External Economies in Market Equilibrium

*Ki-Jun Jeong**

The terms, external economies and external diseconomies are used in various contexts. But it can be said that external economies (technical) are realized if an expansion of the jth firm's output effects (1) total cost or (2) marginal cost of the ith firm in the same industry, its output level remaining constant.

In this note I intend to consider some problems of external economies and diseconomies, and to comment to J. M. Henderson and R. E. Quandt's view on the external economies and diseconomies in the context of market equilibrium. In their book[1] they say:

"Figure 1 (their Figure 4-3) represents the MC curves of the two typical firms in the industry. If the price is $p^{(1)}$, the firms' relevant MC functions are $MC_1^{(1)}$ and $MC_2^{(1)}$, and their outputs are OA_1 and OB_1. Assume that the price rises to p^0. Firm I (Figure 1-a) will want to produce OA_2 and firm II (Figure 1-b), OB_2. However, the rise of I's output by A_1A_2 units shifts II's MC curve to $MC_2^{(2)}$, and the rise in II's output shifts I's MC curve to $MC_1^{(2)}$. The two firms would seem to produce OA_3 and OB_3 respectively. The diminution of their outputs compared to their initial output levels will tend to lower their MC curves. The shifting of the MC curves comes to a stop, and the industry's equilibrium output is determined if $MC_1^{(n)}$ is the relevant MC curve for I when II produces OB_n units and if simultanously $MC_2^{(n)}$ is II's relevant MC curve for an output of OA_n units by I. The final result shows a smaller aggregate output at a higher price. Therefore the aggregate supply curve is *negatively* sloped in this case[2]" (my italics).

Professors Henderson and Quandt's graphical explanation of the effects of

*The author is assistant of College of Commerce, Seoul National University. He is also research assistant of the Institute of Economic Research, Seoul National University.

(1) James M. Henderson and Richard E. Quandt, *Microeconomic Theory, a Mathematical Approach* (McGraw-Hill, 1958).

(2) *Ibid.*, pp. 93-4.

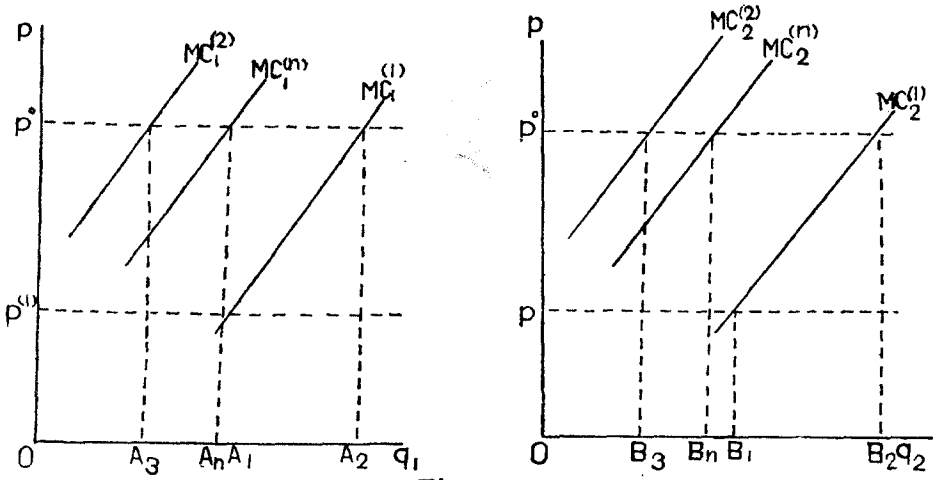

Figure 1

external economies or diseconomies is by nature not exactly analytical, and therefore it is not easy to find out their mistake from the graph. So I intend to formalize their model mathematically as follows.

