at formal analysis and hypothetical simulation results are reported in the Appendix. But what interests us at this point is the perception of a representative consumer-voter. He will observe the level of provision of collec-

(16) It is worth comparing the stability properties of the homogeneous-subpopulation model and the continuously-heterogeneous-population model.

Whenever the "homogeneous-subpopulations" are small relative to, and not divisible by, the optimal club sizes, no stable distribution of club sizes need exist. (Pauly [17, 18]).

Compare incentives arising at the boundaries of the continuously-heterogeneous-population model with this case.

"With an optimal club size of 2/3 of the [homogeneous sub] population, for example, only one such club can exist... Analytically the problem is identical to the cycling problem ...The two farmers forming a winning majority constitute an optimal sized club, but the one farmer left out has an incentive to try and form an optimal club too, and his efforts to form a new club can lead to an unstable equilibrium, a cycle." (Mueller [13, p. 133]).

In a more general *non*continuously-heterogeneous-population framework encompassing both cases, the latter (homogeneous) might be called a "local cycle," while the cycle mentioned in the text might be named "global."

Ellickson's [8] stable equilibrium is only based on his assumption of equal size clubs. Generally, heterogeneous population dictates that optimal club sizes differ, which in turn determines the pattern of instability. In special cases where the clubs take the form of fixed geographic localities, or where jurisdictional mergers and fragmentations are not feasible, then the continuously-heterogeneous-population model has a stable *non*-optimal equilibrium with each club equal in size. This is not equivalent to Ellickson's result, however.

tives in his group forever lowering and his group getting more congested, until he decides to leave "his" group and move to another (smaller, higher production level) group. There he will encounter the same situation, and the cycle will be repeated.

Aside from any historical relevance of such a dynamic process of group structure change, all we are suggesting is that it is very difficult to hope that the free self-selection and self-segregation will lead to the optimal level of provision of local collective goods in an optimal constellation of efficient-sized jurisdictions, either in the short run or in the long run.

V. Summary and Policy Implications

It is argued that due to asymmetrical effects of the free-rider behavior and monopolistic entry restriction, relatively poor consumer-voters who would choose a lower level of local collective goods provided in a larger jurisdiction have inherent disadvantages in realizing their planned resource allocation. This suggests that integration is not always Pareto-inferior to segregation. With a continously-heterogeneous-population model, this argument gets additional support from the asymmetrical effects of excessive group-proliferation incentives, and from the peculiar dynamics of mobility and voting.

With regard to the policy of discouraging or facilitating entry there seems to be an ambivalence, due to the inefficiency resulting from the excessive proliferation of jurisdictions on the one hand, and the monopolistic restrictions on the other. Fortunately by calling attention to the asymmetry between them, one could hope to contrive a selective entry-control policy designed to limit small group entries and encourage large group entries.

When considering dynamics however, we find another ambivalence as to the promotion of or delimitation of "mobility." A sufficient amount of mobility is needed to achieve a free self-selecting segregation optimum, while strict exclusion is important in maintaining the efficient jurisdictional sizes in a stable manner.

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Appendix

Recall the formulation in section I in terms of maximizing utility in (x, n) space. Let n^* be the bliss point solution for n, then we can regard n^* as representing preferences. Assume a uniform distribution of the population over the income-tastes interval $0<n^*<P$, with *p.d.f.* $f(n^*)=1/P$, where P is the size of the population who want to provide the local collective good in question. Now since the population is lined up according to n^*, the

n^{*th} order individual will have the preference n^*. For the first individual, n^* equals one, for the second, two, and so on.

Suppose the population is divided into optimal sized clubs. Then for each club, the size (S_i) equals the preference n^* of the median voter, which in turn equals his order in population. From this efficient production condition, we get $S_1=1,\ S_2=3,\ S_3=9, \cdots, S_i=3^{i-1}$, and, $S_{i+1}=3S_i,\ \ i=1, 2, 3, \cdots$

The problem in section IV can be formulated as follows:

Change in group size=the number of entrants minus leavers

$$\Delta S_i = \left(\frac{S_{i+1}+S_i}{4}-\frac{S_i}{2}\right)-\left(\frac{S_i+S_{i-1}}{4}-\frac{S_{i-1}}{2}\right)$$

$$=\frac{1}{4}(S_{i+1}+S_{i-1}-2S_i),\quad i=2, 3, \cdots, N-1$$

$$\Delta S_1 = \left(\frac{S_2+S_1}{4}-\frac{S_1}{2}\right)=\frac{1}{4}(S_2-S_1)$$

$$\Delta S_N = -\left(\frac{S_N+S_{N-1}}{4}-\frac{S_{N-1}}{2}\right)=\frac{1}{4}(S_{N-1}-S_N),$$

where subscript N denotes the largest group.