Suppose that the industry is composed of two firms I and II, and their supply functions are acffected by the other firm's supply with time lag of half unit. Then we have a set of supply functions:

$$S_{1t}=a_{11}p+a_{12}S_{1t-1/2}+a_{13} \tag{1}$$

$$S_{2t}=a_{21}p+a_{22}S_{1t-1/2}+a_{23} \tag{2}$$

Substituting equation (2) to (1), we have

$$\begin{aligned} S_{1t}&=a_{11}p+a_{12}(a_{21}p+a_{22}S_{1t-1}+a_{23})+a_{13} \\ &=a_{12}a_{22}S_{1t-1}+(a_{11}+a_{12}a_{21})p+a_{12}a_{23}+a_{13} \end{aligned} \tag{3}$$

To solve this difference equation we put

$$\overline{S_1}=\frac{a_{11}+a_{12}a_{21}}{1-a_{12}a_{22}}p+\frac{a_{12}a_{23}+a_{13}}{1-a_{12}a_{22}}$$

and

$$S_{1t}-\overline{S_1}=s_{1t},\quad S_{1t-1}-\overline{S_1}=s_{1t-1}$$

Then at equilibrium,

$$\overline{S_1}=a_{12}a_{22}\,\overline{S_1}+(a_{11}+a_{12}a_{21})p+a_{12}a_{23}+a_{13} \tag{4}$$

Subtracting equation (4) from (3), we get

$$s_{1t}=a_{12}a_{21}s_{1t-1}$$

and

$$s_{1t}=s_{10}(a_{12}a_{22})^t$$

Therefore the general solution of the equation is

$$S_{1t}=\overline{S_1}+s_{10}(a_{12}a_{22})^t \qquad (5)$$

From equation (5), we see that if absolute value of $a_{12}a_{22}$ is smaller than unity the equilibrium is stable, and if it is larger than unity it is unstable.

Similarly we have solution of S_{2t} as follows:

$$S_{2t}=\overline{S_2}+s_{20}(a_{12}a_{22})^t \qquad (6)$$

where

$$\overline{S_2}=\frac{a_{21}+a_{11}a_{22}}{1-a_{12}a_{22}}\,p+\frac{a_{22}a_{13}+a_{23}}{1-a_{12}a_{22}}$$

The stability condition of equation (6) is the same as that of equation (5).

The stable-equilibrium aggregate supply S of the industry is obtained simply by adding $\overline{S_1}$ and $\overline{S_2}$:

$$S=\overline{S_1}+\overline{S_2}=\frac{a_{11}(1+a_{22})+a_{21}(1+a_{12})}{1-a_{12}a_{22}}\,p+\frac{a_{13}(1+a_{22})+a_{23}(1+a_{12})}{1-a_{12}a_{22}} \qquad (7)$$

From epuation (7), we can conclude as follows:

1. when external economies prevail (that is, $a_{12}>0$ and $a_{22}>0$, because a_{11} and a_{21} are normally positive), the slope of the aggregate supply curve is definitely positive;
2. when external diseconomies prevail (that is, $a_{12}<0$ and $a_{22}<0$),
   a. if the absolute values of a_{12} and a_{22} are smaller than unity, the slope of the supply curve is definitely positive, and
   b. if the absolute value of either a_{12} or a_{21}(because of stability condition, both a_{12} and a_{22} cannot be larger than unity) is larger than unity, the slope can be positive or negative according to the magnitudes of a's.

As seen above, the slope of the aggregate supply curve is definitely positive in external economies, and it is positive or negative in external diseconomies. But even in the case of external diseconomies, negative slope is very exceptional. But in their words: "The fact that the firms are realizing external economies is not sufficient to allow the inference that the slope of the aggregate supply function is negative"(3), Henderson and Quandt explicitly say that the slope is normally negative, and then they present an *exceptional* example in which the function has a positive slope *in spite of* external economies. My

(3) *Ibid.*, p. 94

proof, however, shows that it is normally and definitely positive.

When a supply function is graphed on a two-dimensional plane whose two axes represent the quantity of the industry's output and price respectively, the effects of external economies are not represented with its *slope* but with its *shifts*. As shown in Figure 2, suppose that some external economies shift the aggregate supply curve from S_1 to S_2, and equilibrium point from E_1 to E_2. In this case the equilibrium price is lowered from p_1 to p_2, and equilibrium quantity is expanded from q_1 to q_2, But it must remembered that E_1 and E_2 are not on the same supply curve. And the fact that p_2 is lower than p_1, and q_2 is larger than q_1 cannot be used as the basis of the argument of negatively sloped aggregate suppy curve. When external economies prevail, the commonly observed phenomena may be that quantity actually supplied is larger at higher price as the Figure 2 shows. But the observed equilibrium points such as E_1 and E_2 are not on one and the same supply curve but on the same *demand* curve. And what is negatively sloped is not the supply curve but the *demand* curve.