By setting $S_i^t=S_i^{t-1}+\Delta S_i$ and rearranging,

$$S_i^{t+1}=\frac{1}{4}(2S_i^t+S_{i+1}^t+S_{i-1}^t),\quad i=2, 3, \cdots,\ N-1$$

$$S_1^{t+1}=\frac{1}{4}(3S_1^t\ +S_2^t)$$

$$S_N^{t+1}=\frac{1}{4}(3S_N^t+S_{N-1}^t),$$

where superscripts indicate time periods.

This is a first order homogeneous simultaneous difference equation system. But because of additional constraints,

$$S_1^t \geqq S_1^0+S_2^0 \longrightarrow \text{break-down and reindexing of groups}$$

$$S_{N-1}^t+S_N^t \leqq S_N^0 \longrightarrow \text{merge},$$

it is difficult to solve this analytically. Instead, a numerical solution is tried. Below is the result of a simulation with initial values $S_0=1,\ S_1=3,\ S_2=9, \cdots,\ S_i=3^{i-1}, \cdots$. The number of groups is truncated at 10, because the size of the 10th group (19683) seems to be sufficiently large to represent an economically feasible group size in providing a typical local collective good.(17) The simulation was started from an optimal state and carried out in

(17) However, the essential dynamic characteristics described in the text do not change with the population size. Additional experiments with different population sizes support this generality.

Table 1

Period	Jurisdictional Size Distribution								(Population=29524)			
1	1.0	3.0	9.0	27.0	81.0	243.0	729.0	2187.0	6561.0	19683.0		
2	1.5	4.0	12.0	36.0	108.0	324.0	972.0	2916.0	8748.0	16402.5		
3	2.1	5.4	16.0	48.0	144.0	432.0	1296.0	3888.0	9203.6	14488.9		
4	2.9	7.2	21.3	64.0	192.0	576.0	1728.0	4568.9	9196.0	13167.6		
5	1.0	3.0	9.7	28.5	85.3	256.0	768.0	2150.2	5015.5	9032.1	12174.7	
6	1.5	4.2	12.7	38.0	113.8	341.0	985.6	2521.0	5303.3	8813.6	11389.0	
7	2.2	5.6	16.9	50.6	151.7	445.5	1208.4	2832.7	5485.3	19325.1		
8	3.0	7.6	22.5	67.5	199.9	562.8	1423.7	3089.8	8282.1	15865.1		
9	1.0	3.2	10.2	30.0	89.3	257.5	687.3	1625.0	3971.3	8879.8	13969.4	
10	1.5	4.4	13.4	39.9	116.5	322.9	814.3	1977.2	4611.9	8925.1	12697.0	
11	2.3	5.9	17.8	52.4	149.0	394.2	982.2	2345.1	5031.5	8789.7	11754.0	
12	3.2	8.0	23.5	67.9	186.1	479.9	1175.9	2676.0	5299.5	19604.2		
13	1.0	3.4	10.6	30.7	86.4	233.0	580.4	1376.9	2956.8	8219.8	16028.0	
14	1.6	4.6	13.8	39.6	108.4	281.7	691.9	1572.8	3877.6	8856.1	14075.9	
15	2.3	6.2	18.0	50.3	134.5	340.9	809.6	1928.8	4546.0	8916.4	12771.0	
16	3.3	8.2	23.1	63.3	165.1	406.5	972.2	2303.3	4984.3	8787.5	11807.3	
17	1.0	3.5	10.7	29.4	78.7	200.0	487.6	1163.6	2640.8	5264.8	19644.0	
18	1.6	4.7	13.6	37.1	96.7	241.6	584.7	1363.9	2927.5	8203.6	16049.2	
19	2.4	6.1	17.2	46.1	118.0	291.1	693.7	1560.0	3855.6	8846.0	14087.8	
20	3.3	8.0	21.7	56.8	143.3	348.5	809.6	1917.3	4529.3	8908.8	12777.4	
58	1.0	4.0	9.5	21.6	49.8	114.4	261.9	602.3	1371.9	2921.0	8173.9	15992.9
59	1.7	4.6	11.1	25.6	58.9	135.1	310.1	709.6	1566.8	3847.0	8815.4	14038.1
60	2.5	5.5	13.1	30.3	69.6	159.8	366.2	824.0	1922.5	4519.0	8879.0	12732.5
61	3.2	6.6	15.5	35.8	82.3	138.9	429.1	984.2	2297.0	4959.9	8752.4	11769.1
62	1.0	3.1	8.0	18.4	42.4	97.3	222.3	507.8	1173.6	2634.5	5242.3	19573.3
63	1.5	3.8	9.4	21.8	50.1	114.8	262.4	602.9	1372.4	2921.2	8173.1	15990.6
64	2.1	4.6	11.1	25.8	59.2	135.6	310.6	710.1	1567.2	3847.0	8814.5	14036.2
65	2.7	5.6	13.1	30.5	69.9	160.2	366.7	824.5	1922.9	4518.9	8878.1	12730.8
66	3.4	6.8	15.6	36.0	82.6	139.3	429.6	984.7	2297.3	4959.7	8751.5	11767.6
67	1.0	3.3	8.1	18.5	42.6	97.6	222.7	508.3	1174.1	2634.7	5242.0	19571.6
68	1.6	3.9	9.5	21.9	50.3	115.1	262.8	603.3	1372.8	2921.4	8172.5	15988.8
69	2.2	4.7	11.2	25.9	59.4	135.8	311.0	710.6	1567.6	3847.0	8813.8	14034.8
70	2.8	5.7	13.3	30.6	70.1	160.5	367.1	824.9	1923.2	4518.9	8877.3	12729.5
71	3.5	6.9	15.7	36.2	82.9	185.6	429.9	985.0	2297.5	4959.6	8750.8	11766.5
72	1.0	3.4	8.2	18.6	42.7	97.9	223.0	508.6	1174.4	2634.9	5241.9	19569.4
73	1.6	4.0	9.6	22.0	50.5	115.4	263.1	603.7	1373.1	2921.5	8172.0	15987.5
74	2.2	4.8	11.3	26.0	59.6	136.1	311.3	710.9	1567.8	3847.0	8813.3	14033.7
75	2.8	5.8	13.4	30.7	70.3	160.8	367.4	825.2	1923.4	4518.8	8876.8	12728.6
76	3.6	6.9	15.8	36.3	83.0	189.8	430.2	985.3	2297.7	4959.4	8750.2	11765.6
77	1.0	3.4	8.3	18.7	42.9	98.0	223.2	508.9	1174.6	2635.0	5241.7	19568.2
78	1.6	4.0	9.7	22.2	50.6	115.5	263.3	603.9	1373.3	2921.6	8171.7	15986.6
79	2.2	4.8	11.4	26.2	59.7	136.3	311.5	711.1	1568.0	3847.0	8812.9	14032.8
80	2.9	5.8	13.4	30.9	70.5	160.9	367.6	825.4	1923.5	4518.7	8876.4	12727.8
81	3.6	7.0	15.9	36.4	83.2	190.0	430.4	985.5	2297.8	4959.4	8749.8	11765.0
82	1.0	3.5	8.4	18.8	43.0	98.2	223.4	509.1	1174.8	2635.1	5241.6	19567.2
83	1.6	4.1	9.7	22.2	50.7	115.7	263.5	604.1	1373.5	2921.7	8171.4	15985.8
84	2.2	4.9	11.5	26.2	59.8	136.4	311.7	711.3	1568.2	3847.0	8812.6	14032.2
85	2.9	5.9	13.5	30.9	70.6	161.1	367.8	825.6	1923.7	4518.7	8876.1	12727.3
86	3.6	7.0	16.0	36.5	83.3	190.1	430.6	985.7	2297.9	4959.3	8749.5	11764.5
87	1.0	3.5	8.4	18.9	43.1	98.3	223.5	509.2	1175.0	2635.2	5241.5	19566.5
88	1.6	4.1	9.8	22.3	50.8	115.8	263.7	604.2	1373.6	2921.7	8171.2	15985.2