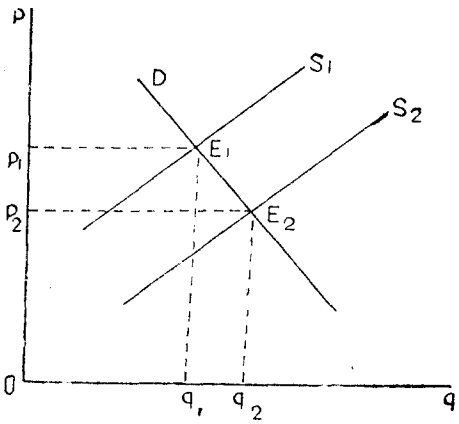

Figure 2

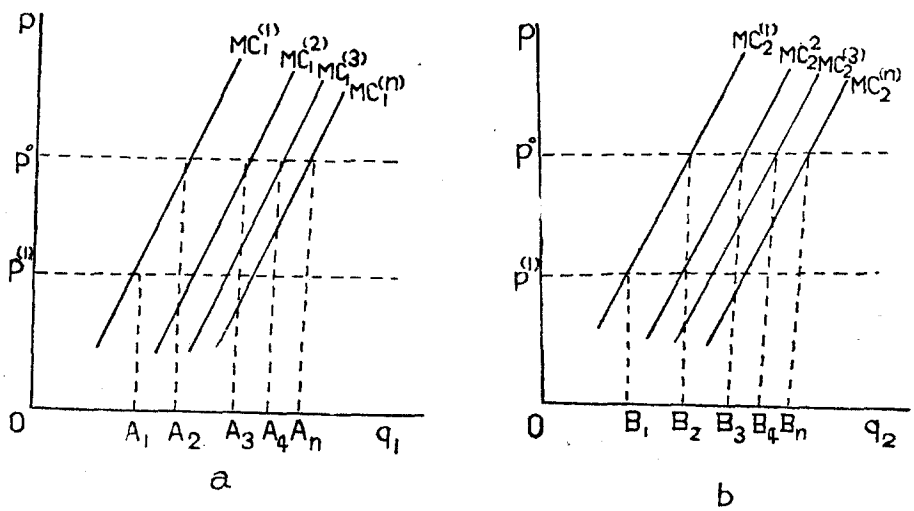

Figure 3

Therefore Henderson and Quandt's example of positively sloped aggregate supply function in spite of external economies, whose marginal cost curve, i.e., supply curve is, moreover, not affected by the external economies, so that there is no shift in supply curve, is trivial in our general result.

Figure 3 shows that the sequences of MC curves and equilibrium points when external economies prevail with the same notion as in Figure 1, and Figure 4, the sequences when external diseconomies prevail. The latter is the case with which Henderson and Quandt intend to show in Figure 1, and therefore Figure 1 should be modified according to Figure 4.

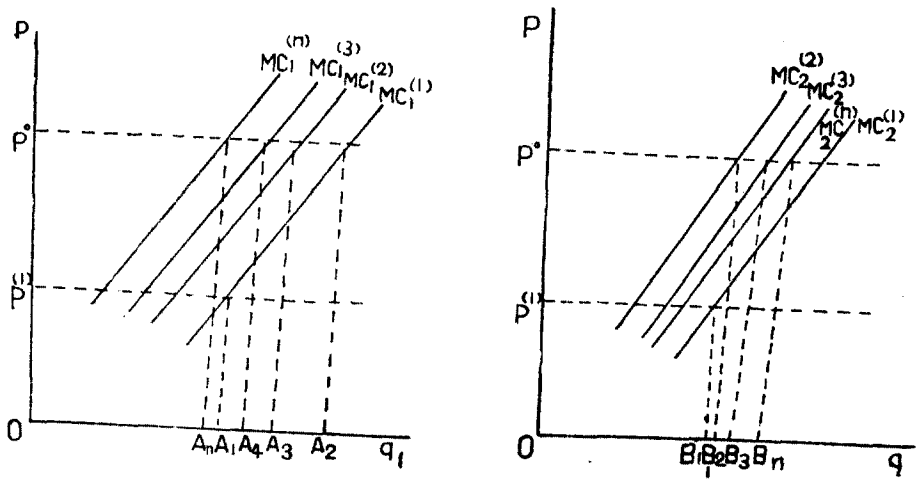

Figure 4