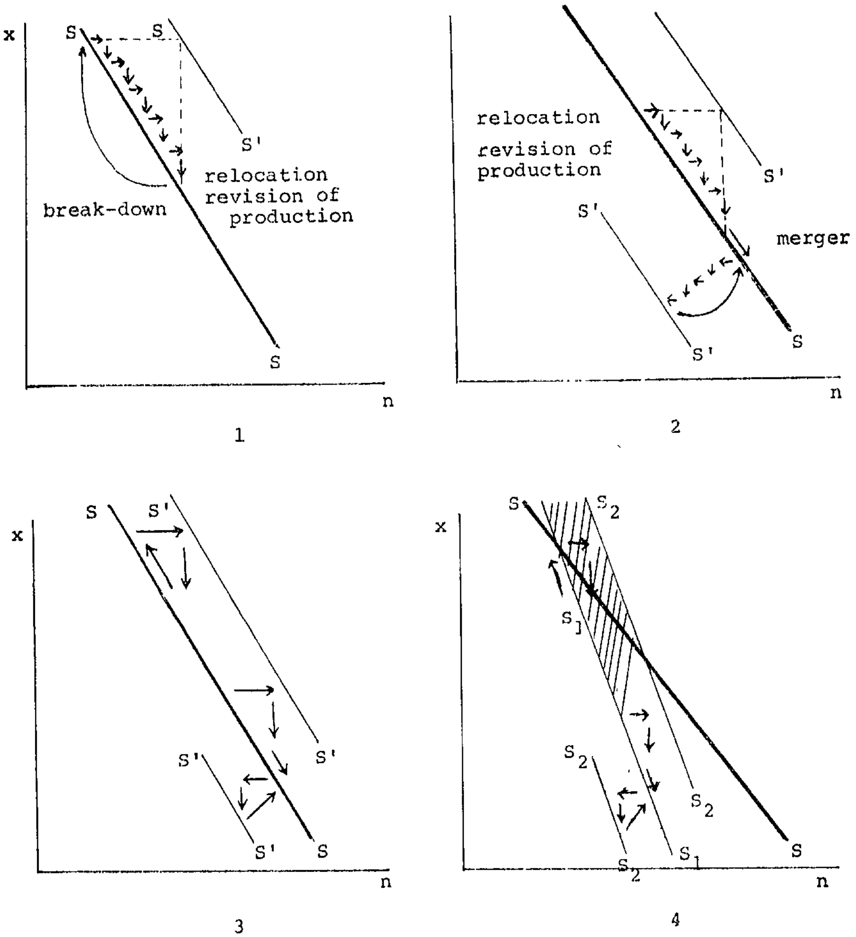

Fig. A.

terms of real numbers instead of integers for group sizes.(18)

According to the results of simulated projection of group size distribution (Table 1), the process of degeneration (congestion with lower provision) is mildly explosive for a certain initial passage of time, and then settles to an approximate oscillation in about 60 periods. Here the unit period indicates the length of the time needed for one round of relocation ("voting-by-feet"), plus the time needed for a one round decision to change (and actual change of) the pattern of local collectives provision.

Observing the end points of the group size distribution, we found it takes four periods

(18) This may appear unrealistic. But it can be justified by allowing people to allocate their life time among different jurisdictions, as in Pestieau [19].

for the smallest group to break down (Fig. A1), while it needs five periods of relocation-redecision for the largest two groups to merge (Fig. A2). This process entails an increasing number of groups over time. However, the length of the smallest group "life-cycle" gets longer (to five), until about 60 periods later, when the explosive force subsides and the process settles to an approximately oscillating pattern.

At this last stage of oscillation, the number of groups is more than optimal (twelve). A typical group has an incentive to attract more members from both directions of the consumer spectrum, but finds entry only from the larger group side, and emigration at the smaller group side. The qualitative nature of the process (growing congestion and lowering production) mentioned in section IV continues, even though most of the groups suffer from insufficient jurisdictional size. In Fig. A4, the range of oscillatory outcome (shaded area) is plotted against the optimal configuration.

A possible concluding argument is that, absence of free-riders, monopolistic restriction, and excessive proliferation from the boundaries notwithstanding, free self-selection leads to a mongrel jurisdictional constellation of too many high-cost groups, where the cyclical change in membership and size distribution gives a typical consumer-voter a perception of degeneration; that is, of ever-growing congestion and ever-lowering provision of local collectives.